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STIELTJES INTERLACING OF ZEROS OF JACOBI POLYNOMIALS FROM DIFFERENT SEQUENCES*

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Abstract. A theorem of Stieltjes proves that, given any sequence $\{p_n\}_{n=0}^{\infty}$ of orthogonal polynomials, there is at least one zero of p_n between any two consecutive zeros of p_k if $k < n$, a property called Stieltjes interlacing. We show that Stieltjes interlacing extends, under certain conditions, to the zeros of Jacobi polynomials from different sequences. In particular, we prove that the zeros of $P_{n+1}^{\alpha,\beta}$ interlace with the zeros of $P_{n-1}^{\alpha+k,\beta}$ and with the zeros of $P_{n-1}^{\alpha,\beta+k}$ for $k \in \{1,2,3,4\}$ as well as with the zeros of $P_{n-1}^{\alpha+t,\beta+k}$ for $t,k \in \{1,2\}$; and, in each case, we identify a point that completes the interlacing process. More generally, we prove that the zeros of the kth derivative of $P_n^{\alpha,\beta}$, together with the zeros of an associated polynomial of degree k, interlace with the zeros of $P_{n+1}^{\alpha,\beta}, k, n \in N, k < n$.

Key words. Interlacing of zeros; Stieltjes' Theorem; Jacobi polynomials.

AMS subject classifications. 33C45, 42C05

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