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Contents

Abstract.
Fractal interpolation functions defined by means of suitable Iterated Function Systems provide a new framework for the approximation of continuous functions defined on a compact real interval. Convergence is one of the desirable properties of a good approximant. The goal of the present paper is to develop fractal approximants, namely Bernstein α-fractal functions, which converge to the given continuous function even if the magnitude of the scaling factors does not approach zero. We use Bernstein α-fractal functions to construct the sequence of Bernstein Müntz fractal polynomials that converges to either $f \in \mathcal{C}(I)$ or $f \in L^p(I), 1 \leq p < \infty$. This gives a fractal analogue of the full Müntz theorems in the aforementioned function spaces. For a given sequence $\{f_n(x)\}_{n=1}^{\infty}$ of continuous functions that converges uniformly to a function $f \in \mathcal{C}(I)$, we develop a double sequence $\left\{\{f_{n,l}^{\alpha}(x)\}_{l=1}^{\infty}\right\}_{n=1}^{\infty}$ of Bernstein α-fractal functions that converges uniformly to f. By establishing suitable conditions on the scaling factors, we solve a constrained approximation problem of Bernstein α-fractal Müntz polynomials. We also study the convergence of Bernstein fractal Chebyshev series.

Key Words.
Bernstein polynomials, fractal approximation, convergence, full Müntz theorems, Chebyshev series, box dimension.

AMS Subject Classifications.
41A30, 28A80, 41A17, 41A50.

Abstract.
We study and develop (stochastic) primal-dual block-coordinate descent methods for convex problems based on the method due to Chambolle and Pock. Our methods have known convergence rates for the iterates and the ergodic gap of $O(1/N^2)$ if each block is strongly convex, $O(1/N)$ if no convexity is present, and more generally a mixed rate $O(1/N^2) + O(1/N)$ for strongly convex blocks if only some blocks are strongly convex. Additional novelties of our methods include blockwise-adapted step lengths and acceleration as well as the ability to update both the primal and dual variables randomly in blocks under a very light compatibility condition. In other words, these variants of our methods are doubly-stochastic. We test the proposed methods on various image processing problems, where we employ pixelwise-adapted acceleration.

Key Words.
PDHGM, Chambolle–Pock method, stochastic, doubly-stochastic, blockwise, primal-dual

AMS Subject Classifications.
49M29, 65K10, 65K15, 90C30, 90C47

Abstract.
A singularly perturbed fourth-order problem with two small parameters is considered in one dimension and on a smooth domain in two dimensions. Using its discretisation by a mixed finite element method on a properly defined Shishkin mesh, we prove convergence in the associated energy norm.

Key Words.
singular perturbation, fourth-order problem, two small parameters, mixed method, layer-adapted meshes

AMS Subject Classifications.
65L11, 65L60, 65N30, 65N50

Abstract.
The matrix pencil method is an eigenvalue-based approach for the parameter identification of sparse exponential sums. We derive a reconstruction algorithm for multivariate exponential sums that is based on simultaneous diagonalization. Randomization is used and quantified to reduce the simultaneous diagonalization to the eigendecomposition of a single random matrix. To verify feasibility, the algorithm is applied to synthetic and experimental fluorescence microscopy data.

Key Words.
frequency analysis, spectral analysis, exponential sum, moment problem, superresolution

AMS Subject Classifications.
65T40, 42C15, 30E05, 65F30

Abstract.
In this paper, a variant of the discretization of the van Roosbroeck equations in the equilibrium state with the composite discontinuous Galerkin method for the rectangular domain is discussed. It is based on the symmetric interior penalty Galerkin (SIPG) method. The proposed method accounts for lower regularity of the solution at the interfaces of the layers of the device. It is shown that the discrete problem is well defined and that the discrete solution is unique. Error estimates are derived. Finally, numerical simulations are presented.

Key Words.
composite discontinuous Galerkin method, drift-diffusion, van Roosbroeck equations

AMS Subject Classifications.
65N30, 65N15, 82D37

Abstract.
This paper investigates a specific ill-posed nonlinear inverse problem that arises in financial markets. Precisely, as a benchmark problem in the context of volatility surface calibration, we consider the simultaneous recovery of implied volatility and interest rate functions over a finite time interval from corresponding call- and put-price functions for idealized continuous families of European vanilla options over the same maturity interval. We prove identifiability of the pair of functions to be identified by showing injectivity of the forward operator in L^2-spaces. To overcome the ill-posedness we employ a two-parameter Tikhonov regularization with heuristic parameter choice rules and demonstrate chances and limitations by means of numerical case studies using synthetic data.

Key Words.
inverse option pricing, simultaneous identification, volatility, interest rate, regularization

AMS Subject Classifications.
65J20, 91G60, 47H30, 47A52

Abstract.
We consider a modification of the Reitzinger-Schöberl algebraic multigrid method for the iterative solution of the curl-curl boundary value problem discretized with edge elements. The Reitzinger-Schöberl method is attractive for its low memory requirements and moderate cost per iteration, but the number of iterations typically tends to increase with the problem size. Here we propose several modifications to the method that aim at curing the size-dependent convergence behavior without significantly affecting the attractive features of the original method. The comparison with an auxiliary space preconditioner, a state-of-the-art solver for the considered problems, further indicates that both methods typically require a comparable amount of work to solve a given discretized problem but that the proposed approach requires less memory.

Key Words.
algebraic multigrid, edge elements, preconditioning, aggregation

AMS Subject Classifications.
65N12, 65N22, 65N55

Abstract.
McDonald, Pestana, and Wathen [SIAM J. Sci. Comput., 40 (2018), pp. A1012–A1033] present a method for preconditioning time-dependent PDEs via an approximation by a nearby time-periodic problem, that is, they employ circulant-related matrices as preconditioners for the non-symmetric block Toeplitz matrices which arise from an all-at-once formulation. They suggest that such an approach might be efficiently implemented in parallel. In this short article, we present parallel numerical results for their preconditioner which exhibit strong scaling. We also extend their preconditioner via a Neumann series approach which also allows for efficient parallel execution. Results are shown for both parabolic and hyperbolic PDEs. Our simple implementation (in C++ and MPI) is available at the Git repository `PARALAAOMPI`.[1]

Key Words.
parallel-in-time, monolithic method, preconditioning

AMS Subject Classifications.
65M20, 65F08, 65Y05

Abstract.
A structured backward error analysis for an approximate eigenpair of structured nonlinear matrix equations with T*-palindromic*, H*-palindromic*, T*-anti-palindromic*, and H*-anti-palindromic* structures is conducted. We construct a minimal structured perturbation in the Frobenius norm such that an approximate eigenpair becomes an exact eigenpair of an appropriately perturbed nonlinear matrix equation. The present work shows that our general framework extends existing results in the literature on the perturbation theory of matrix polynomials.

Key Words.
nonlinear eigenvalue problem, even and odd matrix polynomials, palindromic matrix polynomial

AMS Subject Classifications.
65F15, 15A18, 65F35, 15A12

Abstract.
The topological derivative represents the sensitivity of a domain-dependent functional with respect to a local perturbation of the domain and is a valuable tool in topology optimization. Motivated by an application from electrical engineering, we derive the topological derivative for an optimization problem which is constrained by the quasilinear equation of two-dimensional magnetostatics. Here, the main ingredient is to establish a sufficiently fast decay of the variation of the direct state

[1] `https://github.com/anthonyjamesgoddard/PARALAAOMPI`

at scale 1 as $|x| \to \infty$. In order to apply the method in a bi-directional topology optimization algorithm, we derive both the sensitivity for introducing air inside ferromagnetic material and the sensitivity for introducing material inside an air region. We explicitly compute the arising polarization matrices and introduce a way to efficiently evaluate the obtained formulas. Finally, we employ the derived formulas in a level-set based topology optimization algorithm and apply it to the design optimization of an electric motor.

Key Words.
topological derivative, nonlinear magnetostatics, topology optimization, electrical machine

AMS Subject Classifications.
35J62, 49Q10, 49Q12, 78M35, 78M50

219 Efficient cubature rules.
James R. Van Zandt.

Abstract.
67 new cubature rules are found for three standard multi-dimensional integrals with spherically symmetric regions and weight functions using direct search with a numerical zero-finder. 63 of the new rules have fewer integration points than known rules of the same degree, and 20 are within three points of Möller's lower bound. Most have all positive coefficients, and most have some symmetry, including some supported by one or two concentric spheres. They include degree-7 formulas for the integration over the sphere and Gaussian-weighted integrals over the entire space, each in 6 and 7 dimensions, with 127 and 183 points, respectively.

Key Words.
multiple integrals, Gaussian weight, cubature formula, integration rule, numerical integration, regular simplex

AMS Subject Classifications.
65D30, 65D32, 41A55, 41A63

240 Approximate residual-minimizing shift parameters for the low-rank ADI iteration.
Patrick Kürschner.

Abstract.
The low-rank alternating directions implicit (LR-ADI) iteration is a frequently employed method for efficiently computing low-rank approximate solutions of large-scale Lyapunov equations. In order to achieve a rapid error reduction, the iteration requires shift parameters whose selection and generation is often a difficult task, especially for nonsymmetric matrices in the Lyapunov equation. This article represents a follow up of Benner et al. [Electron. Trans. Numer. Anal., 43 (20142015), pp. 142–162] and investigates self-generating shift parameters based on a minimization principle for the Lyapunov residual norm. Since the involved objective functions are too expensive to evaluate and hence intractable, objective functions are introduced which are efficiently constructed from the available data generated by the LR-ADI iteration. Several numerical experiments indicate that these residual-minimizing shifts using approximated objective functions outperform existing precomputed and dynamic shift parameter selection techniques, although their generation is more involved.

Key Words.
Lyapunov equation, alternating directions implicit, low-rank approximation, shift parameters

AMS Subject Classifications.
15A06, 65F10, 65F30

Abstract.
In this paper, a new ${}^{\gamma}_{\delta}\mathbf{\Phi}$-type eigenvalue inclusion set for tensors is given, and some inclusion relations between this new inclusion set and other ones are presented. In addition, a new sufficient criterion for identifying nonsingular tensors is also provided by using the new ${}^{\gamma}_{\delta}\mathbf{\Phi}$-type eigenvalue inclusion set. Some numerical results are reported to show the superiority of the results.

Key Words.
tensor, eigenvalue, inclusion, nonsingular, ${}^{\gamma}_{\delta}\mathbf{\Phi}$-type.

AMS Subject Classifications.
15A69, 15A18, 65F15, 65H17, 15A15, 65F40

Abstract.
In this paper, we consider a modified Levenberg-Marquardt method for solving an ill-posed inverse problem where the forward mapping is not Gteaux differentiable. By relaxing the standard assumptions for the classical smooth setting, we derive asymptotic stability estimates which are then used to prove convergence of the proposed method. This method can be applied to an inverse source problem for a non-smooth semilinear elliptic PDE where a Bouligand subdifferential can be used in place of the non-existing Frchet derivative, and we show that the corresponding *Bouligand-Levenberg-Marquardt iteration* is an iterative regularization scheme. Numerical examples illustrate the advantage over the corresponding Bouligand-Landweber iteration.

Key Words.
inverse problem, iterative regularization, Levenberg-Marquardt method, non-smooth equation

AMS Subject Classifications.
49K20, 49K40, 90C31

Abstract.
We use a barrier-function technique to prove the parameter-uniform convergence for singularly perturbed convection-diffusion problems discretized on a Bakhvalov-type

Abstract.
A comparison of three Balancing Domain Decomposition by Constraints (BDDC) methods with an approximate coarse space solver using the same software building blocks is attempted for the first time. The comparison is made for a BDDC method with an algebraic multigrid preconditioner for the coarse problem, a three-level BDDC method, and a BDDC method with a vertex-based coarse preconditioner. It is new that all methods are presented and discussed in a common framework. Condition number bounds are provided for all approaches. All methods are implemented in a common highly parallel scalable BDDC software package based on PETSc to allow for a simple and meaningful comparison. Numerical results showing the parallel scalability are presented for the equations of linear elasticity. For the first time, this includes parallel scalability tests for a vertex-based approximate BDDC method.

Key Words.
approximate BDDC, three-level BDDC, multilevel BDDC, vertex-based BDDC

AMS Subject Classifications.
68W10, 65N22, 65N55, 65F08, 65F10, 65Y05

Abstract.
A general framework for oblique projections of non-Hermitian matrices onto rational Krylov subspaces is developed. To obtain this framework we revisit the classical rational Krylov subspace algorithm and prove that the projected matrix can be written efficiently as a structured pencil, where the structure can take several forms such as Hessenberg or inverse Hessenberg. One specific instance of the structures appearing in this framework for oblique projections is a tridiagonal pencil. This is a direct generalization of the classical biorthogonal Krylov subspace method, where the projection becomes a single non-Hermitian tridiagonal matrix and of the Hessenberg pencil representation for rational Krylov subspaces. Based on the compact storage of this tridiagonal pencil in the biorthogonal setting, we can develop short recurrences. Numerical experiments confirm the validity of the approach.

Key Words.
rational Krylov, biorthogonal, short recurrence, oblique projection, matrix pencil

AMS Subject Classifications.
15A22, 47A75, 65F99, 65Q30

Abstract.
We present the Flip-Flop Spectrum-Revealing QR (Flip-Flop SRQR) factorization, a significantly faster and more reliable variant of the QLP factorization of Stewart for low-rank matrix approximations. Flip-Flop SRQR uses SRQR factorization to initialize a partial column-pivoted QR factorization and then computes a partial LQ factorization. As observed by Stewart in his original QLP work, Flip-Flop SRQR tracks the exact singular values with "considerable fidelity". We develop singular