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HOW SHARP IS BERNSTEIN'S INEQUALITY FOR JACOBI POLYNOMIALS?*

WALTER GAUTSCHI†

Dedicated to Richard S. Varga on his 80th birthday

Abstract. Bernstein's inequality for Jacobi polynomials $P_n^{(\alpha,\beta)}$, established in 1987 by P. Baratella for the region $\mathcal{R}_{1/2} = \{|\alpha| \le 1/2,\ |\beta| \le 1/2\}$, and subsequently supplied with an improved constant by Y. Chow, L. Gatteschi, and R. Wong, is analyzed here analytically and, above all, computationally with regard to validity and sharpness, not only in the original region $\mathcal{R}_{1/2}$, but also in larger regions $\mathcal{R}_s = \{-1/2 \le \alpha \le s, -1/2 \le \beta \le s\}$, $s > 1/2$. Computation suggests that the inequality holds with new, somewhat larger, constants in any region $\mathcal{R}_s$. Best constants are provided for $s = 1:.5:4$ and $s = 5:1:10$. Our work also sheds new light on the so-called Erdélyi–Magnus–Nevai conjecture for orthonormal Jacobi polynomials, adding further support for its validity and suggesting $.66198126\ldots$ as the best constant implied in the conjecture.

Key words. Bernstein's inequality, Jacobi polynomials, sharpness, Erdélyi–Magnus–Nevai conjecture

AMS subject classifications. 33C45, 41A17

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†Department of Computer Sciences, Purdue University, West Lafayette, Indiana 47907-2066, USA (wxg@cs.purdue.edu).