

IDR EXPLAINED*

MARTIN H. GUTKNECHT†

Dedicated to Richard S. Varga on the occasion of his 80th birthday.

Abstract. The Induced Dimension Reduction (IDR) method is a Krylov space method for solving linear systems that was developed by Peter Sonneveld around 1979. It was noticed by only a few people, and mainly as the forerunner of Bi-CGSTAB, which was introduced a decade later. In 2007, Sonneveld and van Gijzen reconsidered IDR and generalized it to $IDR(s)$, claiming that $IDR(1) \approx IDR$ is equally fast but preferable to the closely related Bi-CGSTAB, and that $IDR(s)$ with $s > 1$ may be much faster than Bi-CGSTAB. It also turned out that when $s > 1$, $IDR(s)$ is related to $ML(s)BiCGSTAB$ of Yeung and Chan, and that there is quite some flexibility in the IDR approach. This approach differs completely from traditional approaches to Krylov space methods, and therefore it requires an extra effort to get familiar with it and to understand the connections as well as the differences to better-known Krylov space methods. This expository paper aims to provide some help in this and to make the method understandable even to non-experts. After presenting the history of IDR and related methods, we summarize some of the basic facts on Krylov space methods. Then we present the original $IDR(s)$ in detail and put it into perspective with other methods. Specifically, we analyze the differences between the IDR method published in 1980, $IDR(1)$, and Bi-CGSTAB. At the end of the paper, we discuss a recently proposed ingenious variant of $IDR(s)$ whose residuals fulfill extra orthogonality conditions. There we dwell on details that have been left out in the publications of van Gijzen and Sonneveld.

Key words. Krylov space method, iterative method, induced dimension reduction, IDR, CGS, Bi-CGSTAB, $ML(k)BiCGSTAB$, large nonsymmetric linear system

*Received December 28, 2008. Accepted for publication October 16, 2009. Published online February 22, 2010. Recommended by M. Eiermann.

†Seminar for Applied Mathematics, ETH Zurich, CH-8092 Zurich, Switzerland (mhg@math.ethz.ch). Work done while the author was visiting the TU Berlin, supported by the DFG Forschungszentrum MATHEON and the Mercator Visiting Professorship Program of the DFG.