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ON THE MINIMIZATION OF A TIKHONOV FUNCTIONAL WITH A NON-CONVEX SPARSITY CONSTRAINT*

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Abstract. In this paper we present a numerical algorithm for the optimization of a Tikhonov functional with an ℓ_p-sparsity constraints and $p < 1$. Recently, it was proven that the minimization of this functional provides a regularization method. We show that the idea used to obtain these theoretical results can also be utilized in a numerical approach. In particular, we exploit the technique of transforming the Tikhonov functional to a more viable one. In this regard, we consider a surrogate functional approach and show that this technique can be applied straightforwardly. It is proven that at least a critical point of the transformed functional is obtained, which directly translates to the original functional. For a special case, it is shown that a gradient based algorithm can be used to reconstruct the global minimizer of the transformed and the original functional, respectively. Moreover, we apply the developed method to a deconvolution problem and a parameter identification problem in the field of physical chemistry, and we provide numerical evidence for the theoretical results and the desired sparsity promoting features of this method.

Key words. sparsity, surrogate functional, inverse problem, regularization

AMS subject classifications. 65L09, 65J22, 65J20, 47J06, 94A12

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