

CONVERGENCE RATES FOR REGULARIZATION WITH SPARSITY CONSTRAINTS*

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Abstract. Tikhonov regularization with p -powers of the weighted ℓ_p norms as penalties, with $p \in (1, 2)$, have been employed recently in reconstruction of sparse solutions of ill-posed inverse problems. This paper shows convergence rates for such a regularization with respect to the norm of the weighted spaces by assuming that the solutions satisfy a certain smoothness (source) condition. The meaning of the latter is analyzed in some detail. Moreover, converse results are established: Linear convergence rates for the residual, together with convergence of the approximations to the solution, can be achieved only if the solution satisfies a source condition. Further insights for the particular case of a convolution equation are provided by analyzing the equation both theoretically and numerically.

Key words. ill-posed problem, regularization, Bregman distance, sparsity

AMS subject classifications. 47A52, 65J20

*Received March 17, 2009. Accepted for publication October 9, 2009. Published online April 7, 2010. Recommended by D. Calvetti.

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