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Contents

Domain Decomposition by Constraints) domain decomposition methods, the convergence behavior of the iterative scheme can be improved by implementing a coarse space using a transformation of basis and local assembly. This is an alternative to coarse spaces implemented by deflation or balancing. The transformation-of-basis approaches are more robust with respect to inexact solvers than deflation and therefore more suitable for multilevel extensions. In this paper, we show a correspondence of FETI-DP or BDDC methods using a generalized transformation-of-basis approach and of FETI-DP methods using deflation or balancing, where the deflation vectors are obtained from the transformation of basis. These methods then have essentially the same eigenvalues. As opposed to existing theory, this result also applies to general scalings and highly heterogeneous problems.
We note that the new methods differ slightly from the classic FETI-DP and BDDC methods using a transformation of basis and that the classic theory has to be replaced. An important application for the theory presented in this paper are FETI-DP and BDDC methods with adaptive coarse spaces, i.e., where deflation vectors are obtained from approximating local eigenvectors. These methods have recently gained considerable interest.

Key Words.
domain decomposition, FETI-DP, BDDC, coarse space, deflation, transformation of basis, change of variables, elliptic partial differential equations

AMS Subject Classifications.
65N30,65N25,65N50,65N55,74E05

77 Leap-frog method for stochastic functional wave equations.
Henryk Leszczyński, Milena Matusik, and Monika Wrzosek.

Abstract.
We perform a time-space discretisation, known as the leap-frog method, for nonlinear stochastic functional wave equations driven by multiplicative time-space white noise. To prove its stability we apply Cairoli's maximal inequalities for two-parameter martingales and provide a lemma for estimating solutions to a class of stochastic wave equations and a Gronwall-type inequality over cones. The method converges in L^2 at a rate of $O(\sqrt{h})$, where h is a time-space step size.

Key Words.
leap-frog, stochastic wave equation

AMS Subject Classifications.
60H15, 35R60, 35R10

88 The swallowtail integral in the highly oscillatory region II.
Chelo Ferreira, José L. López, and Ester Pérez Sinusía.

Abstract.
We analyze the asymptotic behavior of the swallowtail integral $\int_{-\infty}^{\infty} e^{i(t^5+xt^3+yt^2+zt)}dt$ for large values of $|y|$ and bounded values of $|x|$ and $|z|$. We use the simplified saddle point method introduced in [López et al., J. Math. Anal. Appl., 354 (2009), pp. 347–359]. With this method, the analysis is more straightforward than with the standard saddle point method, and it is possible to derive complete asymptotic expansions of the integral for large $|y|$ and fixed x and z. There are four Stokes lines in the sector $(-\pi, \pi]$ that divide the complex

y-plane into four sectors in which the swallowtail integral behaves differently when $|y|$ is large. The asymptotic approximation is the sum of two asymptotic series whose terms are elementary functions of x, y, and z. One of them is of Poincaré type and is given in terms of inverse powers of $y^{1/2}$. The other one is given in terms of an asymptotic sequence whose terms are of the order of inverse powers of $y^{1/9}$ when $|y| \to \infty$, and it is multiplied by an exponential factor that behaves differently in the four mentioned sectors. Some numerical experiments illustrate the accuracy of the approximation.

Key Words.
swallowtail integral, asymptotic expansions, modified saddle point method

AMS Subject Classifications.
33E20, 41A60

Abstract.
In this paper, we present a stochastic differential game to model the transboundary air pollution problems of the Beijing-Tianjin region with emission permits trading. By using stochastic optimal control theory, we obtain the Hamilton-Jacobi-Bellman equation satisfied by the value function for the cooperative games. Next, we solve the Hamilton-Jacobi-Bellman equation by using a fitted finite volume method. Finally, the efficiency and the usefulness of the fitted finite volume method are illustrated by an empirical study.

Key Words.
transboundary air pollution of the Beijing-Tianjin region, stochastic differential game, emission permits trading, Hamilton-Jacobi-Bellman equation, fitted finite volume method

AMS Subject Classifications.
49J20, 65N30

Abstract.
The interpolation of functions plays a fundamental role in numerical analysis. The highly accurate approximation of non-smooth functions is a challenge in science and engineering as traditional polynomial interpolation cannot characterize the singular features of these functions. This paper aims at designing a fractional Hermite interpolation for non-smooth functions based on the local fractional Taylor expansion and at deriving the corresponding explicit formula and its error remainder. We also present a piecewise hybrid Hermite interpolation scheme, a combination of fractional Hermite interpolation and traditional Hermite interpolation. Some numerical examples are presented to show the high accuracy of the fractional Hermite interpolation method.

Key Words.
non-smooth function, local fractional Taylor expansion, fractional Hermite interpolation, error remainder

AMS Subject Classifications.
26A30, 41A05, 65D05, 97N50

132 A spectral Newton-Schur algorithm for the solution of symmetric generalized eigenvalue problems.
Vassilis Kalantzis.

Abstract.
This paper proposes a numerical algorithm based on spectral Schur complements to compute a few eigenvalues and the associated eigenvectors of symmetric matrix pencils. The proposed scheme follows an algebraic domain decomposition viewpoint and transforms the generalized eigenvalue problem into one of computing roots of scalar functions. These scalar functions are defined so that their roots are equal to the eigenvalues of the original pencil, and these roots are computed by Newton's method. We describe the theoretical aspects of the proposed scheme and demonstrate its performance on a few test problems.

Key Words.
spectral Schur complements, domain decomposition, symmetric generalized eigenvalue problem, Newton's method

AMS Subject Classifications.
65F15, 15A18, 65F50

154 Coercive space-time finite element methods for initial boundary value problems.
Olaf Steinbach and Marco Zank.

Abstract.
We propose and analyse new space-time Galerkin-Bubnov-type finite element formulations of parabolic and hyperbolic second-order partial differential equations in finite time intervals. Using Hilbert-type transformations, this approach is based on elliptic reformulations of first- and second-order time derivatives, for which the Galerkin finite element discretisation results in positive definite and symmetric matrices. For the variational formulation of the heat and wave equations, we prove related stability conditions in appropriate norms, and we discuss the stability of related finite element discretisations. Numerical results are given which confirm the theoretical results.

Key Words.
space-time FEM, heat equation, wave equation

AMS Subject Classifications.
65M60

195 Incomplete beta polynomials.
Manuel Bello-Hernández.

Abstract.
We study some properties of incomplete beta polynomials, in particular, their zero asymptotic distribution. These polynomials satisfy a three-term recurrence relation, so they can be written as the sum of two terms each one leading to their asymptotic behavior in a region.

Key Words.
incomplete beta function, zero distribution, asymptotics, recurrence relations, potential theory

AMS Subject Classifications.
33B20, 41A58, 41A80

Abstract.
The beta-type functions play an important role in many applied sciences. The partial derivatives of the beta function and the incomplete beta function are integrals involving algebraic and logarithmic endpoint singularities. In this paper, some series expansions for these beta-type functions are found, which are easily used to evaluate these functions with prescribed precision. On the other hand, an accurate Gauss-Legendre quadrature formula is designed to compute these beta-type functions and their partial derivatives based on the Puiseux series for the integrands at their singularities. Some numerical examples confirm the high accuracy and high efficiency of the two algorithms, and also show that the algorithms can be used to effectively evaluate the generalized beta-type functions.

Key Words.
beta-type functions, higher-order partial derivatives, series expansion, modified Gauss-Legendre rule

AMS Subject Classifications.
65D20, 33B15, 33B20

Abstract.
We present ADMM-Softmax, an alternating direction method of multipliers (ADMM) for solving multinomial logistic regression (MLR) problems. Our method is geared toward supervised classification tasks with many examples and features. It decouples the nonlinear optimization problem in MLR into three steps that can be solved efficiently. In particular, each iteration of ADMM-Softmax consists of a linear least-squares problem, a set of independent small-scale smooth, convex problems, and a trivial dual variable update. The solution of the least-squares problem can be accelerated by pre-computing a factorization or preconditioner, and the separability in the smooth, convex problem can be easily parallelized across examples. For two image classification problems, we demonstrate that ADMM-Softmax leads to improved generalization compared to a Newton-Krylov, a quasi Newton, and a stochastic gradient descent method.

Key Words.
machine learning, nonlinear optimization, alternating direction method of multipliers, classification, multinomial regression

AMS Subject Classifications.
65J22, 90C25, 49M27

Abstract.
The aim of this paper is to provide a mathematical and numerical analysis for a FEM-BEM coupling approximation of a magnetically nonlinear eddy current formulation by using FEM only on the conducting domain, and by imposing the integral conditions on its boundary. The nonlinear relationship between flux density and the magnetic field intensity is given by a physical parameter called magnetic reluctivity, which is assumed to depend on the Euclidean norm of the magnetic induction in the conducting domain. We use the nonlinear monotone operator theory for parabolic equations to show that the continuous formulation obtained for the coupling is a well-posed problem. Furthermore, we use Nédélec edge elements, standard nodal finite elements, and a backward-Euler time scheme, to obtain a fully discrete formulation and to prove quasi-optimal error estimates.

Key Words.
Time-dependent electromagnetic, eddy current model, nonlinear problems, boundary element method, finite element method.

AMS Subject Classifications.
65M60, 65M38, 78M10, 78M15.

Abstract.
This paper is devoted to the study of approximation of Gaussian functions by their partial Fourier sums of degree $N \in \mathbb{N}$ with respect to the spherical Gauss-Laguerre (SGL) basis in the weighted Hilbert space $L_2(\mathbb{R}^3, \omega_\lambda)$, where $\omega_\lambda(|\boldsymbol{x}|) = \exp(-|\boldsymbol{x}|^2/\lambda)$, $\lambda > 0$. We investigate the behavior of the corresponding error of approximation with respect to the scale factor λ and order of expansion N. As interim results we obtained formulas for the Fourier coefficients of Gaussians with respect to SGL basis in the space $L_2(\mathbb{R}^3, \omega_\lambda)$. Possible application of obtained results to the docking problem are described.

Key Words.
spherical harmonic, Laguerre polynomial, Gaussian, hypergeometric function, molecular docking

AMS Subject Classifications.
33C05, 33C45, 33C55, 42C10

Abstract.
The computation and inversion of the binomial and negative binomial cumulative distribution functions play a key role in many applications. In this paper, we explain how methods used for the central beta distribution function (described in Gil,

AMS Subject Classifications.
65F15, 15A18, 65F35, 15A12

Abstract.
We study finite difference schemes for axisymmetric blow-up solutions of a nonlinear heat equation in higher spatial dimensions. The phenomenology of blow-up in higher-dimensional space is much more complex than that in one space dimension. To obtain a more complete picture for such phenomena from computational results, it is useful to know the technical details of the numerical schemes for higher spatial dimensions. Since first-order differentiation appears in the differential equation, we pay special attention to it. A sufficient condition for stability is derived. In addition to the convergence of the numerical blow-up time, certain blow-up behaviors, such as blow-up sets and blow-up in the L^p-norm, are taken into consideration. It is sometimes experienced that a certain property of solutions of a partial differential equation may be lost by a faithfully constructed convergent numerical scheme. The phenomenon of one-point blow-up is a typical example in the numerical analysis of blow-up problems. We prove that our scheme can preserve such a property. It is also remarkable that the L^p-norm $(1 \leq p < \infty)$ of the solution of the nonlinear heat equation may blow up simultaneously with the L^∞-norm or remains bounded in $[0, T)$, where T denotes the blow-up time of the L^∞-norm. We propose a systematic way to compute numerical evidence of the L^p-norm blow-up. The computational results are also analyzed. Moreover, we prove an abstract theorem which shows the relationship between the numerical L^p-norm blow-up and the exact L^p-norm blow-up. Numerical examples for higher-dimensional blow-up solutions are presented and discussed.

Key Words.
blow-up, finite difference method, nonlinear heat equation, L^p-norm blow-up

AMS Subject Classifications.
65M06, 65M12

Abstract.
In this paper, two numerical schemes for a nonlinear integral equation of Fredholm type with weakly singular kernel are studied. These numerical methods blend collocation, convolution, and approximations based on sinc basis functions with iterative schemes like the steepest descent and Newton's method, involving the solution of a nonlinear system of equations. Exponential rate of convergence for the convolution scheme is shown and collocation method is analyzed. Numerical experiments are presented to illustrate the sharpness of the theoretical estimates and the sensitivity of the solutions with respect to some parameters in the equations. The comparison between the schemes indicates that the sinc convolution method is more effective.

Key Words.
Fredholm integral equation, Urysohn integral operator, weak singularity, convolution method, collocation method

AMS Subject Classifications.
45B05, 45E99, 65J15, 65R60.

Abstract.
We consider a Krylov subspace approximation method for the symmetric differential Riccati equation $\dot{X} = AX + XA^T + Q - XSX$, $X(0) = X_0$. The method we consider is based on projecting the large-scale equation onto a Krylov subspace spanned by the matrix A and the low-rank factors of X_0 and Q. We prove that the method is structure preserving in the sense that it preserves two important properties of the exact flow, namely the positivity of the exact flow and also the property of monotonicity. We provide a theoretical a priori error analysis that shows superlinear convergence of the method. Moreover, we derive an a posteriori error estimate that is shown to be effective in numerical examples.

Key Words.
differential Riccati equations, LQR optimal control problems, large-scale ordinary differential equations, Krylov subspace methods, matrix exponential, exponential integrators, model order reduction, low-rank approximation

AMS Subject Classifications.
65F10, 65F60, 65L20, 65M22, 93A15, 93C05

Abstract.
In this paper, we discuss the well-posedness of a mixed discontinuous Galerkin (DG) scheme for the Poisson and Stokes problems in 2D, considering only piecewise Lagrangian finite elements. The complication here lies in the fact that the classical Babuška-Brezzi theory is difficult to verify for low-order finite elements, so we proceed in a non-standard way. First, we prove uniqueness, and then we apply a discrete version of Fredholm's alternative theorem to ensure existence. The a-priori error analysis is done by introducing suitable projections of the exact solution. As a result, we prove that the method is convergent, and, under standard additional regularity assumptions on the exact solution, the optimal rate of convergence of the method is guaranteed.

Key Words.
discontinuous Galerkin, Lagrange shape functions, a-priori error estimates

AMS Subject Classifications.
65N30; 65N12; 65N15

Abstract.
Pole-swapping algorithms, which are generalizations of the QZ algorithm for the

Key Words.
cancer invasion, density-dependent diffusion, Faedo-Galerkin approximation, finite element method

AMS Subject Classifications.
35D30, 65M60, 35K57