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Contents

Abstract.
We study conformal maps from multiply connected domains in the extended complex plane onto lemniscatic domains. Walsh proved the existence of such maps in 1956 and thus obtained a direct generalization of the Riemann mapping theorem to multiply connected domains. For certain polynomial preimages of simply connected sets, we derive a construction principle for Walsh's conformal map in terms of the Riemann map for the simply connected set. Moreover, we explicitly construct examples of Walsh's conformal map for certain radial slit domains and circular domains.

Key Words.
conformal mapping, multiply connected domains, lemniscatic domains

AMS Subject Classifications.
30C35, 30C20

Abstract.
Implicitly-restarted Lanczos iteration methods are among the most powerful methods for finding a small number of eigenpairs of Hermitian matrices. These methods enjoy strong convergence and have manageable memory requirements. Implicitly-restarted Lanczos methods naturally find approximations to both the largest and smallest eigenpairs of the input matrix, but stagnation of Ritz values may lead to slow convergence, especially when one only wants eigenvalues from one end of the spectrum and memory is constrained. We develop a filtering strategy that breaks Ritz value stagnation for these one-sided eigenvalue problems. We demonstrate the reduction in matrix-vector product costs and note that this new variant has marked advantages when memory is constrained and when matrix-vector products are expensive.

Key Words.
Implicitly-restarted Lanczos, polynomial filtering, Hermitian eigenvalue problems, Krylov subspaces, Chebyshev filtering

AMS Subject Classifications.
65F15, 15A18, 90C99

Abstract.
In this paper we present new double angle theorems for the rotation of the

eigenspaces of Hermitian matrix pairs (H, M), where H is a non-singular matrix which can be factorized as $H = GJG^*$, $J = \mathrm{diag}(\pm 1)$, and M is non-singular. The rotation of the eigenspaces is measured in the matrix-dependent scalar product, and the bounds belong to relative perturbation theory. The quality of the new bounds are illustrated in the numerical examples.

Key Words.
matrix pairs, perturbation of eigenvectors, $\sin 2\Theta$ theorems

AMS Subject Classifications.
15A15, 15A09, 15A23

58 On q-interpolation formulae and their applications.
M. R. Eslahchi and Mohammad Masjed-Jamei.

Abstract.
It is shown that the q-Taylor series corresponding to Jackson's q-difference operator can be generated by the Newton interpolation formulae and the related remainders can be therefore written as the residue of a Newton interpolation formula. The advantage of this approach is that some constraints such as q-integrability in a domain and existence of the q-derivatives of f at zero up to order n are no longer necessary. Two q-quadrature formulae of weighted interpolatory type are derived in this direction and some numerical examples for approximating definite quadratures and solving ordinary differential equations are then given

Key Words.
q-Taylor series, Jackson's q-difference operator, Newton interpolation formulae, q-quadrature rules of weighted interpolatory type

AMS Subject Classifications.
65D05, 65D30, 41A05, 41A55

75 A comparison of adaptive coarse spaces for iterative substructuring in two dimensions.
Axel Klawonn, Patrick Radtke, and Oliver Rheinbach.

Abstract.
The convergence rate of iterative substructuring methods generally deteriorates when large discontinuities occur in the coefficients of the partial differential equations to be solved. In dual-primal Finite Element Tearing and Interconnecting (FETI-DP) and Balancing Domain Decomposition by Constraints (BDDC) methods, sophisticated scalings, e.g., deluxe scaling, can improve the convergence rate when large coefficient jumps occur along or across the interface. For more general cases, additional information has to be added to the coarse space. One possibility is to enhance the coarse space by local eigenvectors associated with subsets of the interface, e.g., edges. At the center of the condition number estimates for FETI-DP and BDDC methods is an estimate related to the P_D-operator which is defined by the product of the transpose of the scaled jump operator B_D^T and the jump operator B of the FETI-DP algorithm. Some enhanced algorithms immediately bring the P_D-operator into focus using related local eigenvalue problems, and some replace a local extension theorem and local Poincaré inequalities by appropriate local eigenvalue problems. Three different strategies, suggested by different authors, are discussed for adapting the coarse space together with suitable scalings. Proofs and numerical results comparing the methods are provided.

Key Words.
FETI-DP, BDDC, eigenvalue problem, coarse space, domain decomposition, multiscale

AMS Subject Classifications.
65F10, 65N30, 65N55

107 Low-rank solvers for fractional differential equations.
Tobias Breiten, Valeria Simoncini, and Martin Stoll.

Abstract.
Many problems in science and technology can be cast using differential equations with both fractional time and spatial derivatives. To accurately simulate natural phenomena using this technology, fine spatial and temporal discretizations are required, leading to large-scale linear systems or matrix equations, especially whenever more than one space dimension is considered. The discretization of fractional differential equations typically involves dense matrices with a Toeplitz structure in the constant coefficient case. We combine the fast evaluation of Toeplitz matrices and their circulant preconditioners with state-of-the-art linear matrix equation methods to efficiently solve these problems, both in terms of CPU time and memory requirements. Additionally, we illustrate how these techniques can be adapted when variable coefficients are present. Numerical experiments on typical differential problems with fractional derivatives in both space and time showing the effectiveness of the approaches are reported.

Key Words.
fractional calculus, fast solvers, Sylvester equations, preconditioning, low-rank methods, tensor equations

AMS Subject Classifications.
65F08, 65F10, 65F50, 92E20, 93C20

133 Any finite convergence curve is possible in the initial iterations of restarted FOM.
Marcel Schweitzer.

Abstract.
We investigate the possible convergence behavior of the restarted full orthogonalization method (FOM) for non-Hermitian linear systems $A\mathbf{x} = \mathbf{b}$. For the GMRES method, it is known that any nonincreasing sequence of residual norms is possible, independent of the eigenvalues of $A \in \mathbb{C}^{n\times n}$. For FOM, however, there has not yet been any literature describing similar results. This paper complements the results for (restarted) GMRES by showing that any finite sequence of residual norms is possible in the first n iterations of restarted FOM, where by finite we mean that we only consider the case that all FOM iterates are defined, and thus no "infinite" residual norms occur. We discuss the relation of our results to known results on restarted GMRES and give a new result concerning the possible convergence behavior of restarted GMRES for iteration counts exceeding the matrix dimension n. In addition, we give a conjecture on an implication of our result with respect to the convergence of the restarted Arnoldi approximation for $g(A)\mathbf{b}$, the action of a matrix function on a vector.

Key Words.
linear systems, restarted Krylov subspace methods, full orthogonalization method, restarted Arnoldi method for matrix functions, GMRES method

AMS Subject Classifications.
65F10, 65F50, 65F60

Abstract.
MacLachlan, Manteuffel, and McCormick [Numer. Linear Algebra Appl., 13 (2006), pp. 599–620] introduced a new algebraic multigrid method, the so-called reduction-based algebraic multigrid method (AMGr). Different from typical multigrid methods, the smoother of the AMGr method is acting only on the fine-grid points. To analyze the AMGr method, different constants and parameters are used. Here, we further analyze the AMGr method. We show that the parameter used by MacLachlan et al. has another important property. We show that it is closely related to the root of a Chebyshev polynomial. This fact also explains the good performance of AMGr. By examining this relation with Chebyshev polynomials, we extend the concept of the AMGr method. We consider algebraic multigrid methods with fine-grid smoothers and AMG methods that are based on polynomial smoothing. We also establish bounds for the error propagation operator. The bound is minimal if Chebyshev polynomials are chosen. If more than one smoothing step is used, the error bound is smaller than the bound given for the AMGr method. For only one smoothing step, the polynomial-based AMG with Chebyshev polynomials coincides with the AMGr method. In this case, our convergence analysis gives some new explanation of the high performance of the AMGr method as well as the parameters used in the AMGr method.

Key Words.
AMG, AMGr, Chebyshev polynomials

AMS Subject Classifications.
65F10, 65F20

Abstract.
This paper develops least squares Legendre and Chebyshev spectral methods for the first order system of Stokes-Darcy equations. The least squares functional is based on the velocity-flux-pressure formulation with the enforcement of the Beavers-Joseph-Saffman interface conditions. Continuous and discrete homogeneous functionals are shown to be equivalent to the combination of weighted H^1 and $H(\mathrm{div})$-norm for the Stokes and Darcy equations. The spectral convergence for the Legendre and Chebyshev methods are derived and numerical experiments are also presented to illustrate the analysis.

Key Words.
Coupled Stokes-Darcy equation, first order system, least squares method, Legendre and Chebyshev pseudo-spectral method, Beavers-Joseph-Saffman law.

AMS Subject Classifications.
65N35, 65N12.

Abstract.
Since solutions of Abel integral equations exhibit singularities, existing spectral methods for these equations suffer from instability and low accuracy. Moreover, for nonlinear problems, solving the resulting complex nonlinear algebraic systems numerically requires high computational costs. To overcome these drawbacks, in this paper we propose an operational Galerkin strategy for solving Abel-Hammerstein integral equations of the second kind which applies Müntz-Legendre polynomials as natural basis functions to discretize the problem and to obtain a sparse nonlinear system with upper-triangular structure that can be solved directly. It is shown that our approach yields a well-posed and easy-to-implement approximation technique with a high order of accuracy regardless of the singularities of the exact solution. The numerical results confirm the superiority and effectiveness of the proposed scheme.

Key Words.
Abel-Hammerstein integral equations, Galerkin method, Müntz-Legendre polynomials, well-posedness

AMS Subject Classifications.
45E10, 41A25

Abstract.
We consider linear systems whose matrices are Laplacians of undirected graphs. We present a new aggregation-based algebraic multigrid method designed to achieve robustness for this class of problems, despite the diversity of connectivity patterns encountered in practical applications. These indeed range from regular mesh graphs to scale-free type of graphs associated with social networks. The method is based on the recursive static elimination of the vertices of degree 1 combined with a new *Degree-aware Rooted Aggregation* (DRA) algorithm. This algorithm always produces aggregates big enough so that the cost per iteration is low, whereas reasonable convergence is observed when the approach is combined with the K-cycle. The robustness of the resulting method is illustrated on a large collection of test problems, and its effectiveness is assessed via the comparison with a state-of-the-art reference method.

Key Words.
graph Laplacian, multigrid, algebraic multigrid, multilevel, preconditioning, aggregation

AMS Subject Classifications.
65F08, 65F10, 65N55, 65F50, 05C50

Abstract.
Non-overlapping domain decomposition methods necessarily have to exchange

Key Words.
biharmonic equation, Hellan-Herrmann-Johnson method, mixed methods, Helmholtz decomposition

AMS Subject Classifications.
65N22, 65F10, 65N55

Abstract.
We present a Legendre spectral method for initial-boundary value problems with variable coefficients and of arbitrary dimensionality, where the computational work in each time step scales linearly with the number of unknowns. Boundary conditions are enforced weakly, allowing for stable solutions of many classes of problems. Working in coefficient space, derivatives can be evaluated recursively in linear time. We show how also the action of variable coefficients can be implemented without transforming back to coordinate space using a recursive, linearly scaling matrix-free algorithm, under the assumption that the coefficients vary on a much longer scale than the solution. We also prove that spectral accuracy is preserved for smooth solutions. Numerical results for the wave equation in two and three dimensions corroborate the theoretical predictions.

Key Words.
spectral methods, matrix-free methods, method of lines, stability, computational wave propagation, boundary conditions

AMS Subject Classifications.
65M12, 65M15, 65M20, 65M70

Abstract.
This paper presents a domain decomposition-type method for solving real symmetric (Hermitian) eigenvalue problems in which we seek all eigenpairs in an interval $[\alpha, \beta]$ or a few eigenpairs next to a given real shift ζ. A Newton-based scheme is described whereby the problem is converted to one that deals with the interface nodes of the computational domain. This approach relies on the fact that the inner solves related to each local subdomain are relatively inexpensive. This Newton scheme exploits spectral Schur complements, and these lead to so-called eigenbranches, which are rational functions whose roots are eigenvalues of the original matrix. Theoretical and practical aspects of domain decomposition techniques for computing eigenvalues and eigenvectors are discussed. A parallel implementation is presented and its performance on distributed computing environments is illustrated by means of a few numerical examples.

Key Words.
Domain decomposition, spectral Schur complements, eigenvalue problems, Newton's method, parallel computing

AMS Subject Classifications.
65F15, 15A18, 65F50

AMS Subject Classifications.
47A52, 65J20

Abstract.
We present and analyze waveform relaxation variants of the Dirichlet-Neumann and Neumann-Neumann methods for parabolic problems. These methods are based on a non-overlapping spatial domain decomposition, and each iteration involves subdomain solves with Dirichlet boundary conditions followed by subdomain solves with Neumann boundary conditions. However, unlike for elliptic problems, each subdomain solve now involves a solution in space and time, and the interface conditions are also time-dependent. We show for the heat equation that when we consider finite time intervals, the Dirichlet-Neumann and Neumann-Neumann methods converge superlinearly for an optimal choice of the relaxation parameter, similar to the case of Schwarz waveform relaxation algorithms. Our analysis is based on Laplace transforms and detailed kernel estimates. The convergence rate depends on the size of the subdomains as well as the length of the time window. For any other choice of the relaxation parameter, convergence is only linear. We illustrate our results with numerical experiments.

Key Words.
waveform relaxation, Dirichlet-Neumann waveform relaxation, Neumann-Neumann waveform relaxation, Schwarz waveform relaxation

AMS Subject Classifications.
65M55, 65Y05, 65M15

Abstract.
The mimetic discretization of a boundary value problem (BVP) seeks to reproduce the same underlying properties that are satisfied by the continuous solution. In particular, the Castillo-Grone mimetic finite difference gradient and divergence fulfill a discrete version of the integration-by-parts theorem on 1-D staggered grids. For the approximation to this integral principle, a boundary flux operator is introduced that also intervenes with the discretization of the given BVP. In this work, we present a tensor formulation of these three mimetic operators on three-dimensional rectangular grids. These operators are used in the formulation of new mimetic schemes for second-order elliptic equations under general Robin boundary conditions. We formally discuss the consistency of these numerical schemes in the case of second-order discretizations and also bound the eigenvalue spectrum of the corresponding linear system. This analysis guarantees the non-singularity of the associated system matrix for a wide range of model parameters and proves the convergence of the proposed mimetic discretizations. In addition, we easily construct fourth-order accurate mimetic operators and extend these discretizations to rectangular grids with a local refinement in any direction. Both of these numerical capabilities are inherited from

AMS Subject Classifications.
65F10, 65F50, 65F08

524 An adaptive choice of primal constraints for BDDC domain decomposition algorithms.
Juan G. Calvo and Olof B. Widlund.

Abstract.
An adaptive choice for primal spaces based on parallel sums is developed for BDDC deluxe methods and elliptic problems in three dimensions. The primal space, which forms the global, coarse part of the domain decomposition algorithm and which is always required for any competitive algorithm, is defined in terms of generalized eigenvalue problems related to subdomain edges and faces; selected eigenvectors associated to the smallest eigenvalues are used to enhance the primal spaces. This selection can be made automatic by using tolerance parameters specified for the subdomain faces and edges. Numerical results verify the results and provide a comparison with primal spaces commonly used. They include results for cubic subdomains as well as subdomains obtained by a mesh partitioner. Different distributions for the coefficients are also considered with constant coefficients, highly random values, and channel distributions.

Key Words.
elliptic problems, domain decomposition, BDDC deluxe preconditioners, adaptive primal constraints

AMS Subject Classifications.
65F08, 65N30, 65N35, 65N55