

## COMPUTATION OF THE MATRIX $P$ TH ROOT AND ITS FRÉCHET DERIVATIVE BY INTEGRALS\*

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**Abstract.** We present new integral representations for the matrix  $p$ th root and its Fréchet derivative and then investigate the computation of these functions by numerical quadrature. Three different quadrature rules are considered: composite trapezoidal, Gauss-Legendre and adaptive Simpson. The problem of computing the matrix  $p$ th root times a vector without the explicit evaluation of the  $p$ th root is also analyzed and bounds for the norm of the matrix  $p$ th root and its Fréchet derivative are derived.

**Key words.** matrix  $p$ th root, Fréchet derivative, quadrature, composite trapezoidal rule, Gauss-Legendre rule, adaptive Simpson rule

**AMS subject classifications.** 65F60, 65D30

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