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POLYNOMIALS AND VANDERMONDE MATRICES OVER THE FIELD OF QUATERNIONS *

GERHARD OPFER†

Abstract. It is known that the space of real valued, continuous functions $\mathrm{C}(B)$ over a multidimensional compact domain $B \subset \mathbb{R}^k$, $k \geq 2$ does not admit Haar spaces, which means that interpolation problems in finite dimensional subspaces V of $\mathrm{C}(B)$ may not have a solutions in $\mathrm{C}(B)$. The corresponding standard short and elegant proof does not apply to complex valued functions over $B \subset \mathbb{C}$. Nevertheless, in this situation Haar spaces $V \subset \mathrm{C}(B)$ exist. We are concerned here with the case of quaternionic valued, continuous functions $\mathrm{C}(B)$ where $B \subset \mathbb{H}$ and $\mathbb{H}$ denotes the skew field of quaternions. Again, the proof is not applicable. However, we show that the interpolation problem is not unisolvent, by constructing quaternionic entries for a Vandermonde matrix $\mathbf{V}$ such that $\mathbf{V}$ will be singular for all orders $n > 2$. In addition, there is a section on the exclusion and inclusion of all zeros in certain balls in $\mathbb{H}$ for general quaternionic polynomials.

Key words. Quaternionic interpolation polynomials, Vandermonde matrix in quaternions, location of zeros of quaternionic polynomials

AMS subject classifications. 11R52, 12E15, 12Y05, 65D05

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† University of Hamburg, Faculty for Mathematics, Informatics, and Natural Sciences [MIN], Bundesstraße 55, 20146 Hamburg, Germany, opfer@math.uni-hamburg.de