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Contents

1 Non–standard orthogonality for Meixner polynomials . *María Álvarez de Morales, Teresa E. Pérez, Miguel A. Piñar, André Ronveaux.*

Abstract.
In this work, we obtain a non–standard orthogonality property for Meixner polynomials $\{M_n^{(\gamma,\mu)}\}_{n\geq 0}$, with $0 < \mu < 1$ and $\gamma \in \mathbb{R}$, that is, we show that they are orthogonal with respect to some discrete inner product involving difference operators. The non–standard orthogonality can be used to recover properties of these Meixner polynomials, e. g., linear relations for the classical Meixner polynomials.

Key Words.
Meixner polynomials, inner product involving difference operators, non–standard orthogonality.

AMS(MOS) Subject Classifications.
33C45.

Files.
vol.9.1999/pp1-25.dir/pp1-25.ps; vol.9.1999/pp1-25.dir/pp1-25.pdf;

Forward References.

26 Computation of Gauss-Kronrod quadrature rules with non-positive weights. *G. S. Ammar, D. Calvetti, and L. Reichel.*

Abstract.
Recently Laurie presented a fast algorithm for the computation of $(2n + 1)$-point Gauss-Kronrod quadrature rules with real nodes and positive weights. We describe modifications of this algorithm that allow the computation of Gauss-Kronrod quadrature rules with complex conjugate nodes and weights or with real nodes and positive and negative weights.

Key Words.
orthogonal polynomials, indefinite measure, fast algorithm, inverse eigenvalue problem.

AMS(MOS) Subject Classifications.
65D32, 65F15, 65F18.

Files.
vol.9.1999/pp26-38.dir/pp26-38.ps; vol.9.1999/pp26-38.dir/pp26-38.pdf;

Forward References.

39 Quadrature formulas for rational functions. *F. Cala Rodriguez, P. Gonzalez–Vera, and M. Jimenez Paiz.*

Abstract.
Let ω be an L_1-integrable function on $[-1,1]$ and let us denote

$$I_\omega(f) = \int_{-1}^{1} f(x)\omega(x)dx,$$

where f is any bounded integrable function with respect to the weight function ω. We consider rational interpolatory quadrature formulas (RIQFs) where all the poles are preassigned and the interpolation is carried out along a table of points contained in $\overline{\mathbb{C}} \setminus [-1,1]$. The main purpose of this paper is the study of the convergence of the RIQFs to $I_\omega(f)$.

Key Words.
weight functions, interpolatory quadrature formulas, orthogonal polynomials, multipoint Padé–type approximants.

AMS(MOS) Subject Classifications.
41A21, 42C05, 30E10.

Files.
vol.9.1999/pp39-52.dir/pp39-52.ps; vol.9.1999/pp39-52.dir/pp39-52.pdf;

Forward References.

53 A footnote on quaternion block-tridiagonal systems. *Cecília Costa and Rogério Serôdio.*

Abstract.
Two direct methods, for solving a system of linear equations having matrices of quaternion entries as coefficients and independent terms, are proposed. Second-kind Chebyshev polynomials, in a matrix argument, are used as a tool to do certain calculations.

Key Words.
block tridiagonal matrices, quaternions, Chebyshev polynomials.

AMS(MOS) Subject Classifications.
65F05.

Files.
vol.9.1999/pp53-55.dir/pp53-55.ps; vol.9.1999/pp53-55.dir/pp53-55.pdf;

Forward References.

56 Sobolev orthogonal polynomials: interpolation and approximation. *Esther M. García–Caballero, Teresa E. Pérez, and Miguel A. Piñar .*

Abstract.
In this paper, we study orthogonal polynomials with respect to the bilinear form

$$(f,g)_S = (f(c_0), f(c_1), \dots, f(c_{N-1}))\mathbf{A}\begin{pmatrix} g(c_0) \\ g(c_1) \\ \vdots \\ g(c_{N-1}) \end{pmatrix} + \langle u, f^{(N)} g^{(N)} \rangle,$$

where u is a quasi-definite (or regular) linear functional on the linear space $\mathbb{P}$ of real polynomials, $c_0, c_1, \ldots, c_{N-1}$ are distinct real numbers, N is a positive integer number, and $\mathbf{A}$ is a real $N \times N$ matrix such that each of its principal submatrices are nonsingular. We show a connection between these non-standard orthogonal polynomials and some standard problems in the theory of interpolation and approximation.

Key Words.
Sobolev orthogonal polynomials, classical orthogonal polynomials, interpolation, approximation.

AMS(MOS) Subject Classifications.
33C45, 42C05.

Files.
vol.9.1999/pp56-64.dir/pp56-64.ps; vol.9.1999/pp56-64.dir/pp56-64.pdf;

Forward References.

Abstract.
Various concepts of orthogonality on the real line are reviewed that arise in connection with quadrature rules. Orthogonality relative to a positive measure and Gauss-type quadrature rules are classical. More recent types of orthogonality include orthogonality relative to a sign-variable measure, which arises in connection with Gauss-Kronrod quadrature, and power (or implicit) orthogonality encountered in Turán-type quadratures. Relevant questions of numerical computation are also considered.

Key Words.
orthogonal polynomials, Gauss-Lobatto, Gauss-Kronrod, and Gauss-Turán rules, computation of Gauss-type quadrature rules.

AMS(MOS) Subject Classifications.
33C45, 65D32, 65F15.

Files.
vol.9.1999/pp65-76.dir/pp65-76.ps; vol.9.1999/pp65-76.dir/pp65-76.pdf;

Forward References.

Abstract.
This article was presented as a Plenary Talk at the IWOP 98, Madrid. An introduction to general purpose computer algebra systems is given, and the possibilities to use computer algebra in the work with orthogonal polynomials and special functions are discussed in detail.

Key Words.
orthogonal polynomials, special functions, computer algebra, Maple V, holonomic functions, differential equations, recurrence equations, hypergeometric functions, Zeilberger's algorithm, Gosper's algorithm, Petkovšek's algorithm.

AMS(MOS) Subject Classifications.
33-02, 33C45, 11B37, 34A30, 33C20, 68Q40.

Files.
vol.9.1999/pp77-101.dir/pp77-101.ps; vol.9.1999/pp77-101.dir/pp77-101.pdf;

Forward References.

102 Creation and annihilation operators for orthogonal polynomials of continuous and discrete variables. *Miguel Lorente.*

Abstract.
We develop general expressions for the raising and lowering operators that belong to the orthogonal polynomials of hypergeometric type with discrete and continuous variable. We construct the creation and annihilation operators that correspond to the normalized polynomials and study their algebraic properties in the case of the Kravchuk/Hermite Meixner/Laguerre polynomials.

Key Words.
orthogonal polynomials, difference equations, creation and annihilation operators.

AMS(MOS) Subject Classifications.
33C45, 33C80, 81R50.

Files.
vol.9.1999/pp102-111.dir/pp102-111.ps; vol.9.1999/pp102-111.dir/pp102-111.pdf;

Forward References.

112 $q-$Classical orthogonal polynomials: a very classical approach . *F. Marcellán and J.C. Medem.*

Abstract.
The $q-$classical orthogonal polynomials defined by Hahn satisfy a Sturm-Liouville type equation in geometric differences. Working with this, we classify the $q-$classical polynomials in twelve families according to the zeros of the polynomial coefficients of the equation and the behavior concerning to q^{-1}. We determine a $q-$analogue of the weight function for the twelve families, and we give a representation of its orthogonality relation and its $q-$integral. We describe this representation in some normal and special cases (indeterminate moment problem and finite orthogonal sequences). Finally, the Sturm-Liouville type equation allows us to establish the correspondence between this classification and the Askey Scheme.

Key Words.
orthogonal $q-$polynomials, classical polynomials.

AMS(MOS) Subject Classifications.
33D25.

Files.
vol.9.1999/pp112-127.dir/pp112-127.ps; vol.9.1999/pp112-127.dir/pp112-127.pdf;

Forward References.

128 An iterative method for computing the eigenvalues of second kind Fredholm operators and applications. *P. Natalini, S. Noschese, and P.E. Ricci.*

Abstract.
An inverse iteration method is used to approximate the eigenvalues of second kind Fredholm operators. Numerical examples are given.

Key Words.
Fredholm Integral equation, eigenvalues, inverse iteration method, Rayleigh-Ritz method.

AMS(MOS) Subject Classifications.
45C05, 45B05, 65R20.

Files.
vol.9.1999/pp128-136.dir/pp128-136.ps; vol.9.1999/pp128-136.dir/pp128-136.pdf;

Forward References.

137 Evaluation of associated Legendre functions off the cut and parabolic cylinder functions. *Javier Segura and Amparo Gil.*

Abstract.
We review a set of algorithms to evaluate associated Legendre functions off the cut; in particular, we consider prolate spheroidal, oblate spheroidal and toroidal harmonics. A similar scheme can be applied to other families of special functions like Bessel and parabolic cylinder functions; we will describe the corresponding algorithm for the evaluation of parabolic cylinder functions.

Key Words.
computation of special functions, Legendre functions, parabolic cylinder functions.

AMS(MOS) Subject Classifications.
65D20, 33-04, 33C05, 33A70.

Files.
vol.9.1999/pp137-146.dir/pp137-146.ps; vol.9.1999/pp137-146.dir/pp137-146.pdf;

Forward References.

147 Proof of a conjecture of Chan, Robbins, and Yuen. *Doron Zeilberger.*

Abstract.
Using the celebrated Morris Constant Term Identity, we deduce a recent conjecture of Chan, Robbins, and Yuen (math.CO/9810154), that asserts that the volume of a certain $n(n-1)/2$-dimensional polytope is given in terms of the product of the first $n-1$ Catalan numbers.

Key Words.
Combinatorics, Catalan numbers, polytope.

AMS(MOS) Subject Classifications.
05-XX, 52B05.

Files.
vol.9.1999/pp147-148.dir/pp147-148.ps; vol.9.1999/pp147-148.dir/pp147-148.pdf;
Forward References.