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ADAPTIVE REDUCTION-BASED MULTIGRID FOR NEARLY SINGULAR AND HIGHLY DISORDERED PHYSICAL SYSTEMS*

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Abstract. Classical multigrid solution of linear systems with matrices that have highly variable entries and are nearly singular is made difficult by the compounding difficulties introduced by these two model features. Efficient multigrid solution of nearly singular matrices is known to be possible, provided the so-called Brandt-McCormick (or eigenvector approximation) criterion is satisfied, which requires building interpolation to fit the near-null-space modes with high accuracy. When these modes are known, traditional multigrid approaches may be very effective. In this paper, we consider the case of matrices describing highly disordered systems, such as those that arise in lattice quantum chromodynamics (QCD), where the near-null modes cannot be easily expressed in closed form. We develop a variational adaptive reduction-based algebraic multigrid preconditioner for such systems and present a two-level convergence theory for the approach for Hermitian and positive-definite systems. The proposed method is applied to a two-dimensional model known as the Gauge Laplacian, a common test problem for development of solvers in quantum dynamics applications, showing promising numerical results. The proposed reduction-based setup uses compatible relaxation coarsening together with a sparse approximation to the so-called ideal interpolation operator to recursively construct the coarse spaces.

Key words. algebraic multigrid, lattice QCD, gauge Laplacian, iterative methods, adaptive multigrid

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