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Contents

Abstract.
In FETI-DP (Finite Element Tearing and Interconnecting) and BDDC (Balancing Domain Decomposition by Constraints) domain decomposition methods, the transformation-of-basis approach is used to improve the convergence by combining the local assembly with a change of basis. Suitable basis vectors can be constructed by the recently introduced adaptive coarse space approaches. The resulting FETI-DP and BDDC methods fulfill a condition number bound independent of heterogeneities in the problem. The adaptive method with a transformation of basis presented here builds on a recently introduced adaptive FETI-DP approach for elliptic problems in three dimensions and uses a coarse space constructed from solving small, local eigenvalue problems on closed faces and on a small number of edges. In contrast to our earlier work on adaptive FETI-DP, the coarse space correction is not implemented by using balancing (or deflation), which requires the use of an exact coarse space solver, but by using local transformations. This will make it simpler to extend the method to a large number of subdomains and large supercomputers. The recently established theory of a generalized transformation-of-basis approach yields a condition number estimate for the preconditioned operator that is independent of jumps of the coefficients across and inside subdomains when using the local adaptive constraints. It is shown that all results are also valid for BDDC. Numerical results are presented in three dimensions for FETI-DP and BDDC. We also provide a comparison of different scalings, i.e., deluxe, rho, stiffness, and multiplicity for our adaptive coarse space in 3D.

Key Words.
domain decomposition, FETI-DP, BDDC, coarse space, adaptive, eigenvalue problem, elliptic partial differential equations

AMS Subject Classifications.
65N30,65N55,65F08,65F10

Abstract.
We present an analysis of the additive average Schwarz preconditioner with two newly proposed adaptively enriched coarse spaces, which were presented at the twenty-third international conference on domain decomposition methods in Korea, for solving second-order elliptic problems with highly varying and discontinuous coefficients. It is shown that the condition number of the preconditioned system is

bounded independently of the variations and the jumps in the coefficient while depending only on a prescribed threshold for the eigenvalues of the coarse space, and it depends linearly on the mesh parameter ratio H/h that is the ratio between the subdomain size and the mesh size thereby retaining the same optimality and scalability of the original additive average Schwarz preconditioner.

Key Words.
domain decomposition preconditioner, additive average Schwarz method, adaptive coarse space, multiscale finite element

AMS Subject Classifications.
65N55, 65N30, 65N22, 65F08

41 Numerical assessment of two-level domain decomposition preconditioners for incompressible Stokes and elasticity equations.
Gabriel R. Barrenechea, Michał Bosy, and Victorita Dolean.

Abstract.
Solving the linear elasticity and Stokes equations by an optimal domain decomposition method derived algebraically involves the use of non-standard interface conditions. The one-level domain decomposition preconditioners are based on the solution of local problems. This has the undesired consequence that the results are not scalable, which means that the number of iterations needed to reach convergence increases with the number of subdomains. This is the reason why in this work we introduce, and test numerically, two-level preconditioners. Such preconditioners use a coarse space in their construction. We consider the nearly incompressible elasticity problems and Stokes equations, and discretise them by using two finite element methods, namely, the hybrid discontinuous Galerkin and Taylor-Hood discretisations.

Key Words.
Stokes problem, nearly incompressible elasticity, Taylor-Hood, hybrid discontinuous Galerkin methods, domain decomposition, coarse space, optimized restricted additive Schwarz methods.

AMS Subject Classifications.
65F10, 65N22, 65N30, 65N55.

64 BDDC and FETI-DP algorithms with a change of basis formulation on adaptive primal constraints.
Hyea Hyun Kim, Eric Chung, and Junxian Wang.

Abstract.
BDDC (Balancing Domain Decomposition by Constraints) and FETI-DP (Dual-Primal Finite Element Tearing and Interconnecting) algorithms with adaptively enriched primal constraints are considered. The coarse component of the two algorithms is built on the set of primal unknowns consisting of those at subdomain vertices and those from the adaptive primal constraints after a change of basis. For the FETI-DP algorithm, a more general form of a preconditioner is proposed to extend the algorithm to the set of primal unknowns including those from the adaptive primal constraints. In addition, it can be shown that the two algorithms share the same spectra except those equal to one or zero when the same set of adaptive primal constraints are employed. Numerical results are included for both two and three dimensional model problems.

Key Words.
FETI-DP, BDDC, adaptive primal constraints, change of basis, condition numbers

AMS Subject Classifications.
65F10, 65N30, 65N55

81 Isogeometric Schwarz preconditioners for the biharmonic problem.
D. Cho, L. F. Pavarino, and S. Scacchi.

Abstract.
A scalable overlapping Schwarz preconditioner for the biharmonic Dirichlet problem discretized by isogeometric analysis is constructed, and its convergence rate is analyzed. The proposed preconditioner is based on solving local biharmonic problems on overlapping subdomains that form a partition of the CAD domain of the problem and on solving an additional coarse biharmonic problem associated with the subdomain coarse mesh. An h-analysis of the preconditioner shows an optimal convergence rate bound that is scalable in the number of subdomains and is cubic in the ratio between subdomain and overlap sizes. Numerical results in 2D and 3D confirm this analysis and also illustrate the good convergence properties of the preconditioner with respect to the isogeometric polynomial degree p and regularity k.

Key Words.
domain decomposition methods, overlapping Schwarz, biharmonic problem, scalable preconditioners, isogeometric analysis, finite elements, B-splines, NURBS

AMS Subject Classifications.
65N55, 65N30, 65F10

103 A full-space quasi-Lagrange-Newton-Krylov algorithm for trajectory optimization problems.
Hsuan-Hao Wang, Yi-Su Lo, Feng-Tai Hwang, and Feng-Nan Hwang.

Abstract.
The objectives of this work are to study and to apply the full-space quasi-Lagrange-Newton-Krylov (FQLNK) algorithm for solving trajectory optimization problems arising from aerospace industrial applications. As its name suggests, in this algorithm we first convert the constrained optimization problem into an unconstrained one by introducing the augmented Lagrangian parameters. The next step is to find the optimal candidate solution by solving the Karush-Kuhn-Tucker (KKT) system with a Newton-Krylov method. To reduce the computational cost of constructing the KKT system, we employ the Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula to build an approximation of the (1,1) subblock of the KKT matrix, which is the most expensive part of the overall computation. The BFGS-based FQLNK algorithm exhibits a superior speedup compared to some of the alternatives. We demonstrate our FQLNK algorithm to be a practical approach for designing an optimal trajectory of a launch vehicle in space missions.

Key Words.
launch vehicle mission, trajectory optimization, KKT system, BFGS, Lagrange-Newton-Krylov solver

AMS Subject Classifications.
65H10, 49M15

Abstract.
In this paper, we present a new time-multipatch discontinuous Galerkin Isogeometric Analysis (IgA) technology for solving parabolic initial-boundary problems in space and time simultaneously. We prove coercivity of the IgA variational problem with respect to a suitably chosen norm that together with boundedness, consistency, and approximation results yields a priori discretization error estimates in this norm. Furthermore, we provide efficient parallel generation and parallel multigrid solution technologies. Finally, we present first numerical results on massively parallel computers.

Key Words.
parabolic initial-boundary value problems, space-time isogeometric analysis, time discontinuous Galerkin methods, space-time multigrid solvers

AMS Subject Classifications.
35K20, 65M12, 65M15, 65M55

Abstract.
We propose and analyze a posteriori estimates for global-in-time, nonoverlapping domain decomposition methods for heterogeneous and anisotropic porous media diffusion problems. We consider mixed formulations with a lowest-order Raviart-Thomas-Nédélec discretization often used for such problems. Optimized Robin transmission conditions are employed on the space-time interface between subdomains, and different time grids are used to adapt to different time scales in the subdomains. Our estimators allow to distinguish the spatial discretization, the temporal discretization, and the domain decomposition error components. We design an adaptive space-time domain decomposition algorithm, wherein the iterations are stopped when the domain decomposition error does not affect significantly the global error. Overall, a guaranteed bound for the overall error is obtained at each iteration of the space-time domain decomposition algorithm, and simultaneously important savings in terms of the number of domain decomposition iterations can be achieved. Numerical results for two-dimensional problems with strong heterogeneities and local time-stepping are presented to illustrate the performance of our adaptive domain decomposition algorithm.

Key Words.
mixed finite element method, global-in-time domain decomposition, nonconforming time grids, Robin interface conditions, a posteriori error estimate, stopping criteria

AMS Subject Classifications.
65N15, 65N22, 65N55, 65F10, 76S05

Abstract.
Optimized Schwarz methods enhance convergence using optimized transmission conditions between subdomains. The optimization is usually performed for a model problem on an unbounded domain and two subdomains represented by half spaces. The influence of the domain decomposition geometry on the convergence and the optimized parameters is thus lost in the process, and it is not even theoretically clear if the results published for the unbounded domain still hold in concrete applications where the domains are bounded. We prove here rigorously for a two-subdomain decomposition that the asymptotic performance of optimized Schwarz methods derived from an unbounded domain analysis still holds in the case of a bounded domain, but the constants in the best choice of parameters and convergence rate estimates are influenced by the domain truncation. We obtain accurate estimates for this influence and show theoretically that the domain truncation has more remarkable influence for the slowly converging optimized Schwarz methods than for those converging fast. When the subdomain size is very small, our new optimized parameters lead to much faster algorithms than those obtained from an unbounded domain analysis. We illustrate our theoretical results with numerical experiments.

Key Words.
optimized Schwarz methods, domain decomposition methods, transmission conditions, influence of domain truncation

AMS Subject Classifications.
65N55, 65F10

210 Analysis of the parallel Schwarz method for growing chains of fixed-sized subdomains: Part III.
Gabriele Ciaramella and Martin J. Gander.

Abstract.
In the ddCOSMO solvation model for the numerical simulation of molecules (chains of atoms), the unusual observation was made that the associated Schwarz domain-decomposition method converges independently of the number of subdomains (atoms) and this without coarse correction, i.e., the one-level Schwarz method is scalable. We analyzed this unusual property for the simplified case of a rectangular molecule and square subdomains using Fourier analysis, leading to robust convergence estimates in the L^2-norm and later also for chains of subdomains represented by disks using maximum principle arguments, leading to robust convergence estimates in L^∞. A convergence analysis in the more natural H^1-setting proving convergence independently of the number of subdomains was, however, missing. We close this gap in this paper using tools from the theory of alternating projection methods and estimates introduced by P.-L. Lions for the study of domain decomposition methods. We prove that robust convergence independently of the number of subdomains is possible also in H^1 and show furthermore that even for certain two-dimensional domains with holes, Schwarz methods can be scalable without coarse-space corrections. As a by-product, we review some of the results of P.-L. Lions [On the Schwarz alternating method. I, in Domain Decomposition Methods for Partial Differential Equations, SIAM, Philadelphia, 1988, pp. 1–42] and in some cases provide simpler proofs.

Key Words.
domain decomposition methods, Schwarz methods, chain of subdomains, elliptic

PDE, Laplace equation, COSMO solvation model

AMS Subject Classifications.
65N55, 65F10, 65N22, 70-08, 35J05, 35J57

Abstract.
New nonlinear BDDC (Balancing Domain Decomposition by Constraints) domain decomposition methods using inexact solvers for the subdomains and the coarse problem are proposed. In nonlinear domain decomposition methods, the nonlinear problem is decomposed before linearization to improve concurrency and robustness. For linear problems, the new methods are equivalent to known inexact BDDC methods. The new approaches are therefore discussed in the context of other known inexact BDDC methods for linear problems. Relations are pointed out, and the advantages of the approaches chosen here are highlighted. For the new approaches, using an algebraic multigrid method as a building block, parallel scalability is shown for more than half a million (524 288) MPI ranks on the JUQUEEN IBM BG/Q supercomputer (JSC Jülich, Germany) and on up to 193 600 cores of the Theta Xeon Phi supercomputer (ALCF, Argonne National Laboratory, USA), which is based on the recent Intel Knights Landing (KNL) many-core architecture. One of our nonlinear inexact BDDC domain decomposition methods is also applied to three-dimensional plasticity problems. Comparisons to standard Newton-Krylov-BDDC methods are provided.

Key Words.
nonlinear BDDC, nonlinear domain decomposition, nonlinear elimination, Newton's method, nonlinear problems, parallel computing, inexact BDDC, nonlinear elasticity, plasticity

AMS Subject Classifications.
68W10, 68U20, 65N55, 65F08, 65Y05

Abstract.
When the obstacle problem of clamped Kirchhoff plates is discretized by a partition of unity method, the resulting discrete variational inequalities can be solved by a primal-dual active set algorithm. In this paper we develop and analyze additive Schwarz preconditioners for the systems that appear in each iteration of the primal-dual active set algorithm. Numerical results that corroborate the theoretical estimates are also presented.

Key Words.
partition of unity, additive Schwarz, displacement obstacle problem for clamped Kirchhoff plates, fourth-order variational inequality

AMS Subject Classifications.
65N30, 65K15