

## Contents

- 1 On conformal maps from multiply connected domains onto lemniscatic domains.  
*Olivier Sète and Jörg Liesen.*

**Abstract.**

We study conformal maps from multiply connected domains in the extended complex plane onto lemniscatic domains. Walsh proved the existence of such maps in 1956 and thus obtained a direct generalization of the Riemann mapping theorem to multiply connected domains. For certain polynomial preimages of simply connected sets, we derive a construction principle for Walsh's conformal map in terms of the Riemann map for the simply connected set. Moreover, we explicitly construct examples of Walsh's conformal map for certain radial slit domains and circular domains.

**Key Words.**

conformal mapping, multiply connected domains, lemniscatic domains

**AMS Subject Classifications.**

30C35, 30C20

- 16 New filtering strategies for implicitly restarted Lanczos iteration.  
*Alex Breuer.*

**Abstract.**

Implicitly-restarted Lanczos iteration methods are among the most powerful methods for finding a small number of eigenpairs of Hermitian matrices. These methods enjoy strong convergence and have manageable memory requirements. Implicitly-restarted Lanczos methods naturally find approximations to both the largest and smallest eigenpairs of the input matrix, but stagnation of Ritz values may lead to slow convergence, especially when one only wants eigenvalues from one end of the spectrum and memory is constrained. We develop a filtering strategy that breaks Ritz value stagnation for these one-sided eigenvalue problems. We demonstrate the reduction in matrix-vector product costs and note that this new variant has marked advantages when memory is constrained and when matrix-vector products are expensive.

**Key Words.**

Implicitly-restarted Lanczos, polynomial filtering, Hermitian eigenvalue problems, Krylov subspaces, Chebyshev filtering

**AMS Subject Classifications.**

65F15, 15A18, 90C99

- 33 Double angle theorems for definite matrix pairs.  
*Luka Grubišić, Suzana Miodragović, and Ninoslav Truhar.*

**Abstract.**

In this paper we present new double angle theorems for the rotation of the

eigenspaces of Hermitian matrix pairs  $(H, M)$ , where  $H$  is a non-singular matrix which can be factorized as  $H = GJG^*$ ,  $J = \text{diag}(\pm 1)$ , and  $M$  is non-singular. The rotation of the eigenspaces is measured in the matrix-dependent scalar product, and the bounds belong to relative perturbation theory. The quality of the new bounds are illustrated in the numerical examples.

**Key Words.**

matrix pairs, perturbation of eigenvectors,  $\sin 2\Theta$  theorems

**AMS Subject Classifications.**

15A15, 15A09, 15A23

**58** On  $q$ -interpolation formulae and their applications.

*M. R. Eslahchi and Mohammad Masjed-Jamei.*

**Abstract.**

It is shown that the  $q$ -Taylor series corresponding to Jackson's  $q$ -difference operator can be generated by the Newton interpolation formulae and the related remainders can be therefore written as the residue of a Newton interpolation formula. The advantage of this approach is that some constraints such as  $q$ -integrability in a domain and existence of the  $q$ -derivatives of  $f$  at zero up to order  $n$  are no longer necessary. Two  $q$ -quadrature formulae of weighted interpolatory type are derived in this direction and some numerical examples for approximating definite quadratures and solving ordinary differential equations are then given

**Key Words.**

$q$ -Taylor series, Jackson's  $q$ -difference operator, Newton interpolation formulae,  $q$ -quadrature rules of weighted interpolatory type

**AMS Subject Classifications.**

65D05, 65D30, 41A05, 41A55

**75** A comparison of adaptive coarse spaces for iterative substructuring in two dimensions.

*Axel Klawonn, Patrick Radtke, and Oliver Rheinbach.*

**Abstract.**

The convergence rate of iterative substructuring methods generally deteriorates when large discontinuities occur in the coefficients of the partial differential equations to be solved. In dual-primal Finite Element Tearing and Interconnecting (FETI-DP) and Balancing Domain Decomposition by Constraints (BDDC) methods, sophisticated scalings, e.g., deluxe scaling, can improve the convergence rate when large coefficient jumps occur along or across the interface. For more general cases, additional information has to be added to the coarse space. One possibility is to enhance the coarse space by local eigenvectors associated with subsets of the interface, e.g., edges. At the center of the condition number estimates for FETI-DP and BDDC methods is an estimate related to the  $P_D$ -operator which is defined by the product of the transpose of the scaled jump operator  $B_D^T$  and the jump operator  $B$  of the FETI-DP algorithm. Some enhanced algorithms immediately bring the  $P_D$ -operator into focus using related local eigenvalue problems, and some replace a local extension theorem and local Poincaré inequalities by appropriate local eigenvalue problems. Three different strategies, suggested by different authors, are discussed for adapting the coarse space together with suitable scalings. Proofs and numerical results comparing the methods are provided.

**Key Words.**

FETI-DP, BDDC, eigenvalue problem, coarse space, domain decomposition, multi-scale

**AMS Subject Classifications.**

65F10, 65N30, 65N55

**107** Low-rank solvers for fractional differential equations.

*Tobias Breiten, Valeria Simoncini, and Martin Stoll.*

**Abstract.**

Many problems in science and technology can be cast using differential equations with both fractional time and spatial derivatives. To accurately simulate natural phenomena using this technology, fine spatial and temporal discretizations are required, leading to large-scale linear systems or matrix equations, especially whenever more than one space dimension is considered. The discretization of fractional differential equations typically involves dense matrices with a Toeplitz structure in the constant coefficient case. We combine the fast evaluation of Toeplitz matrices and their circulant preconditioners with state-of-the-art linear matrix equation methods to efficiently solve these problems, both in terms of CPU time and memory requirements. Additionally, we illustrate how these techniques can be adapted when variable coefficients are present. Numerical experiments on typical differential problems with fractional derivatives in both space and time showing the effectiveness of the approaches are reported.

**Key Words.**

fractional calculus, fast solvers, Sylvester equations, preconditioning, low-rank methods, tensor equations

**AMS Subject Classifications.**

65F08, 65F10, 65F50, 92E20, 93C20

**133** Any finite convergence curve is possible in the initial iterations of restarted FOM.

*Marcel Schweitzer.*

**Abstract.**

We investigate the possible convergence behavior of the restarted full orthogonalization method (FOM) for non-Hermitian linear systems  $Ax = b$ . For the GMRES method, it is known that any nonincreasing sequence of residual norms is possible, independent of the eigenvalues of  $A \in \mathbb{C}^{n \times n}$ . For FOM, however, there has not yet been any literature describing similar results. This paper complements the results for (restarted) GMRES by showing that any finite sequence of residual norms is possible in the first  $n$  iterations of restarted FOM, where by finite we mean that we only consider the case that all FOM iterates are defined, and thus no “infinite” residual norms occur. We discuss the relation of our results to known results on restarted GMRES and give a new result concerning the possible convergence behavior of restarted GMRES for iteration counts exceeding the matrix dimension  $n$ . In addition, we give a conjecture on an implication of our result with respect to the convergence of the restarted Arnoldi approximation for  $g(A)b$ , the action of a matrix function on a vector.

**Key Words.**

linear systems, restarted Krylov subspace methods, full orthogonalization method, restarted Arnoldi method for matrix functions, GMRES method

**AMS Subject Classifications.**

65F10, 65F50, 65F60

- 146** On AMG methods with F-smoothing based on Chebyshev polynomials and their relation to AMGr.

*Florian Gossler and Reinhard Nabben.*

**Abstract.**

MacLachlan, Manteuffel, and McCormick [Numer. Linear Algebra Appl., 13 (2006), pp. 599–620] introduced a new algebraic multigrid method, the so-called reduction-based algebraic multigrid method (AMGr). Different from typical multigrid methods, the smoother of the AMGr method is acting only on the fine-grid points. To analyze the AMGr method, different constants and parameters are used. Here, we further analyze the AMGr method. We show that the parameter used by MacLachlan et al. has another important property. We show that it is closely related to the root of a Chebyshev polynomial. This fact also explains the good performance of AMGr. By examining this relation with Chebyshev polynomials, we extend the concept of the AMGr method. We consider algebraic multigrid methods with fine-grid smoothers and AMG methods that are based on polynomial smoothing. We also establish bounds for the error propagation operator. The bound is minimal if Chebyshev polynomials are chosen. If more than one smoothing step is used, the error bound is smaller than the bound given for the AMGr method. For only one smoothing step, the polynomial-based AMG with Chebyshev polynomials coincides with the AMGr method. In this case, our convergence analysis gives some new explanation of the high performance of the AMGr method as well as the parameters used in the AMGr method.

**Key Words.**

AMG, AMGr, Chebyshev polynomials

**AMS Subject Classifications.**

65F10, 65F20

- 160** Least squares spectral method for velocity-flux form of the coupled Stokes-Darcy equations.

*Peyman Hessari and Bongsoo Jang.*

**Abstract.**

This paper develops least squares Legendre and Chebyshev spectral methods for the first order system of Stokes-Darcy equations. The least squares functional is based on the velocity-flux-pressure formulation with the enforcement of the Beavers-Joseph-Saffman interface conditions. Continuous and discrete homogeneous functionals are shown to be equivalent to the combination of weighted  $H^1$  and  $H(\text{div})$ -norm for the Stokes and Darcy equations. The spectral convergence for the Legendre and Chebyshev methods are derived and numerical experiments are also presented to illustrate the analysis.

**Key Words.**

Coupled Stokes-Darcy equation, first order system, least squares method, Legendre and Chebyshev pseudo-spectral method, Beavers-Joseph-Saffman law.

**AMS Subject Classifications.**

65N35, 65N12.

- 183 Operational Müntz-Galerkin approximation for Abel-Hammerstein integral equations of the second kind.

*P. Mokhtary.*

**Abstract.**

Since solutions of Abel integral equations exhibit singularities, existing spectral methods for these equations suffer from instability and low accuracy. Moreover, for nonlinear problems, solving the resulting complex nonlinear algebraic systems numerically requires high computational costs. To overcome these drawbacks, in this paper we propose an operational Galerkin strategy for solving Abel-Hammerstein integral equations of the second kind which applies Müntz-Legendre polynomials as natural basis functions to discretize the problem and to obtain a sparse nonlinear system with upper-triangular structure that can be solved directly. It is shown that our approach yields a well-posed and easy-to-implement approximation technique with a high order of accuracy regardless of the singularities of the exact solution. The numerical results confirm the superiority and effectiveness of the proposed scheme.

**Key Words.**

Abel-Hammerstein integral equations, Galerkin method, Müntz-Legendre polynomials, well-posedness

**AMS Subject Classifications.**

45E10, 41A25

- 201 An efficient multigrid method for graph Laplacian systems.

*Artem Napov and Yvan Notay.*

**Abstract.**

We consider linear systems whose matrices are Laplacians of undirected graphs. We present a new aggregation-based algebraic multigrid method designed to achieve robustness for this class of problems, despite the diversity of connectivity patterns encountered in practical applications. These indeed range from regular mesh graphs to scale-free type of graphs associated with social networks. The method is based on the recursive static elimination of the vertices of degree 1 combined with a new *Degree-aware Rooted Aggregation* (DRA) algorithm. This algorithm always produces aggregates big enough so that the cost per iteration is low, whereas reasonable convergence is observed when the approach is combined with the K-cycle. The robustness of the resulting method is illustrated on a large collection of test problems, and its effectiveness is assessed via the comparison with a state-of-the-art reference method.

**Key Words.**

graph Laplacian, multigrid, algebraic multigrid, multilevel, preconditioning, aggregation

**AMS Subject Classifications.**

65F08, 65F10, 65N55, 65F50, 05C50

- 219 Cross-points in domain decomposition methods with a finite element discretization.

*Martin J. Gander and Kevin Santugini.*

**Abstract.**

Non-overlapping domain decomposition methods necessarily have to exchange

Dirichlet and Neumann traces at interfaces in order to allow for convergence to the underlying mono-domain solution. Well-known such non-overlapping methods are the Dirichlet-Neumann method, the FETI and Neumann-Neumann methods, and optimized Schwarz methods. For all these methods, cross-points in the domain decomposition configuration where more than two subdomains meet do not pose any problem at the continuous level, but care must be taken when the methods are discretized. We show in this paper two possible approaches for the consistent discretization of Neumann conditions at cross-points in a finite element setting: the *auxiliary variable method* and *complete communication*.

**Key Words.**

domain decomposition, cross-points, finite element discretization, auxiliary variables, complete communication

**AMS Subject Classifications.**

65N55, 65N30, 65F10

- 241 Maps for global separation of roots.  
*Mário M. Graça.*

**Abstract.**

The global separation of the fixed-points of a real-valued function  $g$  on an interval  $D = [a, b]$  is considered by introducing the notions of quasi-step maps associated to  $g$  and quasi-step maps educated by two predicates. The process of ‘education’ by the predicates is an a priori global technique which does not require initial guesses. The main properties of these maps are studied and the theoretical results are illustrated by some examples where appropriate quasi-step maps for Newton and Halley methods are applied.

**Key Words.**

step function, fixed-point, iteration map, Newton map, Halley map, sieve of Eratosthenes

**AMS Subject Classifications.**

65H05, 65H20, 65S05

- 257 A decomposition result for biharmonic problems and the Hellan-Herrmann-Johnson method.  
*Wolfgang Krendl, Katharina Rafetseder, and Walter Zulehner.*

**Abstract.**

For the first biharmonic problem a mixed variational formulation is introduced which is equivalent to a standard primal variational formulation on arbitrary polygonal domains. Based on a Helmholtz decomposition for an involved nonstandard Sobolev space it is shown that the biharmonic problem is equivalent to three (consecutively to solve) second-order elliptic problems. Two of them are Poisson problems, the remaining one is a planar linear elasticity problem with Poisson ratio 0. The Hellan-Herrmann-Johnson mixed method and a modified version are discussed within this framework. The unique feature of the proposed solution algorithms for the Hellan-Herrmann-Johnson method and its modified variant is that they are solely based on standard Lagrangian finite element spaces and standard multigrid methods for second-order elliptic problems and that they are of optimal complexity.

**Key Words.**

biharmonic equation, Hellan-Herrmann-Johnson method, mixed methods, Helmholtz decomposition

**AMS Subject Classifications.**

65N22, 65F10, 65N55

- 283** A matrix-free Legendre spectral method for initial-boundary value problems.  
*Bernd Brumm and Emil Kieri.*

**Abstract.**

We present a Legendre spectral method for initial-boundary value problems with variable coefficients and of arbitrary dimensionality, where the computational work in each time step scales linearly with the number of unknowns. Boundary conditions are enforced weakly, allowing for stable solutions of many classes of problems. Working in coefficient space, derivatives can be evaluated recursively in linear time. We show how also the action of variable coefficients can be implemented without transforming back to coordinate space using a recursive, linearly scaling matrix-free algorithm, under the assumption that the coefficients vary on a much longer scale than the solution. We also prove that spectral accuracy is preserved for smooth solutions. Numerical results for the wave equation in two and three dimensions corroborate the theoretical predictions.

**Key Words.**

spectral methods, matrix-free methods, method of lines, stability, computational wave propagation, boundary conditions

**AMS Subject Classifications.**

65M12, 65M15, 65M20, 65M70

- 305** Spectral Schur complement techniques for symmetric eigenvalue problems.  
*Vassilis Kalantzis, Ruipeng Li, and Yousef Saad.*

**Abstract.**

This paper presents a domain decomposition-type method for solving real symmetric (Hermitian) eigenvalue problems in which we seek all eigenpairs in an interval  $[\alpha, \beta]$  or a few eigenpairs next to a given real shift  $\zeta$ . A Newton-based scheme is described whereby the problem is converted to one that deals with the interface nodes of the computational domain. This approach relies on the fact that the inner solves related to each local subdomain are relatively inexpensive. This Newton scheme exploits spectral Schur complements, and these lead to so-called eigenbranches, which are rational functions whose roots are eigenvalues of the original matrix. Theoretical and practical aspects of domain decomposition techniques for computing eigenvalues and eigenvectors are discussed. A parallel implementation is presented and its performance on distributed computing environments is illustrated by means of a few numerical examples.

**Key Words.**

Domain decomposition, spectral Schur complements, eigenvalue problems, Newton's method, parallel computing

**AMS Subject Classifications.**

65F15, 15A18, 65F50

- 330** A new geometric acceleration of the von Neumann-Halperin projection method.  
*Williams López.*
- Abstract.**  
We develop a geometrical acceleration scheme for the von Neumann-Halperin alternating projection method, when applied to the problem of finding the projection of a point onto the intersection of a finite number of closed subspaces of a Hilbert space. We study the convergence properties of the new scheme. We also present some encouraging preliminary numerical results to illustrate the performance of the new scheme when compared with a well-known geometrical acceleration scheme, and also with the original von Neumann-Halperin alternating projection method.
- Key Words.**  
von Neumann-Halperin algorithm, alternating projection methods, orthogonal projections, acceleration schemes
- AMS Subject Classifications.**  
52A20, 46C07, 65H10, 47J25
- 342** Robust a posteriori error bounds for spline collocation applied to singularly perturbed reaction-diffusion problems.  
*Torsten Linss and Goran Radojević.*
- Abstract.**  
Collocation with arbitrary order  $C^1$ -splines for a singularly perturbed reaction-diffusion problem in one dimension is studied. Robust a posteriori error bounds are derived for the collocation method on arbitrary meshes. These bounds are used to drive an adaptive mesh moving algorithm. Numerical results are presented.
- Key Words.**  
reaction-diffusion, spline collocation, singular perturbations, a posteriori error estimation
- AMS Subject Classifications.**  
65L10, 65L11, 65L60
- 354** A BDDC algorithm for second-order elliptic problems with hybridizable discontinuous Galerkin discretizations.  
*Xuemin Tu and Bin Wang.*
- Abstract.**  
A balancing domain decomposition by constraints (BDDC) algorithm is applied to the linear system arising from a hybridizable discontinuous Galerkin (HDG) discretization of the second-order elliptic problems. Edge/face constraints are enforced across the subdomain interface and the similar condition number bound is obtained as those for conforming finite element discretization. Numerical experiments demonstrate the convergence rate of the proposed algorithm.
- Key Words.**  
discontinuous Galerkin, HDG, domain decomposition, BDDC
- AMS Subject Classifications.**  
65F10, 65N30, 65N55

- 371 Gauss-Kronrod quadrature formulae — A survey of fifty years of research.  
*Sotirios E. Notaris.*

**Abstract.**

Kronrod in 1964, trying to estimate economically the error of the  $n$ -point Gauss quadrature formula for the Legendre weight function, developed a new formula by adding to the  $n$  Gauss nodes  $n + 1$  new ones, which are determined, together with all weights, such that the new formula has maximum degree of exactness. It turns out that the new nodes are zeros of a polynomial orthogonal with respect to a variable-sign weight function, considered by Stieltjes in 1894, without though making any reference to quadrature. We survey the considerable research work that has been emerged on this subject, during the past fifty years, after Kronrod's original idea.

**Key Words.**

Gauss quadrature formula, Gauss-Kronrod quadrature formula, Stieltjes polynomials

**AMS Subject Classifications.**

65D32, 33C45

- 405 Internality of generalized averaged Gauss rules and their truncations for Bernstein-Szegő weights.  
*D. Lj. Djukić, L. Reichel, M. M. Spalević, and J. D. Tomanović.*

**Abstract.**

Generalized averaged Gauss quadrature formulas may have nodes outside the interval of integration. Quadrature rules with nodes outside the interval of integration cannot be applied to approximate integrals with an integrand that is defined on the interval of integration only. This paper investigates when generalized averaged Gauss quadrature rules for Bernstein-Szegő weight functions have all nodes in the interval of integration. Also truncated variants of these quadrature rules are considered. The relation between generalized averaged Gauss quadrature formulas and Gauss-Kronrod rules is explored.

**Key Words.**

Gauss quadrature, averaged Gauss quadrature, truncated generalized averaged Gauss quadrature, internality of quadrature rule

**AMS Subject Classifications.**

65D32, 65D30

- 420 A note on optimal rates for Lavrentiev regularization with adjoint source conditions.  
*Andreas Neubauer.*

**Abstract.**

In a recent paper, Plato, Mathé, and Hofmann proved several convergence rate results for Lavrentiev regularization. Especially, they also proved new results for the case when the exact solution  $u$  of an ill-posed linear problem  $Au = f$  satisfies the adjoint source condition  $u \in \mathcal{R}((A^*)^p)$ ,  $0 < p \leq \frac{1}{2}$ . In this note we slightly improve the rate for  $p = \frac{1}{2}$  and also prove the rate  $O(\delta^{\frac{1}{3}})$  if  $p > \frac{1}{2}$ .

**Key Words.**

Lavrentiev regularization, convergence rates

**AMS Subject Classifications.**

47A52, 65J20

- 424** Dirichlet-Neumann and Neumann-Neumann waveform relaxation algorithms for parabolic problems.

*Martin J. Gander, Felix Kwok, and Bankim C. Mandal.*

**Abstract.**

We present and analyze waveform relaxation variants of the Dirichlet-Neumann and Neumann-Neumann methods for parabolic problems. These methods are based on a non-overlapping spatial domain decomposition, and each iteration involves subdomain solves with Dirichlet boundary conditions followed by subdomain solves with Neumann boundary conditions. However, unlike for elliptic problems, each subdomain solve now involves a solution in space and time, and the interface conditions are also time-dependent. We show for the heat equation that when we consider finite time intervals, the Dirichlet-Neumann and Neumann-Neumann methods converge superlinearly for an optimal choice of the relaxation parameter, similar to the case of Schwarz waveform relaxation algorithms. Our analysis is based on Laplace transforms and detailed kernel estimates. The convergence rate depends on the size of the subdomains as well as the length of the time window. For any other choice of the relaxation parameter, convergence is only linear. We illustrate our results with numerical experiments.

**Key Words.**

waveform relaxation, Dirichlet-Neumann waveform relaxation, Neumann-Neumann waveform relaxation, Schwarz waveform relaxation

**AMS Subject Classifications.**

65M55, 65Y05, 65M15

- 457** Tensor formulation of 3-D mimetic finite differences and applications to elliptic problems.

*J. Blanco, O. Rojas, C. Chacón, J. M. Guevara-Jordan, and J. Castillo.*

**Abstract.**

The mimetic discretization of a boundary value problem (BVP) seeks to reproduce the same underlying properties that are satisfied by the continuous solution. In particular, the Castillo-Grone mimetic finite difference gradient and divergence fulfill a discrete version of the integration-by-parts theorem on 1-D staggered grids. For the approximation to this integral principle, a boundary flux operator is introduced that also intervenes with the discretization of the given BVP. In this work, we present a tensor formulation of these three mimetic operators on three-dimensional rectangular grids. These operators are used in the formulation of new mimetic schemes for second-order elliptic equations under general Robin boundary conditions. We formally discuss the consistency of these numerical schemes in the case of second-order discretizations and also bound the eigenvalue spectrum of the corresponding linear system. This analysis guarantees the non-singularity of the associated system matrix for a wide range of model parameters and proves the convergence of the proposed mimetic discretizations. In addition, we easily construct fourth-order accurate mimetic operators and extend these discretizations to rectangular grids with a local refinement in any direction. Both of these numerical capabilities are inherited from

the original tensor formulation. As a numerical assessment, we solve a boundary-layer test problem with increasing difficulty as a sensitivity parameter is gradually adjusted. Results on uniform grids show optimal convergence rates while the solutions computed after a smooth grid clustering exhibit a significant gain in accuracy for the same number of grid cells.

**Key Words.**

mimetic finite differences, tensor products, locally refined grids, elliptic equations

**AMS Subject Classifications.**

65H17, 65N06, 40A30

**476** Weighted Hermite quadrature rules.

*Mohammad Masjed-Jamei and Gradimir V. Milovanović.*

**Abstract.**

In this paper, a new representation of Hermite osculatory interpolation is presented in order to construct weighted Hermite quadrature rules. Then, explicit forms of several special cases of the established quadrature are obtained such as weighted Hermite quadrature rules with arithmetic and geometric nodes as well as standard Gauss-Christoffel quadrature rules and Gaussian quadratures rules using only function derivatives. Some numerical examples are also given for the above mentioned cases.

**Key Words.**

weighted Hermite quadrature rule, Hermite interpolation, Gaussian quadrature, divided differences, distribution of nodes

**AMS Subject Classifications.**

65D05, 65D30, 41A55, 65D32

**499** Two recursive GMRES-type methods for shifted linear systems with general preconditioning.

*Kirk M. Soodhalter.*

**Abstract.**

We present two minimum residual methods for solving sequences of shifted linear systems, the right-preconditioned shifted GMRES and shifted Recycled GMRES algorithms which use a seed projection strategy often employed to solve multiple related problems. These methods are compatible with a general preconditioning of all systems, and, when restricted to right preconditioning, require no extra applications of the operator or preconditioner. These seed projection methods perform a minimum residual iteration for the base system while improving the approximations for the shifted systems at little additional cost. The iteration continues until the base system approximation is of satisfactory quality. The method is then recursively called for the remaining unconverged systems. We present both methods inside of a general framework which allows these techniques to be extended to the setting of flexible preconditioning and inexact Krylov methods. We present some analysis of such methods and numerical experiments demonstrating the effectiveness of the proposed algorithms.

**Key Words.**

Krylov subspace methods, shifted linear systems, parameterized linear systems, quantum chromodynamics

**AMS Subject Classifications.**

65F10, 65F50, 65F08

- 524** An adaptive choice of primal constraints for BDDC domain decomposition algorithms.

*Juan G. Calvo and Olof B. Widlund.*

**Abstract.**

An adaptive choice for primal spaces based on parallel sums is developed for BDDC deluxe methods and elliptic problems in three dimensions. The primal space, which forms the global, coarse part of the domain decomposition algorithm and which is always required for any competitive algorithm, is defined in terms of generalized eigenvalue problems related to subdomain edges and faces; selected eigenvectors associated to the smallest eigenvalues are used to enhance the primal spaces. This selection can be made automatic by using tolerance parameters specified for the subdomain faces and edges. Numerical results verify the results and provide a comparison with primal spaces commonly used. They include results for cubic subdomains as well as subdomains obtained by a mesh partitioner. Different distributions for the coefficients are also considered with constant coefficients, highly random values, and channel distributions.

**Key Words.**

elliptic problems, domain decomposition, BDDC deluxe preconditioners, adaptive primal constraints

**AMS Subject Classifications.**

65F08, 65N30, 65N35, 65N55