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Contents

Abstract.
Gauss quadrature can be naturally generalized in order to approximate quasi-definite linear functionals, where the interconnections with (formal) orthogonal polynomials, (complex) Jacobi matrices, and the Lanczos algorithm are analogous to those in the positive definite case. In this survey we review these relationships with giving references to the literature that presents them in several related contexts. In particular, the existence of the n-weight (complex) Gauss quadrature corresponds to successfully performing the first n steps of the Lanczos algorithm for generating biorthogonal bases of the two associated Krylov subspaces. The Jordan decomposition of the (complex) Jacobi matrix can be explicitly expressed in terms of the Gauss quadrature nodes and weights and the associated orthogonal polynomials. Since the output of the Lanczos algorithm can be made real whenever the input is real, the value of the Gauss quadrature is a real number whenever all relevant moments of the quasi-definite linear functional are real.

Key Words.
quasi-definite linear functionals, Gauss quadrature, formal orthogonal polynomials, complex Jacobi matrices, matching moments, Lanczos algorithm.

AMS Subject Classifications.
65D15, 65D32, 65F10, 47B36

Abstract.
We consider the Kronrod extension of generalizations of the Micchelli-Rivlin quadrature formula for the Fourier-Chebyshev coefficients with the highest algebraic degree of precision. For analytic functions, the remainder term of these quadrature formulas can be represented as a contour integral with a complex kernel. We study the kernel on elliptic contours with foci at the points ∓ 1 and the sum of semi-axes $\rho > 1$ for the mentioned quadrature formulas. We derive L^∞-error bounds and L^1-error bounds for these quadrature formulas. Finally, we obtain explicit bounds by expanding the remainder term. Numerical examples that compare these error bounds are included.

Key Words.
Kronrod extension of generalizations of the Micchelli-Rivlin quadrature formula, Chebyshev weight function of the first kind, error bound, remainder term for analytic functions, contour integral representation

AMS Subject Classifications.
65D32, 65D30, 41A55

36 Polynomial approximation with Pollaczeck-Laguerre weights on the real semiaxis. A survey.
Giuseppe Mastroianni, Gradimir V. Milovanović, and Incoronata Notarangelo.

Abstract.
This paper summarizes recent results on weighted polynomial approximations for functions defined on the real semiaxis. The function may grow exponentially both at 0 and at $+\infty$. We discuss orthogonal polynomials, polynomial inequalities, function spaces with new moduli of smoothness, estimates for the best approximation, Gaussian rules, and Lagrange interpolation with respect to the weight $w(x) = x^{\gamma}\mathrm{e}^{-x^{-\alpha}-x^{\beta}}$ ($\alpha > 0$, $\beta > 1$, $\gamma \geq 0$).

Key Words.
orthogonal polynomials, weighted polynomial approximation, polynomial inequalities, Gaussian quadrature rules, Lagrange interpolation, Pollaczeck-Laguerre exponential weights

AMS Subject Classifications.
41A05, 41A10, 41A17, 41A25, 65D05, 65D32

52 Binet-type polynomials and their zeros.
Walter Gautschi and Gradimir V. Milovanović.

Abstract.
Procedures based on moments are developed for computing the three-term recurrence relations for orthogonal polynomials relative to the Binet, generalized Binet, squared Binet, and related subrange weight functions. Monotonicity properties for the zeros of the respective orthogonal polynomials are also established.

Key Words.
Binet weight function, orthogonal polynomials, zeros, monotonicity

AMS Subject Classifications.
33C47, 65D20

71 Computation of induced orthogonal polynomial distributions.
Akil Narayan.

Abstract.
We provide a robust and general algorithm for computing distribution functions associated to induced orthogonal polynomial measures. We leverage several tools for orthogonal polynomials to provide a spectrally-accurate method for a broad class of measures, encompassing those associated to classical orthogonal polynomial families, which is stable for polynomial degrees up to at least 1000. Paired with other standard tools such as a numerical root-finding algorithm and inverse transform sampling, this provides a methodology for generating random samples from an induced orthogonal polynomial measure. Generating samples from this measure is one ingredient in optimal numerical methods for certain types of multivariate polynomial approximation. For example, sampling from induced distributions for weighted discrete least-squares approximation has recently been shown to yield convergence

and the existence of a Nash-equilibrium is established. The particular case of $n = 2$ tosses is completely solved.

Key Words.
minimax optimization, biased probability, polynomial interpolation and approximation, Nash-equilibrium

AMS Subject Classifications.
65C50, 41A10, 91A05, 41A05

129 A product integration rule for hypersingular integrals on $(0, +\infty)$.
Maria Carmela De Bonis and Donatella Occorsio.

Abstract.
In the present paper we propose a product integration rule for hypersingular integrals on the positive semi-axis. The rule is based on an approximation of the density function f by a suitable truncated Lagrange polynomial. We discuss theoretical aspects by proving stability and convergence of the procedure for density functions f belonging to weighted uniform spaces. Moreover, we give some computational details for the effective construction of the rule coefficients. For the sake of completeness, we present some numerical tests that support the theoretical estimates and some comparisons with other numerical methods.

Key Words.
Hadamard finite part integrals, approximation by polynomials, orthogonal polynomials, product integration rules

AMS Subject Classifications.
65D32, 65R20, 41A10

144 The extended global Lanczos method for matrix function approximation.
A. H. Bentbib, M. El Ghomari, C. Jagels, K. Jbilo, and L. Reichel.

Abstract.
The need to compute the trace of a large matrix that is not explicitly known, such as the matrix $\exp(A)$, where A is a large symmetric matrix, arises in various applications including in network analysis. The global Lanczos method is a block method that can be applied to compute an approximation of the trace. When the block size is one, this method simplifies to the standard Lanczos method. It is known that for some matrix functions and matrices, the extended Lanczos method, which uses subspaces with both positive and negative powers of A, can give faster convergence than the standard Lanczos method, which uses subspaces with nonnegative powers of A only. This suggests that it may be beneficial to use an extended global Lanczos method instead of the (standard) global Lanczos method. This paper describes an extended global Lanczos method and discusses properties of the associated Gauss-Laurent quadrature rules. Computed examples that illustrate the performance of the extended global Lanczos method are presented.

Key Words.
extended Krylov subspace, extended moment matching, Laurent polynomial, global Lanczos method, matrix function, Gauss quadrature rule

AMS Subject Classifications.
65F25, 65F30, 65F60, 33C47

164 Construction of the optimal set of quadrature rules in the sense of Borges.
Aleksandar N. Jovanović, Marija P. Stanić, and Tatjana V. Tomović.

Abstract.
In this paper we give a numerical method for the construction of an optimal set of quadrature rules in the sense of Borges [Numer. Math., 67 (1994), pp. 271–288] for definite integrals with the same integrand and interval of integration but with different weight functions related to an arbitrary multi-index. We present a numerical method for the construction of an optimal set of quadrature rules in the sense of Borges for four weight functions and explain how to perform an analogous construction for an arbitrary number of weight functions.

Key Words.
multi-index, optimal set of quadrature rules, multiple orthogonal polynomials

AMS Subject Classifications.
65D32, 42C05

182 Multiple Hermite polynomials and simultaneous Gaussian quadrature.
Walter Van Assche and Anton Vuerinckx.

Abstract.
Multiple Hermite polynomials are an extension of the classical Hermite polynomials for which orthogonality conditions are imposed with respect to $r > 1$ normal (Gaussian) weights $w_j(x) = e^{-x^2+c_j x}$ with different means $c_j/2$, $1 \leq j \leq r$. These polynomials have a number of properties such as a Rodrigues formula, recurrence relations (connecting polynomials with nearest neighbor multi-indices), a differential equation, etc. The asymptotic distribution of the (scaled) zeros is investigated, and an interesting new feature is observed: depending on the distance between the c_j, $1 \leq j \leq r$, the zeros may accumulate on s disjoint intervals, where $1 \leq s \leq r$. We will use the zeros of these multiple Hermite polynomials to approximate integrals of the form $\int_{-\infty}^{\infty} f(x) \exp(-x^2+c_j x)\, dx$ simultaneously for $1 \leq j \leq r$ for the case $r = 3$ and the situation when the zeros accumulate on three disjoint intervals. We also give some properties of the corresponding quadrature weights.

Key Words.
multiple Hermite polynomials, simultaneous Gauss quadrature, zero distribution, quadrature coefficients

AMS Subject Classifications.
33C45, 41A55, 42C05, 65D32