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Contents

Abstract.
In this paper, we discuss the acceleration of the regularized alternating least-squares (RALS) algorithm for tensor approximations. We propose a fast iterative method using an Aitken-Stefensen-like update for the regularized algorithm. Through numerical experiments, a faster convergence rate for the accelerated version is demonstrated in comparison to both the standard and regularized alternating least-squares algorithms. In addition, we analyze global convergence based on the Kurdyka-Łojasiewicz inequality, and we show that the RALS algorithm has a linear local convergence rate.

Key Words.
alternating least-squares, Kurdyka-Łojasiewicz inequality, tensor approximation

AMS Subject Classifications.
15A69, 65F30

Abstract.
In this paper, we construct a new quadratic spline-wavelet basis on the interval and on the unit square satisfying homogeneous Dirichlet boundary conditions of the first order. The wavelets have one vanishing moment and the shortest support among quadratic spline wavelets with at least one vanishing moment adapted to the same type of boundary conditions. The stiffness matrices arising from the discretization of the second-order elliptic problems using the constructed wavelet basis have uniformly bounded condition numbers, and the condition numbers are small. We present some quantitative properties of the constructed basis. We provide numerical examples to show that the Galerkin method and the adaptive wavelet method using our wavelet basis require fewer iterations than methods with other quadratic spline wavelet bases. Moreover, due to the small support of the wavelets, when using these methods with the new wavelet basis, the system matrix is sparser, and thus one iteration requires a smaller number of floating point operations than for other quadratic spline wavelet bases.

Key Words.
wavelet, quadratic spline, homogeneous Dirichlet boundary conditions, condition number, elliptic problem

AMS Subject Classifications.
46B15, 65N12, 65T60

40 Convergence of the multiplicative Schwarz method for singularly perturbed convection-diffusion problems discretized on a Shishkin mesh.
Carlos Echeverría, Jörg Liesen, Daniel B. Szyld, and Petr Tichý.

Abstract.
We analyze the convergence of the multiplicative Schwarz method applied to nonsymmetric linear algebraic systems obtained from discretizations of one-dimensional singularly perturbed convection-diffusion equations by upwind and central finite differences on a Shishkin mesh. Using the algebraic structure of the Schwarz iteration matrices we derive bounds on the infinity norm of the error that are valid from the first step of the iteration. Our bounds for the upwind scheme prove rapid convergence of the multiplicative Schwarz method for all relevant choices of parameters in the problem. The analysis for the central difference is more complicated, since the submatrices that occur are nonsymmetric and sometimes even fail to be M-matrices. Our bounds still prove the convergence of the method for certain parameter choices.

Key Words.
singularly perturbed problems, Shishkin mesh discretization, multiplicative Schwarz method, iterative methods, convergence analysis

AMS Subject Classifications.
15A60, 65F10, 65F35

63 Asymptotic consistent exponential-type integrators for Klein-Gordon-Schrödinger systems from relativistic to non-relativistic regimes.
Simon Baumstark, Georgia Kokkala, and Katharina Schratz.

Abstract.
In this paper we propose asymptotic consistent exponential-type integrators for the Klein-Gordon-Schrödinger system. This novel class of integrators allows us to solve the system from slowly varying relativistic up to challenging highly oscillatory non-relativistic regimes without any step size restriction. In particular, our first- and second-order exponential-type integrators are asymptotically consistent in the sense of asymptotically converging to the corresponding decoupled free Schrödinger limit system.

Key Words.
highly oscillatory, Klein-Gordon-Schrödinger, asymptotic consistency, exponential-type integrators

AMS Subject Classifications.
35C20, 65M12, 35L05

81 Eigenfunction expansion method for multiple solutions of fourth-order ordinary differential equations with cubic polynomial nonlinearity.
Abdrhaman Mahmoud, Bo Yu, and Xuping Zhang.

Abstract.
Multiple solutions of fourth-order ordinary differential equations (ODEs) with cubic polynomial nonlinearity are presented in this paper. The ODEs are discretized by the eigenfunction expansion method. Discretization error estimates are derived. We construct an efficient polynomial homotopy to find all solutions for the system

of polynomial equations on the coarse level by recursion. Two kinds of filters are suggested for removing possible spurious solutions of the discretized system of polynomial equations. Numerical experiments are included to verify the error estimates and efficiency of the proposed homotopy.

Key Words.
fourth-order ODEs, eigenfunction expansion method, system of polynomial equations, multiple solutions, homotopy continuation method

AMS Subject Classifications.
34L10, 34B15, 65H04

97 Adaptive refinement strategies for the simulation of gas flow in networks using a model hierarchy.
Pia Domschke, Aseem Dua, Jeroen J. Stolwijk, Jens Lang, and Volker Mehrmann.

Abstract.
A model hierarchy that is based on the one-dimensional isothermal Euler equations of fluid dynamics is used for the simulation and optimisation of natural gas flow through a pipeline network. Adaptive refinement strategies have the aim of bringing the simulation error below a prescribed tolerance while keeping the computational costs low. While spatial and temporal stepsize adaptivity is well studied in the literature, model adaptivity is a new field of research. The problem of finding an optimal refinement strategy that combines these three types of adaptivity is a generalisation of the unbounded knapsack problem. A refinement strategy that is currently used in gas flow simulation software is compared to two novel greedy-like strategies. Both a theoretical experiment and a realistic gas flow simulation show that the novel strategies significantly outperform the current refinement strategy with respect to the computational cost incurred.

Key Words.
gas supply networks, model hierarchy, error estimators, model adaptivity, refinement strategies

AMS Subject Classifications.
65K99, 65Z99, 65M22, 35Q31

114 Numerical analysis of a dual-mixed problem in non-standard Banach spaces.
Jessika Camaño, Cristian Muñoz, and Ricardo Oyarzúa.

Abstract.
In this paper we analyze the numerical approximation of a saddle-point problem posed in non-standard Banach spaces $\mathrm{H}(\mathrm{div}_p\,,\Omega) \times L^q(\Omega)$, where $\mathrm{H}(\mathrm{div}_p\,,\Omega) := \{\boldsymbol{\tau} \in [L^2(\Omega)]^n \colon \mathrm{div}\boldsymbol{\tau} \in L^p(\Omega)\}$, with $p > 1$ and $q \in \mathbb{R}$ being the conjugate exponent of p and $\Omega \subseteq \mathbb{R}^n$ ($n \in \{2, 3\}$) a bounded domain with Lipschitz boundary Γ. In particular, we are interested in deriving the stability properties of the forms involved (inf-sup conditions, boundedness), which are the main ingredients to analyze mixed formulations. In fact, by using these properties we prove the well-posedness of the corresponding continuous and discrete saddle-point problems by means of the classical Babuška-Brezzi theory, where the associated Galerkin scheme is defined by Raviart-Thomas elements of order $k \geq 0$ combined with piecewise polynomials of degree k. In addition we prove optimal convergence of the numerical approximation in the associated Lebesgue norms. Next, by employing the theory

developed for the saddle-point problem, we analyze a mixed finite element method for a convection-diffusion problem, providing well-posedness of the continuous and discrete problems and optimal convergence under a smallness assumption on the convective vector field. Finally, we corroborate the theoretical results with suitable numerical results in two and three dimensions.

Key Words.
mixed finite element method, Raviart-Thomas, Lebesgue spaces, Lp data, convection-diffusion

AMS Subject Classifications.
65N15, 65N12, 65N30, 74S05

131 Runge-Kutta methods revisited for a class of structured strangeness-free differential-algebraic equations.
Vu Hoang Linh and Nguyen Duy Truong.

Abstract.
Numerical methods for a class of nonlinear differential-algebraic equations (DAEs) of the strangeness-free form are investigated. Half-explicit and implicit Runge-Kutta methods are revisited as they are applied to a reformulated form of the original DAEs. It is shown that the methods preserve the same convergence order and the same stability properties as if they were applied to ordinary differential equations (ODEs). Thus, a wide range of explicit Runge-Kutta methods and implicit ones, which are not necessarily stiffly accurate, can efficiently solve the class of DAEs under consideration. Implementation issues and a perturbation analysis are also discussed. Numerical experiments are presented to illustrate the theoretical results.

Key Words.
differential-algebraic equation, strangeness-free form, Runge-Kutta method, half-explicit method, convergence, stability

AMS Subject Classifications.
65L80, 65L05, 65L06, 65L20

156 Multiscale Coarse Spaces for Overlapping Schwarz Methods Based on the ACMS Space in 2D.
Alexander Heinlein, Axel Klawonn, Jascha Knepper, and Oliver Rheinbach.

Abstract.
Two-level overlapping Schwarz domain decomposition methods for second-order elliptic problems in two dimensions are proposed using coarse spaces constructed from the Approximate Component Mode Synthesis (ACMS) multiscale discretization approach. These coarse spaces are based on eigenvalue problems using Schur complements on subdomain edges. It is then shown that the convergence of the resulting preconditioned Krylov method can be controlled by a user-specified tolerance and thus can be made independent of heterogeneities in the coefficient of the partial differential equation. The relations of this new approach to other known adaptive coarse space approaches for overlapping Schwarz methods are also discussed. Compared to one of the competing adaptive approaches, the new coarse space can be significantly smaller. Compared to other competing approaches, the eigenvalue problems are significantly cheaper to solve, i.e., the dimension of the eigenvalue

Abdeslem Hafid Bentbib, Abderrahman Bouhamadi, and Karim Kreit.

Abstract.
In this paper, we consider the problem of image denoising by total variation regularization. We combine the conditional gradient method with the total variation regularization in the dual formulation to derive a new method for denoising images. The convergence of this method is proved. Some numerical examples are given to illustrate the effectiveness of the proposed method.

Key Words.
ill-posed problem, regularization, total variation, conditional gradient, image restoration, image denoising, convex programming

AMS Subject Classifications.
65F10, 65R30

329 High-order exponentially fitted difference schemes for singularly perturbed two-point boundary value problems.
Miljenko Marušić.

Abstract.
We introduce a family of exponentially fitted difference schemes of arbitrary order as numerical approximations to the solution of a singularly perturbed two-point boundary value problem: $\varepsilon y'' + by' + cy = f$. The difference schemes are derived from interpolation formulae for exponential sums. The so-defined k-point differentiation formulae are exact for functions that are a linear combination of $1, x, \ldots, x^{k-2}, \exp(-\rho x)$. The parameter ρ is chosen from the asymptotic behavior of the solution in the boundary layer. This approach allows a construction of the method with arbitrary order of consistency. Using an estimate for the interpolation error, we prove consistency of all the schemes from the family. The truncation error is bounded by Ch^{k-2}, where C is a constant independent of ε and h. Therefore, the order of consistency for the k-point scheme is $k-2$ ($k \geq 3$) in case of a small perturbation parameter ε. There is no general proof of stability for the proposed schemes. Each scheme has to be considered separately. In the paper, stability, and therefore convergence, is proved for three-point schemes in the case when $c < 0$ and $b \neq 0$.

Key Words.
difference scheme, singular perturbation, ODE, interpolation, exponential sum

AMS Subject Classifications.
65L12, 65L11, 65L10, 65L20, 41A30, 65D25

348 Multigrid methods combined with low-rank approximation for tensor-structured Markov chains.
Matthias Bolten, Karsten Kahl, Daniel Kressner, Francisco Macedo, and Sonja Sokolović.

Abstract.
Markov chains that describe interacting subsystems suffer from state space explosion but lead to highly structured matrices. In this work, we propose a novel tensor-based algorithm to address such tensor-structured Markov chains. Our algorithm combines a tensorized multigrid method with AMEn, an optimization-based low-rank tensor solver, for addressing coarse grid problems. Numerical experiments

demonstrate that this combination overcomes the limitations incurred when using each of the two methods individually. As a consequence, Markov chain models of unprecedented size from a variety of applications can be addressed.

Key Words.
Multigrid method, SVD, Tensor Train format, Markov chains, singular linear system, alternating optimization

AMS Subject Classifications.
65F10, 65F50, 60J22, 65N55

Abstract.
It is well known that the entries of the inverse of a Hermitian positive definite, banded matrix exhibit a decay away from the main diagonal if the condition number of the matrix is not too large compared to the matrix size. There is a rich literature on bounds which predict and explain this decay behavior. However, all the widely known results on exponential decay lead to a Toeplitz matrix of bounds, i.e., they yield the same bound for all entries along a sub- or superdiagonal. In general, there is no reason to expect the inverse of A to have a Toeplitz structure so that this is an obvious shortcoming of these decay bounds. We construct an example of a tridiagonal matrix for which the difference between these decay bounds and the actual decay is especially pronounced and then show how these bounds can be adapted to better reflect the actual decay by investigating certain (modified) submatrices of A. As a by-product, we also investigate how the distribution of all eigenvalues of A rather than just the spectral interval influences the decay behavior. Here, our results hold for matrices with a general, not necessarily banded, sparsity structure.

Key Words.
matrix inverse, tridiagonal matrix, off-diagonal decay, Sherman–Morrison formula, Toeplitz matrix

AMS Subject Classifications.
15A16, 65F50, 65F60

Abstract.
The generalized sinc-Gaussian sampling operator is established by Asharabi (2016) to approximate two classes of analytic functions. In this paper, we use this operator to construct a new sampling method to approximate the eigenvalues of the periodic (semi-periodic) Dirac system of differential equations problem. The convergence rate of this method is of exponential type, i.e., $\mathrm{e}^{-\alpha_r N}/\sqrt{N}$, $\alpha_r = ((r+1)\pi - \sigma h)/2$. The sinc-Gaussian and Hermite-Gauss methods are special cases of this method. We estimate the amplitude error associated to this operator, which gives us the possibility to establish the error analysis of this method. Various illustrative examples are presented and they show a good agreement with our theoretical analysis.

Abstract.
Imbalances in rotating machines cause vibrations of the system and may lead to an early wearout of the machine. In this paper, we consider the development of an algorithm for the detection (and subsequent correction) of imbalances from vibrational measurements at certain nodes of the system. Since, e.g., modern wind turbines operate with variable speed, the vibration data are usually collected during changing rotational speed. Based on a mathematical model that connects the measured vibrations at different rotational speeds with the imbalance distribution, we propose an algorithm for its reconstruction. The reconstruction algorithm is based on a tensor product formulation of the forward model. Test examples with artificial data are used to verify our approach.

Key Words.
integral equation, Kronecker product, inverse problem, tensor product, regularization

AMS Subject Classifications.
65N21, 47A52, 15A05

450 Uniform representations of the incomplete beta function in terms of elementary functions.
Chelo Ferreira, José L. López, and Ester Pérez Sinusía.

Abstract.
We consider the incomplete beta function $B_z(a,b)$ in the maximum domain of analyticity of its three variables: $a,b,z \in \mathbb{C}$, $-a \notin \mathbb{N}$, $z \notin [1,\infty)$. For $\Re b \leq 1$ we derive a convergent expansion of $z^{-a}B_z(a,b)$ in terms of the function $(1-z)^b$ and of rational functions of z that is uniformly valid for z in any compact set in $\mathbb{C} \setminus [1,\infty)$. When $-b \in \mathbb{N} \cup \{0\}$, the expansion also contains a logarithmic term of the form $\log(1-z)$. For $\Re b \geq 1$ we derive a convergent expansion of $z^{-a}(1-z)^b B_z(a,b)$ in terms of the function $(1-z)^b$ and of rational functions of z that is uniformly valid for z in any compact set in the exterior of the circle $|z-1| = r$ for arbitrary $r > 0$. The expansions are accompanied by realistic error bounds. Some numerical experiments show the accuracy of the approximations.

Key Words.
incomplete beta function, convergent expansions, uniform expansions

AMS Subject Classifications.
33B20, 41A58, 41A80

462 Conformal modulus and planar domains with strong singularities and cusps.
Harri Hakula, Antti Rasila, and Matti Vuorinen.

Abstract.
We study the problem of computing the conformal modulus of rings and quadrilaterals with strong singularities and cusps at their boundary. We reduce this problem to the numerical solution of the associated Dirichlet and Dirichlet-Neumann-type boundary values problems for the Laplace equation. Several experimental results, with error estimates, are reported. In particular, we consider domains with dendrite-like boundaries where an analytic formula for the conformal modulus can be derived. The boundary value problems are solved using an hp-finite element method.

Key Words.
conformal capacity, conformal modulus, quadrilateral modulus, hp-FEM, numerical conformal mapping

AMS Subject Classifications.
65E05, 31A15, 30C85