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IMPROVED PREDICTOR SCHEMES FOR LARGE SYSTEMS OF LINEAR ODES*

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Abstract. When solving linear systems of ordinary differential equations (ODEs) with constant coefficients by implicit schemes such as implicit Euler, Crank-Nicolson, or implicit Runge-Kutta, one is faced with the difficulty of correctly solving the repeated linear systems that arise in the implicit scheme. These systems often have the same matrix but different right-hand sides. When the size of the matrix is large, iterative methods based on Krylov subspaces can be used. However, the effectiveness of these methods strongly depends on the initial guesses. The closer the initial guesses are to the exact solutions, the faster the convergence. This paper presents an approach that computes good initial guesses to these linear systems. It can be viewed as an improved predictor method. It is based on a Petrov-Galerkin process and multistep schemes and consists of building, throughout the iterations, an approximation subspace using the previous computations, where good initial guesses to the next linear systems can be found. It is shown that the quality of the computed initial guess depends only on the stepsize of the discretization and the dimension of the approximation subspace. The approach can be applied to most of the common implicit schemes. It is tested on several examples.

Key words. Convergence acceleration, implicit scheme, predictor, Petrov-Galerkin, GMRES

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