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Contents

Abstract.
A procedure for counting the number of eigenvalues of a matrix in a region surrounded by a closed curve is presented. It is based on the application of the residual theorem. The quadrature is performed by evaluating the principal argument of the logarithm of a function. A strategy is proposed for selecting a path length that insures that the same branch of the logarithm is followed during the integration. Numerical tests are reported for matrices obtained from conventional matrix test sets.

Key Words.
eigenvalue, resolvent, determinant, complex logarithm

AMS Subject Classifications.
65F15, 65F40, 65F50, 65E05

Abstract.
Recently, an extension of the class of matrices admitting a Francis type of multishift QR algorithm was proposed by the authors. These so-called condensed matrices admit a storage cost identical to that of the Hessenberg matrix and share all of the properties essential for the development of an effective implicit QR type method. This article continues along this trajectory by discussing the generalized eigenvalue problem. The novelty does not lie in the almost trivial extension of replacing the Hessenberg matrix in the pencil by a condensed matrix, but in the fact that both pencil matrices can be partially of condensed form. Again, the storage cost and crucial features of the Hessenberg-upper triangular pencil are retained, giving rise to an equally viable QZ-like method. The associated implicit algorithm also relies on bulge chasing and exhibits a sort of bulge hopping from one to the other matrix. This article presents the reduction to a condensed pencil form and an extension of the QZ algorithm. Relationships between these new ideas and some known algorithms are also discussed.

Key Words.
condensed matrices, generalized eigenvalues, QZ algorithm, QR algorithm, extended Krylov

AMS Subject Classifications.
65F15, 15A18

36   Inexact and truncated parareal-in-time Krylov subspace methods for parabolic optimal control problems. *Xiuhong Du, Marcus Sarkis, Christian E. Schaerer, and Daniel B. Szyld.*

Abstract.
We study the use of inexact and truncated Krylov subspace methods for the solution of the linear systems arising in the discretized solution of the optimal control of a parabolic partial differential equation. An all-at-once temporal discretization and a reduction approach are used to obtain a symmetric positive definite system for the control variables only, where a Conjugate Gradient (CG) method can be used at the cost of the solution of two very large linear systems in each iteration. We propose to use inexact Krylov subspace methods, in which the solution of the two large linear systems are not solved exactly, and their approximate solutions can be progressively less exact. The option we propose is the use of the parareal-in-time algorithm for approximating the solution of these two linear systems. The use of less parareal iterations makes it possible to reduce the time integration costs and to improve the time parallel scalability. We also show that truncated methods could be used without much delay in convergence but with important savings in storage. Spectral bounds are provided and numerical experiments with inexact versions of CG, the full orthogonalization method (FOM), and of truncated FOM are presented, illustrating the potential of the proposed methods.

Key Words.
parabolic optimal control, reduced system, saddle point problem, inexact Krylov subspace methods, truncated Krylov subspace methods, parareal approximation, spectral bounds

AMS Subject Classifications.
65F10, 65F50, 65N22, 35B37, 15A42, 35A15

58   Discretization independent convergence rates for noise level-free parameter choice rules for the regularization of ill-conditioned problems. *Stefan Kindermann.*

Abstract.
We develop a convergence theory for noise level-free parameter choice rules for Tikhonov regularization of finite-dimensional, linear, ill-conditioned problems. In particular, we derive convergence rates with bounds that do not depend on troublesome parameters such as the small singular values of the system matrix. The convergence analysis is based on specific qualitative assumptions on the noise, the noise conditions, and on certain regularity conditions. Furthermore, we derive several sufficient noise conditions both in the discrete and infinite-dimensional cases. This leads to important conclusions for the actual implementation of such rules in practice. For instance, we show that for the case of random noise, the regularization parameter can be found by minimizing a parameter choice functional over a subinterval of the spectrum (whose size depends on the smoothing properties of the forward operator), yielding discretization independent convergence rate estimates, which are of optimal order under regularity assumptions for the exact solution.

Key Words.
regularization, parameter choice rule, Hanke-Raus rule, quasioptimality rule, generalized cross validation

AMS Subject Classifications.
65J20, 47A52, 65J22

82   On Sylvester's law of inertia for nonlinear eigenvalue problems. *Aleksandra Kostić and Heinrich Voss*.

Abstract.

For Hermitian matrices and generalized definite eigenproblems, the LDL^H factorization provides an easy tool to slice the spectrum into two disjoint intervals. In this note we generalize this method to nonlinear eigenvalue problems allowing for a minmax characterization of (some of) their real eigenvalues. In particular we apply this approach to several classes of quadratic pencils.

Key Words.

eigenvalue, variational characterization, minmax principle, Sylvester's law of inertia

AMS Subject Classifications.

15A18, 65F15

94   Discrete Poincaré inequalities for arbitrary meshes in the discrete duality finite volume context. *Anh Ha Le and Pascal Omnes*.

Abstract.

We establish discrete Poincaré type inequalities on a two-dimensional polygonal domain covered by arbitrary, possibly nonconforming meshes. On such meshes, discrete scalar fields are defined by their values both at the cell centers and vertices, while discrete gradients are associated with the edges of the mesh, like in the discrete duality finite volume scheme. We prove that the constants that appear in these inequalities depend only on the domain and on the angles between the diagonals of the diamond cells constructed by joining the two vertices of each mesh edge and the centers of the cells that share that edge.

Key Words.

Poincaré inequalities, finite volumes, discrete duality, arbitrary meshes

AMS Subject Classifications.

65N08, 46E35

120   A combinatorial approach to nearly uncoupled Markov chains I: Reversible Markov chains. *Ryan M. Tifenbach*.

Abstract.

A Markov chain is a sequence of random variables $x_0, x_1, \ldots$ that take values in a state space $\mathcal{S}$. A set $\mathcal{E} \subseteq \mathcal{S}$ is referred to as an almost invariant aggregate if transitions from x_t to x_{t+1} where $x_t \in \mathcal{E}$ and $x_{t+1} \notin \mathcal{E}$ are exceedingly rare. A Markov chain is referred to as nearly uncoupled if there are two or more disjoint almost invariant aggregates contained in its state space. Nearly uncoupled Markov chains are characterised by long periods of relatively constant behaviour punctuated by sudden, extreme changes. We present an algorithm for producing almost invariant aggregates of a nearly uncoupled reversible Markov chain. This algorithm utilises the stochastic complement to iteratively reduce the order of the given state space.

Key Words.

nearly uncoupled Markov chain, reversible Markov chain, stochastic complement, stochastic matrix

AMS Subject Classifications.

15A18, 15A51, 60J10, 60J20, 65F15

Abstract.

In this paper we present a global-in-time non-overlapping Schwarz method applied to the one-dimensional unsteady diffusion equation. We address specifically the problem with discontinuous diffusion coefficients, our approach is therefore especially designed for subdomains with heterogeneous properties. We derive efficient interface conditions by solving analytically the minmax problem associated with the search for optimized conditions in a *Robin-Neumann* case and in a *two-sided Robin-Robin* case. The performance of the proposed schemes are illustrated by numerical experiments.

Key Words.

optimized Schwarz methods, waveform relaxation, alternating and parallel Schwarz methods

AMS Subject Classifications.

65M55, 65F10, 65N22, 35K15

Abstract.

This paper is the second part of a study dealing with the application of a global-in-time Schwarz method to a one-dimensional diffusion problem defined on two non-overlapping subdomains. In the first part, we considered the case that the diffusion coefficients were constant and possibly discontinuous. In the present study, we address the problem for spatially variable coefficients with a discontinuity at the interface between subdomains. For this particular case, we derive a new approach to analytically determine the convergence factor of the associated algorithm. The theoretical results are illustrated by numerical experiments with *Dirichlet-Neumann* and *Robin-Robin* interface conditions. In the *Robin-Robin* case, thanks to the convergence factor found at the analytical level, we can optimize the convergence speed of the Schwarz algorithm.

Key Words.

optimized Schwarz methods, waveform relaxation, alternating and parallel Schwarz methods

AMS Subject Classifications.

65M55, 65F10, 65N22, 35K15, 76F40

Abstract.

The Lyapunov matrix equation $AX + XA^* = C$ arises in many applications, particularly in the context of stability of matrices or solutions of ordinary differential equations. In this paper we present a method, based on interval arithmetic, which computes with mathematical rigor an interval matrix containing the exact solution

of the Lyapunov equation. We work out two options which can be used to verify, again with mathematical certainty, that the exact solution of the equation is positive definite. This allows to prove stability of the (non-Hermitian) matrix A if we chose C as a negative definite Hermitian matrix. Our algorithm has computational cost comparable to that of a state-of-the art algorithm for computing a floating point approximation of the solution because we can cast almost all operations as matrix-matrix operations for which interval arithmetic can be implemented very efficiently.

Key Words.
stability analysis, Lyapunov matrix equation, interval arithmetic, Krawczyk's method, verified computation

AMS Subject Classifications.
65F05, 65G20

204   Parameter estimation for multivariate exponential sums. *Daniel Potts and Manfred Tasche*.

Abstract.
The recovery of signal parameters from noisy sampled data is an essential problem in digital signal processing. In this paper, we discuss the numerical solution of the following parameter estimation problem. Let h_0 be a multivariate exponential sum, i.e., h_0 is a finite linear combination of complex exponentials with distinct frequency vectors. Determine all parameters of h_0, i.e., all frequency vectors, all coefficients, and the number of exponentials, if finitely many sampled data of h_0 are given. Using Ingham-type inequalities, the Riesz stability of finitely many multivariate exponentials with well-separated frequency vectors is discussed in continuous as well as discrete norms. Furthermore, we show that a rectangular Fourier-type matrix has a bounded condition number, if the frequency vectors are well-separated and if the number of samples is sufficiently large. Then we reconstruct the parameters of an exponential sum h_0 by a novel algorithm, the so-called sparse approximate Prony method (SAPM), where we use only some data sampled along few straight lines. The first part of SAPM estimates the frequency vectors using the approximate Prony method in the univariate case. The second part of SAPM computes all coefficients by solving an overdetermined linear Vandermonde-type system. Numerical experiments show the performance of our method.

Key Words.
parameter estimation, multivariate exponential sum, multivariate exponential fitting problem, harmonic retrieval, sparse approximate Prony method, sparse approximate representation of signals

AMS Subject Classifications.
65D10, 65T40, 41A45, 41A63, 65F20, 94A12

225   Preconditioners based on strong subgraphs. *Iain S. Duff and Kamer Kaya*.

Abstract.
This paper proposes an approach for obtaining block diagonal and block triangular preconditioners that can be used for solving a linear system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a large, nonsingular, real, $n \times n$ sparse matrix. The proposed approach uses Tarjan's algorithm for hierarchically decomposing a digraph into its strong subgraphs. To

the best of our knowledge, this is the first work that uses this algorithm for preconditioning purposes. We describe the method, analyse its performance, and compare it with preconditioners from the literature such as `ILUT` and `XPABLO` and show that it is highly competitive with them in terms of both memory and iteration count. In addition, our approach shares with `XPABLO` the benefit of being able to exploit parallelism through a version that uses a block diagonal preconditioner.

Key Words.
sparse matrices, strong subgraphs, strong components, preconditioners

AMS Subject Classifications.
05C50, 05C70, 65F50

249   Adaptive regularization and discretization of bang-bang optimal control problems. *Daniel Wachsmuth*.

Abstract.
In this article, Tikhonov regularization of control-constrained optimal control problems is investigated. Typically the solutions of such problems exhibit a so-called bang-bang structure. We develop a parameter choice rule that adaptively selects the Tikhonov regularization parameter depending on a posteriori computable quantities. We prove that this choice leads to optimal convergence rates with respect to the discretization parameter. The article is complemented by numerical results.

Key Words.
optimal control, bang-bang control, Tikhonov regularization, parameter choice rule

AMS Subject Classifications.
49K20, 49N45, 65K15

268   Implicit-explicit predictor-corrector methods combined with improved spectral methods for pricing European style vanilla and exotic options. *Edson Pindza, Kailash C. Patidar, and Edgard Ngounda*.

Abstract.
In this paper we present a robust numerical method to solve several types of European style option pricing problems. The governing equations are described by variants of Black-Scholes partial differential equations (BS-PDEs) of the reaction-diffusion-advection type. To discretise these BS-PDEs numerically, we use the spectral methods in the asset (spatial) direction and couple them with a third-order implicit-explicit predictor-corrector (IMEX-PC) method for the discretisation in the time direction. The use of this high-order time integration scheme sustains the better accuracy of the spectral methods for which they are well-known. Our spectral method consists of a pseudospectral formulation of the BS-PDEs by means of an improved Lagrange formula. On the other hand, in the IMEX-PC methods, we integrate the diffusion terms implicitly whereas the reaction and advection terms are integrated explicitly. Using this combined approach, we first solve the equations for standard European options and then extend this approach to digital options, butterfly spread options, and European calls in the Heston model. Numerical experiments

illustrate that our approach is highly accurate and very efficient for pricing financial options such as those described above.

Key Words.
European options, butterfly spread options, digital options, Black-Scholes equation, barycentric interpolation, implicit-explicit predictor-corrector methods

AMS Subject Classifications.
39A05, 65M06, 65M12, 91G60

294   Fast iterative solvers for convection-diffusion control problems. *John W. Pearson and Andrew J. Wathen*.

Abstract.
In this manuscript, we describe effective solvers for the optimal control of stabilized convection-diffusion control problems. We employ the Local Projection Stabilization, which results in the same matrix system whether the discretize-then-optimize or optimize-then-discretize approach for this problem is used. We then derive two effective preconditioners for this problem, the first to be used with MINRES and the second to be used with the Bramble-Pasciak Conjugate Gradient method. The key components of both preconditioners are an accurate mass matrix approximation, a good approximation of the Schur complement, and an appropriate multigrid process to enact this latter approximation. We present numerical results to illustrate that these preconditioners result in convergence in a small number of iterations, which is robust with respect to the step-size h and the regularization parameter β for a range of problems.

Key Words.
PDE-constrained optimization, convection-diffusion control, preconditioning, Local Projection Stabilization, Schur complement

AMS Subject Classifications.
49M25, 65F08, 65F10, 65N30

311   Chebyshev acceleration of the GeneRank algorithm.   *Michele Benzi and Verena Kuhlemann*.

Abstract.
The ranking of genes plays an important role in biomedical research. The GeneRank method of Morrison et al. [BMC Bioinformatics, 6:233 (2005)] ranks genes based on the results of microarray experiments combined with gene expression information, for example from gene annotations. The algorithm is a variant of the well known PageRank iteration, and can be formulated as the solution of a large, sparse linear system. Here we show that classical Chebyshev semi-iteration can considerably speed up the convergence of GeneRank, outperforming other acceleration schemes such as conjugate gradients.

Key Words.
GeneRank, computational genomics, Chebyshev semi-iteration, polynomials of best uniform approximation, conjugate gradients

AMS Subject Classifications.
65F10, 65F50; 9208, 92D20

321   Convergence analysis of Galerkin POD for linear second order evolution equations. *Sabrina Herkt, Michael Hinze, and Rene Pinnau.*

Abstract.
In this paper, we investigate the proper orthogonal decomposition (POD) discretization method for linear second order evolution equations. We present error estimates for two different choices of snapshot sets, one consisting of solution snapshots only and one consisting of solution snapshots and their derivatives up to second order. We show that the results of [Numer. Math., 90 (2001), pp. 117–148] for parabolic equations can be extended to linear second order evolution equations, and that the derivative snapshot POD method behaves better than the classical one for small time steps. Numerical comparisons of the different approaches are presented, illustrating the theoretical results.

Key Words.
wave equation, POD, error estimates

AMS Subject Classifications.
34A05

338   Energy backward error: interpretation in numerical solution of elliptic partial differential equations and behaviour in the conjugate gradient method. *Serge Gratton, Pavel Jiránek, and Xavier Vasseur.*

Abstract.
Backward error analysis is of great importance in the analysis of the numerical stability of algorithms in finite precision arithmetic, and backward errors are also often employed in stopping criteria of iterative methods for solving systems of linear algebraic equations. The backward error measures how far we must perturb the data of the linear system so that the computed approximation solves it exactly. We assume that the linear systems are algebraic representations of partial differential equations discretised using the Galerkin finite element method. In this context, we try to find reasonable interpretations of the perturbations of the linear systems which are consistent with the problem they represent and consider the optimal backward perturbations with respect to the energy norm, which is naturally present in the underlying variational formulation. We also investigate its behaviour in the conjugate gradient method by constructing approximations in the underlying Krylov subspaces which actually minimise such a backward error.

Key Words.
symmetric positive definite systems, elliptic problems, finite element method, conjugate gradient method, backward error

AMS Subject Classifications.
65F10, 65F50

356   Solving regularized linear least-squares problems by the alternating direction method with applications to image restoration. *Jianjun Zhang and Benedetta Morini.*

Abstract.
We present and analyze ways to apply the Alternating Direction Method (ADM) to bound-constrained quadratic problems including ℓ_1 and ℓ_2 regularized linear least-squares problems. The resulting ADM schemes require the solution of two subproblems at each iteration: the first one is a linear system, the second one is a bound-constrained optimization problem with closed-form solution. Numerical results on image deblurring problems are provided and comparisons are made with a Newton-based method and a first-order method for bound-constrained optimization.

Key Words.
linear least-squares problems, ℓ_1 and ℓ_2 regularization, bound-constraints, alternating direction method, image deblurring

AMS Subject Classifications.
65F22, 65K10, 65T50, 68U10, 90C25

373   A note on the relation between the Newton homotopy method and the damped Newton method. *Xuping Zhang and Bo Yu.*

Abstract.
The homotopy continuation method and the damped Newton method are two known methods for circumventing the drawback of local convergence of the standard Newton method. Although some relations between these two methods have already been obtained, these relations are mainly for the differential equations which determine the paths followed by the two methods, rather than the sequences generated by the algorithms. In this paper, these sequences are investigated and some further relations are explored in terms of the marching directions and the step sizes during the iteration processes. Numerical solution of a semilinear elliptic equation is included to illustrate the relations discovered.

Key Words.
homotopy continuation, damped Newton method, domain of convergence, nonlinear algebraic equations, semilinear elliptic equations, finite element method

AMS Subject Classifications.
65H10, 65H20

381   Parallelism and robustness in GMRES with a Newton basis and deflated restarting. *Desire Nuentsa Wakam and Jocelyne Erhel.*

Abstract.
The GMRES iterative method is widely used as a Krylov subspace technique for solving sparse linear systems when the coefficient matrix is nonsymmetric and indefinite. The Newton basis implementation has been proposed on distributed memory computers as an alternative to the classical approach with the Arnoldi process. The aim of our work here is to introduce a modification based on deflation techniques. This approach builds an augmented subspace in an adaptive way to accelerate the convergence of the restarted formulation. In our numerical experiments, we

show the benefits of using this implementation with hybrid direct/iterative methods to solve large linear systems.

Key Words.

AMS Subject Classifications.

407   On computing stabilizability radii of linear time-invariant continuous systems.
D. C. Khanh, H. T. Quyen, and D. D. X. Thanh.

Abstract.
In this paper we focus on a non-convex and non-smooth singular value optimization problem. Our framework encompasses the distance to stabilizability of a linear system (A, B) when both A and B or only one of them are perturbed. We propose a trisection algorithm for the numerical solution of the singular value optimization problem. This method requires $O(n^4)$ operations on average, where n is the order of the system. Numerical experiments indicate that the method is reliable in practice.

Key Words.

AMS Subject Classifications.

414   Computing approximate extended Krylov subspaces without explicit inversion.
Thomas Mach, Miroslav S. Pranić, and Raf Vandebril.

Abstract.
It is shown that extended Krylov subspaces—under some assumptions—can be computed approximately without any explicit inversion or system solves involved. Instead, the necessary computations are done in an implicit way using the information from an enlarged standard Krylov subspace. For both the classical and extended Krylov spaces, the matrices capturing the recurrence coefficients can be retrieved by projecting the original matrix on a particular orthogonal basis of the associated (extended) Krylov space. It is also well-known that for (extended) Krylov spaces of full dimension, i.e., equal to the matrix size, the matrix of recurrences can be obtained directly by executing similarity transformations on the original matrix. In practice, however, for large dimensions, computing time is saved by making use of iterative procedures to gradually gather the recurrences in a matrix. Unfortunately, for extended Krylov spaces, one is obliged to frequently solve systems of equations. In this paper the iterative and the direct similarity approach are integrated, thereby avoiding system solves. At first, an orthogonal basis of a standard Krylov subspace of dimension $m_\ell + m_r + p$ and the matrix of recurrences are constructed iteratively. After that, cleverly chosen unitary similarity transformations are executed to alter the matrix of recurrences, thereby also changing the orthogonal basis vectors spanning the large Krylov space. Finally, only the first $m_\ell + m_r - 1$ new basis vectors are retained resulting in an orthogonal basis approximately spanning the extended Krylov subspace

$$\mathcal{K}_{m_\ell, m_r}(A, v) = \operatorname{span}\left\{A^{-m_r+1}v, \ldots, A^{-1}v, v, Av, A^2v, \ldots, A^{m_\ell-1}v\right\}.$$

Numerical experiments support the claim that this approximation is very good if the large Krylov subspace approximately contains $\mathrm{span}\left\{A^{-m_r+1}v, \ldots, A^{-1}v\right\}$. This can culminate in significant dimensionality reduction and as such can also lead to time savings when approximating or solving, e.g., matrix functions or equations.

Key Words.
Krylov, extended Krylov, iterative methods, Ritz values, polynomial approximation, rotations, QR factorization

AMS Subject Classifications.
65F60, 65F10, 47J25, 15A16

Abstract.
We study the problem of computing the exterior modulus of a bounded quadrilateral. We reduce this problem to the numerical solution of the Dirichlet-Neumann problem for the Laplace equation. Several experimental results, with error estimates, are reported. Our main method makes use of an hp-FEM algorithm, which enables computations in the case of complicated geometry. For simple geometries, good agreement with computational results based on the SC Toolbox, is observed. We also use the reciprocal error estimation method introduced in our earlier paper to validate our numerical results. In particular, exponential convergence, in accordance with the theory of Babuška and Guo, is demonstrated.

Key Words.
conformal capacity, conformal modulus, quadrilateral modulus, hp-FEM, numerical conformal mapping

AMS Subject Classifications.
65E05, 31A15, 30C85

Abstract.
For the solution of linear ill-posed problems, in this paper we introduce a simple algorithm for the choice of the regularization parameters when performing multi-parameter Tikhonov regularization through an iterative scheme. More specifically, the new technique is based on the use of the Arnoldi-Tikhonov method and the discrepancy principle. Numerical experiments arising from the discretization of integral equations are presented.

Key Words.
multi-parameter regularization, Arnoldi-Tikhonov method, discrepancy principle

AMS Subject Classifications.
65F10, 65F22

476   Efficient MCMC-based image deblurring.   *Johnathan M. Bardsley, Marylesa Howard, and James G. Nagy.*

Abstract.

The problem of uncertainty quantification (UQ) for inverse problems has become of significant recent interest. However, UQ requires more than the classical methods for computing solutions of inverse problems. In this paper, we take a Bayesian approach for the solution of ill-posed deconvolution problems with a symmetric convolution kernel and Neumann boundary conditions. The prior is modeled as a Gaussian Markov random field (GMRF) with the same boundary conditions and symmetry assumptions. These assumptions yield better results in certain instances and also allow for the use of the discrete cosine transform for fast computations. Moreover, we use a hierarchical model for the noise precision (inverse-variance) and prior precision parameters. This leads to a posterior density function from which we can compute samples using a basic Markov Chain Monte Carlo (MCMC) method. The resulting samples can then be used for both estimation (using, e.g., the sample mean) and uncertainty quantification (using, e.g., histograms, the sample variance, or a movie created from the image samples). We provide a numerical experiment showing that the method is effective, computationally efficient, and that for certain problems, the boundary conditions can yield significantly better results than if a periodic boundary is assumed. The novelty in the work lies in the combination of the MCMC method, Neumann boundary conditions, GMRF priors, and in the use of a movie to visualize uncertainty in the unknown image.

Key Words.

image deblurring, inverse problems, Bayesian inference, Gaussian Markov random fields, Markov chain Monte Carlo methods, Neumann boundary conditions

AMS Subject Classifications.

15A29, 62F15, 65F22, 94A08

489   Vector extrapolation applied to algebraic Riccati equations arising in transport theory. *Rola El-Moallem and Hassane Sadok.*

Abstract.

We apply the reduced rank extrapolation method (RRE) to an iterative method for computing the minimal positive solution of a nonsymmetric algebraic Riccati equation that arises in transport theory. The computations yield the minimal positive solution of a vector equation, which is derived from the special form of solutions of the Riccati equation and by exploiting the special structure of the coefficient matrices of the Riccati equation. Numerical experiments and comparisons illustrate the effectiveness of the new approach.

Key Words.

nonsymmetric algebraic Riccati equation, transport theory, minimal positive solution, iterative methods, vector sequences, polynomial vector extrapolation methods, convergence acceleration, reduced rank extrapolation

AMS Subject Classifications.

15A24, 65F10, 65B05