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## TWO EFFICIENT SVD/KRYLOV ALGORITHMS FOR MODEL ORDER REDUCTION OF LARGE SCALE SYSTEMS\*

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Abstract. We present two efficient algorithms to produce a reduced order model of a time-invariant linear dynamical system by approximate balanced truncation. Attention is focused on the use of the structure and the iterative construction via Krylov subspaces of both controllability and observability matrices to compute low-rank approximations of the Gramians or the Hankel operator. This allows us to take advantage of any sparsity in the system matrices and indeed the cost of our two algorithms is only linear in the system dimension. Both algorithms efficiently produce good low-rank approximations (in the least square sense) of the Cholesky factor of each Gramian and the Hankel operator. The first algorithm computes low-rank approximation of each Gramian independently. The second algorithm works directly on the Hankel operator, and it has the advantage that it is independent of the chosen realization. Moreover, it is also an approximate Hankel norm method. The two reduced order models produced by our methods are guaranteed to be stable and balanced. We study the convergence of our iterative algorithms and the properties of the fixed point iteration. We also discuss the stopping criteria and the choice of the reduced order.

**Key words.** model order reduction, approximate balanced truncation, Stein equations, Hankel map, Krylov subspaces, approximate Hankel norm method, low-rank approximations

## AMS subject classifications. 15A24, 65P99, 93B40, 93C55, 93D99

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