Electronic Transactions on Numerical Analysis. Volume 36, pp. 99-112, 2010. Copyright © 2010, Kent State University. ISSN 1068-9613.

THE STRUCTURED DISTANCE TO NEARLY NORMAL MATRICES*

LAURA SMITHIES †

Dedicated to Richard S. Varga on the occasion of his 80th birthday

Abstract. In this note we examine the algebraic variety \mathcal{I}_{Λ} of complex tridiagonal $n \times n$ matrices T, such that $T^*T - TT^* = \Lambda$, where Λ is a fixed real diagonal matrix. If $\Lambda = \mathbf{0}$ then \mathcal{I}_{Λ} is $\mathcal{N}_{\mathbf{T}}$, the set of tridiagonal normal matrices. For $\Lambda \neq \mathbf{0}$, we identify the structure of the matrices in \mathcal{I}_{Λ} and analyze the suitability for eigenvalue estimation using normal matrices for elements of \mathcal{I}_{Λ} . We also compute the Frobenius norm of elements of \mathcal{I}_{Λ} , describe the algebraic subvariety \mathcal{M}_{Λ} consisting of elements of \mathcal{I}_{Λ} with minimal Frobenius norm, and calculate the distance from a given complex tridiagonal matrix to \mathcal{I}_{Λ} .

Key words. nearness to normality, tridiagonal matrix, Krein spaces, eigenvalue estimation, Gersgorin type sets

AMS subject classifications. 65F30, 65F35, 15A57, 15A18, 47A25

(smithies@math.kent.edu).

99

^{*}Received March 13, 2009. Accepted for publication August 9, 2009. Published online on January 20, 2010. Recommended by L. Reichel.

[†]Department of Mathematical Sciences, Kent State University, Kent, OH 44242