

## AN ANALYSIS OF LOW-RANK MODIFICATIONS OF PRECONDITIONERS FOR SADDLE POINT SYSTEMS\*

CHEN GREIF<sup>†</sup> AND MICHAEL L. OVERTON<sup>‡</sup>

Dedicated to the memory of our mentor and friend, Gene H. Golub

**Abstract.** We characterize the spectral behavior of a primal Schur-complement-based block diagonal preconditioner for saddle point systems, subject to low-rank modifications. This is motivated by a desire to reduce as much as possible the computational cost of matrix-vector products with the (1,1) block, while keeping the eigenvalues of the preconditioned matrix reasonably clustered. The formulation leads to a perturbed hyperbolic quadratic eigenvalue problem. We derive interlacing results, highlighting the differences between this problem and perturbed linear eigenvalue problems. As an example, we consider primal-dual interior point methods for semidefinite programs, and express the eigenvalues of the preconditioned matrix in terms of the centering parameter.

**Key words.** saddle point systems, preconditioners, Schur complement, semidefinite programming

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<sup>†</sup>Department of Computer Science, The University of British Columbia, Vancouver, BC ([greif@cs.ubc.ca](mailto:greif@cs.ubc.ca)). The work of this author was partially supported by the Natural Sciences and Engineering Research Council of Canada.

<sup>‡</sup>Courant Institute of Mathematical Sciences, New York University, New York, NY 10012 ([overton@cs.nyu.edu](mailto:overton@cs.nyu.edu)). The work of this author was partially supported by the National Science Foundation under award DMS-0714321, and partially by the Pacific Institute for the Mathematical Sciences.