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VARIATIONAL ENSEMBLE KALMAN FILTERING USING LIMITED MEMORY BFGS*

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Abstract. The extended Kalman filter (EKF) is one of the most used nonlinear state estimation methods. However, in large-scale problems, the CPU and memory requirements of EKF are prohibitively large. Recently, Auvinen et al. proposed a promising approximation to EKF called the variational Kalman filter (VKF). The implementation of VKF requires the tangent linear and adjoint codes for propagating error covariances in time. However, the trouble of building the codes can be circumvented by using ensemble filtering techniques, where an ensemble of states is propagated in time using the full nonlinear model, and the statistical information needed in EKF formulas is estimated from the ensemble. In this paper, we show how the VKF ideas can be used in the ensemble filtering context. Following VKF, we obtain the state estimate and its covariance by solving a minimization problem using the limited memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) method, which provides low-storage approximations to the state covariances. The resulting hybrid method, the variational ensemble Kalman filter (VEnKF), has several attractive features compared to existing ensemble methods. The model error and observation error covariances can be inserted directly into the minimization problem instead of randomly perturbing model states and observations as in the standard ensemble Kalman filter. New ensembles can be directly generated from the LBFGS covariance approximation without the need of a square root (Cholesky) matrix decomposition. The frequent resampling from the full state space circumvents the problem of ensemble in-breeding frequently associated with ensemble filters. Numerical examples are used to show that the proposed approach performs better than the standard ensemble Kalman filter, especially when the ensemble size is small.

Key words. Data assimilation, state estimation, Kalman filtering, ensemble filtering, LBFGS

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