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Contents

Abstract.
First-order system least squares methods have been recently proposed and analyzed for second order elliptic equations and systems. They produce symmetric and positive definite discrete systems by using standard finite element spaces which are not required to satisfy the inf-sup condition. In this paper, several domain decomposition algorithms for these first-order least squares methods are studied. Some representative overlapping and substructuring algorithms are considered in their additive and multiplicative variants. The theoretical and numerical results obtained show that the classical convergence bounds (on the iteration operator) for standard Galerkin discretizations are also valid for least squares methods. Therefore, domain decomposition algorithms provide parallel and scalable preconditioners also for least squares discretizations.

Key words.
domain decomposition, first-order system least squares.

AMS(MOS) subject classification.
65N30, 65N55.

Files.
vol.8.1999/pp1-14.dir/pp1-14.ps;
vol.8.1999/pp1-14.dir/pp1-14.pdf.

Forward References.

Abstract.
Recently, two Geršgorin-type matrix questions were raised. These are answered here, using ovals of Cassini.

Key words.
Geršgorin circle theorem, ovals of Cassini.

AMS(MOS) subject classification.
15A18.

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vol.8.1999/pp15-20.dir/pp15-20.ps;
vol.8.1999/pp15-20.dir/pp15-20.pdf.

Forward References.
vol.12.2001/pp113-133.dir/pp113-133.ps;
vol.12.2001/pp113-133.dir/pp113-133.pdf.

21 An optimum iteration for the matrix polar decomposition. *A. A. Dubrulle.*

Abstract.
It is shown that an acceleration parameter derived from the Frobenius norm makes Newton's iteration for the computation of the polar decomposition optimal and monotonic in norm. A simple machine-independent stopping criterion ensues. These features are extended to Gander's formulas for full-rank rectangular matrices.

Key words.
matrix polar decomposition, Newton iteration.

AMS(MOS) subject classification.
65F30, 65F35.

Files.
vol.8.1999/pp21-25.dir/pp21-25.ps;
vol.8.1999/pp21-25.dir/pp21-25.pdf.

Forward References.

26 Preconditioners for least squares problems by LU factorization. *A. Björck and J. Y. Yuan.*

Abstract.
Iterative methods are often suitable for solving least-squares problems $\min \|Ax - b\|_2$, where $A \in \mathbf{R}^{m\times n}$ is large and sparse. The use of the conjugate gradient method with a nonsingular square submatrix $A_1 \in \mathbf{R}^{n\times n}$ of A as preconditioner was first suggested by Läuchli in 1961. This conjugate gradient method has recently been extended by Yuan to generalized least-squares problems.
In this paper we consider the problem of finding a suitable submatrix A_1 and its LU factorization for a sparse rectangular matrix A. We give three algorithms based on the sparse LU factorization algorithm by Gilbert and Peierls.
Numerical results are given, which indicate that our preconditioners can be effective.

Key words.
Linear least squares, preconditioner, conjugate gradient method, LU factorization.

AMS(MOS) subject classification.
65F10, 65F20.

Files.
vol.8.1999/pp26-35.dir/pp26-35.ps;
vol.8.1999/pp26-35.dir/pp26-35.pdf.

Forward References.

36 On a posteriori error estimators in the finite element method on anisotropic meshes. *Manfred Dobrowolski, Steffen Gräf, and Christoph Pflaum.*

Abstract.
On anisotropic finite element meshes, the standard residual based error indicator is derived and it is proved that it is not efficient if the aspect ratio deteriorates. For a nonlocal error indicator it is proved that it is reliable and efficient independent of the aspect ratio. This is also confirmed by some numerical calculations.

Key words.
finite elements, a posteriori estimators, anisotropic meshes.

AMS(MOS) subject classification.
65N30, 65N15.

Files.
vol.8.1999/pp36-45.dir/pp36-45.ps;
vol.8.1999/pp36-45.dir/pp36-45.pdf.

Forward References.

46 On the convergence of multigrid methods for flow problems. *Ingemar Persson, Klas Samuelsson, and Anders Szepessy.*

Abstract.
We prove two theorems on the residual damping in multigrid methods when solving convection dominated diffusion equations and shock wave problems, discretized by the streamline diffusion finite element method. The first theorem shows that a V-cycle, including sufficiently many pre and post smoothing steps, damps the residual in L_1^{loc} for a constant coefficient convection problem with small diffusion in two space dimensions, without the assumption that the coarse grid is sufficiently fine. The proof is based on discrete Green's functions for the smoothing and correction operators on a uniform unbounded mesh aligned with the characteristic. The second theorem proves a similar result for a certain continuous version of a two grid method, with isotropic artificial diffusion, applied to a two dimensional Burgers shock wave problem. We also present numerical experiments that verify the residual damping dependence on the equation, the choice of artificial diffusion and the number of smoothing steps. In particular numerical experiments show improved convergence of the multigrid method, with damped Jacobi smoothing steps, for the compressible Navier-Stokes equations in two space dimensions by using the theoretically suggested exponential increase of the number of smoothing steps on coarser meshes, as compared to the same amount of work with constant number of smoothing steps on each level.

Key words.
multigrid methods, convergence, convection-diffusion, conservation laws, Green's function, shock waves.

AMS(MOS) subject classification.
65N55, 65N30, 35L65.

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vol.8.1999/pp46-87.dir/pp46-87.ps;
vol.8.1999/pp46-87.dir/pp46-87.pdf.

Forward References.

88 Numerical experiments with parallel orderings for ILU preconditioners. *Michele Benzi, Wayne Joubert, and Gabriel Mateescu.*

Abstract.
Incomplete factorization preconditioners such as ILU, ILUT and MILU are well-known robust general-purpose techniques for solving linear systems on serial computers. However, they are difficult to parallelize efficiently. Various techniques have been used to parallelize these preconditioners, such as multicolor orderings and subdomain preconditioning. These techniques may degrade the performance and robustness of ILU preconditionings. The purpose of this paper is to perform numerical

experiments to compare these techniques in order to assess what are the most effective ways to use ILU preconditioning for practical problems on serial and parallel computers.

Key words.
Krylov subspace methods, preconditioning, incomplete factorizations, sparse matrix orderings, additive Schwarz methods, parallel computing.

AMS(MOS) subject classification.
65F10, 65F15.

Files.
vol.8.1999/pp88-114.dir/pp88-114.ps;
vol.8.1999/pp88-114.dir/pp88-114.pdf.

Forward References.

115 A note on the numerical solution of complex Hamiltonian and skew-Hamiltonian eigenvalue problems. *Peter Benner, Volker Mehrmann and Hongguo Xu.*

Abstract.
In this paper we describe a simple observation that can be used to extend two recently proposed structure preserving methods for the eigenvalue problem for real Hamiltonian matrices to the case of complex Hamiltonian and skew-Hamiltonian matrices.

Key words.
eigenvalue problem, Hamiltonian matrix, skew-Hamiltonian matrix, algebraic Riccati equation, invariant subspace.

AMS(MOS) subject classification.
65F15, 93B40, 93B36, 93C60.

Files.
vol.8.1999/pp115-126.dir/pp115-126.ps;
vol.8.1999/pp115-126.dir/pp115-126.pdf.

Forward References.

127 Bounds for the minimum eigenvalue of a symmetric Toeplitz matrix. *Heinrich Voss.*

Abstract.
In a recent paper Melman [12] derived upper bounds for the smallest eigenvalue of a real symmetric Toeplitz matrix in terms of the smallest roots of rational and polynomial approximations of the secular equation $f(\lambda) = 0$, the best of which being constructed by the $(1, 2)$-Padé approximation of f. In this paper we prove that this bound is the smallest eigenvalue of the projection of the given eigenvalue problem onto a Krylov space of T_n^{-1} of dimension 3. This interpretation of the bound suggests enhanced bounds of increasing accuracy. They can be substantially improved further by exploiting symmetry properties of the principal eigenvector of T_n.

Key words.
Toeplitz matrix, eigenvalue problem, symmetry.

AMS(MOS) subject classification.
65F15.

Files.
vol.8.1999/pp127-137.dir/pp127-137.ps;
vol.8.1999/pp127-137.dir/pp127-137.pdf.

Forward References.

138 Discrete wavelet transforms accelerated sparse preconditioners for dense boundary element systems. *Ke Chen.*

Abstract.
We consider a construction of efficient preconditioners, using discrete and fast wavelet transforms, for dense and unsymmetric linear systems that arise from boundary elements. The wavelet compression property combined with operator splitting result in much improved preconditioners, in terms of both eigenspectra clustering and inverse approximations, taking the form of band matrices with wrap-around boundaries. With our new non-standard wavelet transform, the transformed matrix is permuted to band forms. It is shown that, to have band matrices, one has to use a smaller number of wavelet levels. Numerical experiments using the iterative methods of conjugate gradients based on the normal equations (CGN) and generalised minimal residuals (GMRES) are reported.

Key words.
fast wavelet transforms, dense linear systems, sparse preconditioners, conjugate gradient, boundary elements.

AMS(MOS) subject classification.
65F10, 65N38, 45E05.

Files.
vol.8.1999/pp138-153.dir/pp138-153.ps;
vol.8.1999/pp138-153.dir/pp138-153.pdf.

Forward References.

154 Whitney elements on pyramids. *V. Gradinaru and R. Hiptmair.*

Abstract.
Conforming finite elements in $\boldsymbol{H}(\mathrm{div};\Omega)$ and $\boldsymbol{H}(\mathbf{curl};\Omega)$ can be regarded as discrete differential forms (Whitney–forms). The construction of such forms is based on an interpolation idea, which boils down to a simple extension of the differential form to the interior of elements. This flexible approach can accommodate elements of more complicated shapes than merely tetrahedra and bricks. The pyramid serves as an example for the successful application of the construction: New Whitney forms are derived for it and they display all desirable properties of conforming finite elements.

Key words.
Whitney elements, edge elements, pyramidal element.

AMS(MOS) subject classification.
65N30 41A10 58A15.

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vol.8.1999/pp154-168.dir/pp154-168.ps;
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Forward References.