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Contents

Abstract.
Bernstein's inequality for Jacobi polynomials $P_n^{(\alpha,\beta)}$, established in 1987 by P. Baratella for the region $\mathcal{R}_{1/2} = \{|\alpha| \leq 1/2, |\beta| \leq 1/2\}$, and subsequently supplied with an improved constant by Y. Chow, L. Gatteschi, and R. Wong, is analyzed here analytically and, above all, computationally with regard to validity and sharpness, not only in the original region $\mathcal{R}_{1/2}$, but also in larger regions $\mathcal{R}_s = \{-1/2 \leq \alpha \leq s, -1/2 \leq \beta \leq s\}$, $s > 1/2$. Computation suggests that the inequality holds with new, somewhat larger, constants in any region $\mathcal{R}_s$. Best constants are provided for $s = 1 : .5 : 4$ and $s = 5 : 1 : 10$. Our work also sheds new light on the so-called Erdélyi–Magnus–Nevai conjecture for orthonormal Jacobi polynomials, adding further support for its validity and suggesting $.66198126\ldots$ as the best constant implied in the conjecture.

Key Words.
Bernstein's inequality, Jacobi polynomials, sharpness, Erdélyi–Magnus–Nevai conjecture

AMS Subject Classifications.
33C45, 41A17

Abstract.
It is known that the space of real valued, continuous functions $\mathrm{C}(B)$ over a multidimensional compact domain $B \subset \mathbb{R}^k$, $k \geq 2$ does not admit Haar spaces, which means that interpolation problems in finite dimensional subspaces V of $\mathrm{C}(B)$ may not have a solutions in $\mathrm{C}(B)$. The corresponding standard short and elegant proof does not apply to complex valued functions over $B \subset \mathbb{C}$. Nevertheless, in this situation Haar spaces $V \subset \mathrm{C}(B)$ exist. We are concerned here with the case of quaternionic valued, continuous functions $\mathrm{C}(B)$ where $B \subset \mathbb{H}$ and $\mathbb{H}$ denotes the skew field of quaternions. Again, the proof is not applicable. However, we show that the interpolation problem is not unisolvent, by constructing quaternionic entries for a Vandermonde matrix $\mathbf{V}$ such that $\mathbf{V}$ will be singular for all orders $n > 2$. In addition, there is a section on the exclusion and inclusion of all zeros in certain balls in $\mathbb{H}$ for general quaternionic polynomials.

Key Words.
quaternionic interpolation polynomials, Vandermonde matrix in quaternions, location of zeros of quaternionic polynomials

AMS Subject Classifications.
11R52, 12E15, 12Y05, 65D05

Abstract.
The main result of this paper is the parametrization of ideal projectors onto an arbitrary finite-dimensional linear subspace $G \subset \Bbbk[\mathbf{x}]$. This parametrization extends the previous ones by B. Mourrain and by M. Kreuzer and L. Robbiano. We also give applications of the technique developed in this paper to a question of similarity between a sequence of commuting matrices and its transpose and to the existence of real solutions to a system of polynomial equations.

Key Words.
ideal projector, commuting operators, border schemes

AMS Subject Classifications.
Primary: 41A63, 41A10, 41A35; Secondary: 13P10

Abstract.
We examine the asymptotic behavior of the zeros of sections of the binomial expansion, that is, we consider the distribution of zeros of $B_{r,n}(z) = \sum_{k=0}^{r} \binom{n}{k} z^k$, where $1 \leq r \leq n$.

Key Words.
binomial expansion, partial sums, zeros

AMS Subject Classifications.
30C15

Abstract.
We study the convergence of P-regular splitting iterative methods for non-Hermitian positive definite linear systems. Our main result is that P-regular splittings of the form $A = M - N$, where $N = N^*$, are convergent. Natural examples of splittings satisfying the convergence conditions are constructed, and numerical experiments are performed to illustrate the convergence results obtained.

Key Words.
non-Hermitian positive definite matrices, P-regular splitting, convergence, SOR methods, preconditioned GMRES

AMS Subject Classifications.
65F10, 15A15, 15F10

Abstract.
The Nonnegative Matrix Factorization (NMF) technique has been used in many areas of science, engineering, and technology. In this paper, we propose four algorithms for solving the nonsmooth nonnegative matrix factorization (nsNMF) problems. The nsNMF uses a smoothing parameter $\theta \in [0, 1]$ to control the sparseness

in its matrix factors and it reduces to the original NMF if $\theta = 0$. Each of our algorithms alternately solves a nonnegative linear least squares subproblem in matrix form using a projected Barzilai–Borwein method with a nonmonotone line search or no line search. We have tested and compared our algorithms with the projected gradient method of Lin on a variety of randomly generated NMF problems. Our numerical results show that three of our algorithms, namely, APBB1, APBB2, and APBB3, are significantly faster than Lin's algorithm for large-scale, difficult, or exactly factorable NMF problems in terms of CPU time used. We have also tested and compared our APBB2 method with the multiplicative algorithm of Lee and Seung and Lin's algorithm for solving the nsNMF problem resulted from the ORL face database using both $\theta = 0$ and $\theta = 0.7$. The experiments show that when $\theta = 0.7$ is used, the APBB2 method can produce sparse basis images and reconstructed images which are comparable to the ones by the Lin and Lee–Seung methods in considerably less time. They also show that the APBB2 method can reconstruct better quality images and obtain sparser basis images than the methods of Lee–Seung and Lin when each method is allowed to run for a short period of time. Finally, we provide a numerical comparison between the APBB2 method and the Hierarchical Alternating Least Squares (HALS)/Rank-one Residue Iteration (RRI) method, which was recently proposed by Cichocki, Zdunek, and Amari and by Ho, Van Dooren, and Blondel independently.

Key Words.
nonnegative matrix factorization, smoothing matrix, nonnegative least squares problem, projected Barzilai–Borwein method, nonmonotone line search

AMS Subject Classifications.
15A48, 15A23, 65F30, 90C30

Abstract.
Given a linear functional $\mathcal{L}$ in the linear space $\mathbb{P}$ of polynomials with complex coefficients, we analyze those linear functionals $\tilde{\mathcal{L}}$ such that, for a fixed $\alpha \in \mathbb{C}$, $\langle \tilde{\mathcal{L}}, (z + z^{-1} - (\alpha + \bar{\alpha}))p \rangle = \langle \mathcal{L}, p \rangle$ for every $p \in \mathbb{P}$. We obtain the relation between the corresponding Carathéodory functions in such a way that a linear spectral transform appears. If $\mathcal{L}$ is a positive definite linear functional, the necessary and sufficient conditions in order for $\tilde{\mathcal{L}}$ to be a quasi-definite linear functional are given. The relation between the corresponding sequences of monic orthogonal polynomials is presented.

Key Words.
orthogonal polynomials, linear functionals, Laurent polynomials, linear spectral transformations

AMS Subject Classifications.
42C05

Abstract.
In this note we examine the algebraic variety $\mathcal{I}_\Lambda$ of complex tridiagonal $n \times n$ matrices T, such that $T^*T - TT^* = \Lambda$, where Λ is a fixed real diagonal matrix. If $\Lambda = \mathbf{0}$

then $\mathcal{I}_\Lambda$ is $\mathcal{N}_\mathbf{T}$, the set of tridiagonal normal matrices. For $\Lambda \neq \mathbf{0}$, we identify the structure of the matrices in $\mathcal{I}_\Lambda$ and analyze the suitability for eigenvalue estimation using normal matrices for elements of $\mathcal{I}_\Lambda$. We also compute the Frobenius norm of elements of $\mathcal{I}_\Lambda$, describe the algebraic subvariety $\mathcal{M}_\Lambda$ consisting of elements of $\mathcal{I}_\Lambda$ with minimal Frobenius norm, and calculate the distance from a given complex tridiagonal matrix to $\mathcal{I}_\Lambda$.

Key Words.
nearness to normality, tridiagonal matrix, Kreĭn spaces, eigenvalue estimation, Geršgorin type sets

AMS Subject Classifications.
65F30, 65F35, 15A57, 15A18, 47A25

113 On an unsymmetric eigenvalue problem governing free vibrations of fluid–solid structures. *Markus Stammberger and Heinrich Voss.*

Abstract.
In this paper we consider an unsymmetric eigenvalue problem occurring in fluid-solid vibrations. We present some properties of this eigenvalue problem and a Rayleigh functional which allows for a min-max-characterization. With this Rayleigh functional the one-sided Rayleigh functional iteration converges cubically, and a Jacobi-Davidson-type method improves the local and global convergence properties.

Key Words.
eigenvalue, variational characterization, minmax principle, fluid–solid interaction, Rayleigh quotient iteration, Jacobi–Davidson method

AMS Subject Classifications.
65F15

126 IDR explained. *Martin H. Gutknecht.*

Abstract.
The Induced Dimension Reduction (IDR) method is a Krylov space method for solving linear systems that was developed by Peter Sonneveld around 1979. It was noticed by only a few people, and mainly as the forerunner of Bi-CGSTAB, which was introduced a decade later. In 2007, Sonneveld and van Gijzen reconsidered IDR and generalized it to IDR(s), claiming that IDR$(1) \approx$ IDR is equally fast but preferable to the closely related Bi-CGSTAB, and that IDR(s) with $s > 1$ may be much faster than Bi-CGSTAB. It also turned out that when $s > 1$, IDR(s) is related to ML(s)BiCGSTAB of Yeung and Chan, and that there is quite some flexibility in the IDR approach. This approach differs completely from traditional approaches to Krylov space methods, and therefore it requires an extra effort to get familiar with it and to understand the connections as well as the differences to better-known Krylov space methods. This expository paper aims to provide some help in this and to make the method understandable even to non-experts. After presenting the history of IDR and related methods, we summarize some of the basic facts on Krylov space methods. Then we present the original IDR(s) in detail and put it into perspective with other methods. Specifically, we analyze the differences between the IDR method published in 1980, IDR(1), and Bi-CGSTAB. At the end of the paper, we discuss a recently proposed ingenious variant of IDR(s) whose residuals fulfill

extra orthogonality conditions. There we dwell on details that have been left out in the publications of van Gijzen and Sonneveld.

Key Words.
Krylov space method, iterative method, induced dimension reduction, IDR, CGS, Bi-CGSTAB, ML(k)BiCGSTAB, large nonsymmetric linear system

Abstract.
Discrete maximum principles are established for finite element approximations of nonlinear parabolic problems. The conditions on the space and time discretizations are similar to the usual conditions for linear problems.

Key Words.
nonlinear parabolic problems, discrete maximum principle, finite element method

AMS Subject Classifications.
65M60, 65M50, 35B50

Abstract.
Various forms of preconditioners for elliptic finite element matrices are studied, based on suitable block matrix partitionings. Bounds for the resulting condition numbers are given, including a study of sensitivity to jumps in the coefficients and to the constant in the strengthened Cauchy-Schwarz-Bunyakowski inequality.

Key Words.
preconditioning, Schur complement, domain decomposition, Poincaré–Steklov operator, approximate block factorization, strengthened Cauchy-Schwarz-Bunyakowski inequality

AMS Subject Classifications.
65F10, 65N22

Abstract.
We review recent derivations of formulas for conformal maps from finitely connected domains with circular holes to canonical radial or circular slit domains. The formulas are infinite products based on simple reflection arguments. An earlier similar derivation of the Schwarz-Christoffel formula for the bounded multiply connected case and recent progress in its numerical implementation are also reviewed. We give some sample calculations with a reflection method and an estimate of its accuracy. We also discuss the relation of our approach to that of D. Crowdy and J. Marshall. In addition, a slit map calculation using Laurent series computed by the

least squares method in place of the reflection method is given as an example of a possible direction for future improvements in the numerics.

Key Words.
conformal mapping, Schwarz-Christoffel transformation, multiply connected domains, canonical slit domains, Schottky-Klein prime function

AMS Subject Classifications.
30C30, 65E05