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On Class of Hypergeometric Meromorphic Functions with Fixed Second Positive Coefficients

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Abstract

In the present paper, we consider the class of hypergeometric meromorphic functions $\Sigma^*(A, B, k, c)$ with fixed second positive coefficient. The object of the present paper is to obtain the coefficient estimates, convex linear combinations, distortion theorems, and radii of starlikeness and convexity for f in the class $\Sigma^*(A, B, k, c)$.

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1 Introduction

Let Σ denote the class of meromorphic functions f normalized by

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{1}$$

that are analytic and univalent in the punctured unit disk $U = \{z : 0 < |z| < 1\}$. For $0 \leq \beta < 1$, we denote by $S^*(\beta)$ and $k(\beta)$, the subclasses of Σ consisting of all meromorphic functions that are, respectively, starlike of order β and convex of order β in U (cf. e.g., [[1, 3, 5, 16]]).

For functions $f_j(z)(j = 1; 2)$ defined by

$$f_j(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,j} z^n, \tag{2}$$

we denote the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$ by

$$(f_1 * f_2) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^n. \tag{3}$$

Let us define the function $\tilde{\phi}(a, c; z)$ by

$$\tilde{\phi}(a, c; z) = \frac{1}{z} + \sum_{n=0}^{\infty} \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| a_n z^n, \tag{4}$$

for $c \neq 0, -1, -2, ...,$ and $a \in \mathbb{C}/\{0\}$, where $(\lambda)_n = \lambda(\lambda + 1)_{n+1}$ is the Pochhammer symbol. We note that

$$\tilde{\phi}(a, c; z) = \frac{1}{z} {}_2F_1(1, a, c; z)$$

where

$${}_2F_1(b, a, c; z) = \sum_{n=0}^{\infty} \frac{(b)_n (a)_n}{(c)_n} \frac{z^n}{n!}$$

is the well-known Gaussian hypergeometric function. Corresponding to the function $\tilde{\phi}(a,c;z)$, using the Hadamard product for $f \in \Sigma$, we define a new linear operator $L^*(a,c)$ on Σ by

$$L^*\left(a,c\right)f\left(z\right)=\tilde{\phi}\left(a,c;z\right)*f\left(z\right)=\frac{1}{z}+\sum_{n=1}^{\infty}\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right|a_n z^n. \tag{5}$$

The meromorphic functions with the generalized hypergeometric functions were considered recently by Dziok and Srivastava [6], [7], Liu [10], Liu and Srivastava [11], [12],[13], Cho and Kim [4] .

For a function $f \in L^*\left(a,c\right)f\left(z\right)$ we define

$$I^0\left(L^*\left(a,c\right)f\left(z\right)\right)=L^*\left(a,c\right)f\left(z\right),$$

and for $k = 1, 2, 3, ...,$

$$\begin{aligned} I^k\left(L^*\left(a,c\right)f\left(z\right)\right) &= z\left(I^{k-1}L^*\left(a,c\right)f\left(z\right)\right)'+\frac{2}{z} \\ &= \frac{1}{z}+\sum_{n=1}^{\infty}n^k\left|\frac{(a)_{n+1}}{(c)_{n+1}}\right|a_n z^n. \end{aligned} \tag{6}$$

We note that $I^k\left(L^*\left(a,a\right)f\left(z\right)\right)$ studied by Frasin and Darus [8].

It follows from (5) that

$$z\left(L(a,c)f(z)\right)'=aL(a+1,c)f(z)-\left(a+1\right)L(a,c)f(z). \tag{7}$$

Also, from (6) and (7) we get

$$z\left(I^kL(a,c)f(z)\right)'=aI^kL(a+1,c)f(z)-\left(a+1\right)\ I^kL(a,c)f(z). \tag{8}$$

Now, let $-1 \le B < A \le 1$ and for all $z \in U$, a function $f \in \Sigma$ is said to be a member of the class $\Sigma^*\left(A,B,k\right)$ if it satisfies

$$\left|\frac{z\left(I^kL^*\left(a,c\right)f(z)\right)'+\ I^kL^*\left(a,c\right)f(z)}{Bz\left(I^kL^*\left(a,c\right)f(z)\right)'+A\ \left(I^kL^*\left(a,c\right)f(z)\right)}\right|<1.$$

Note that, for $a = c$, $\Sigma^* (1 - 2\alpha, -1, k)$ with $0 \le \alpha < 1$, is the class introduced and studied in [8]. In the following section, we will state a result studied previously by Ghanim, Darus and Swaminathan [9].

2 Preliminary results

For the class $\Sigma^*(A, B, k)$, Ghanim, Darus and Swaminathan [9] showed:

Theorem 1 *Let the function f be defined by* (5). *If*

$$\sum_{n=1}^{\infty} n^k \left| \frac{(a)_{n+1}}{(c)_{n+1}} \right| (n(1-B) + (1-A)) |a_n| \le A - B, \tag{9}$$

where $k \in N_0$, $-1 \le B < A \le 1$, then $f \in \Sigma^*(A, B, k)$.

In view of Theorem 1, we can see that the function f given by (5) is in the class $\Sigma^*(A, B, k)$ satisfying

$$a_n \le \frac{|(c)_{n+1}| (A - B)}{|(a)_{n+1}| n^k (n(1-B) + (1-A))}, \quad (n \ge 1,\ k \in N_0). \tag{10}$$

In view of (9), we can see that the function f defined by (5) is in the class $\Sigma^*(A, B, k)$ satisfying the coefficient inequality

$$\frac{|(a)_2|}{|(c)_2|} a_1 \le \frac{(A - B)}{(2 - (A + B))}. \tag{11}$$

Hence we may take

$$\frac{|(a)_2|}{|(c)_2|} a_1 = \frac{(A - B)\, c}{(2 - (A + B))}, \quad \text{for some c}\,(0 < c < 1). \tag{12}$$

Making use of (12), we now introduce the following class of functions:
Let $\Sigma^*(A,B,k,c)$ denote the class of functions f in $\Sigma^*(A,B,k)$ of the form

$$f(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))}z + \sum_{n=2}^{\infty} \frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\left|a_n\right| z^n \tag{13}$$

with $0 < c < 1$.

In this paper we obtain coefficient estimates, convex linear combination, distortion theorem, and radii of starlikeness and convexity for f to be in the class $\Sigma^*(A,B,k,c)$.

There are many studies regarding the fixed second coefficient see for example: Aouf and Darwish [2], Silverman and Silvia [14], and Uralegaddi [15], few to mention. We shall use similar techniques to prove our results.

3 Coefficient inequalities

Theorem 2 *A function f defined by (13) is in the class $\Sigma^*(A,B,k,c)$, if and only if,*

$$\sum_{n=2}^{\infty} n^k \frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\left(n\left(1-B\right)+\left(1-A\right)\right)\left|a_n\right| \leq \left(A-B\right)\left(1-c\right). \tag{14}$$

The result is sharp.

Proof. By putting

$$\frac{\left|(a)_2\right|}{\left|(c)_2\right|}a_1 = \frac{(A-B)\,c}{(2-(A+B))}, \qquad 0 < c < 1 \tag{15}$$

in (9), the result is easily derived. The result is sharp for function

$$f_n(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))}z + \tag{16}$$

$$\frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\frac{(A-B)\,(1-c)}{n^k\,(n\,(1-B)+(1-A))}z^n, \qquad n \geq 2.$$

Corollary 1 *Let the function f given by (13) be in the class $\Sigma^*(A,B,k,c)$, then*

$$a_n \leq \frac{(c)_{n+1}}{(a)_{n+1}}\frac{(A-B)\,(1-c)}{n^k\,(n\,(1-B)+(1-A))}, \qquad n \geq 2. \tag{17}$$

The result is sharp for the function f given by (16).

4 Growth and distortion theorems

A growth and distortion property for function f to be in the class $\Sigma^*(A,B,k,c)$ is given as follows:

Theorem 3 *If the function f defined by (13) is in the class $\Sigma^*(A,B,k,c)$ for $0 < |\mathrm{z}| = r < 1$, then we have*

$$\frac{1}{r} - \frac{(A-B)\,c}{(2-(A+B))}r - \frac{(A-B)\,(1-c)}{(3-(2B+A))}r^2 \leq |f(z)|$$

$$\leq \frac{1}{r} + \frac{(A-B)\,c}{(2-(A+B))}r + \frac{(A-B)\,(1-c)}{(3-(2B+A))}r^2$$

with equality for

$$f_2(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))}z + \frac{(A-B)\,(1-c)}{(3-(2B+A))}z^2.$$

Proof. Since $\Sigma^*(A,B,k,c)$, Theorem 2 yields to the inequality

$$\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n \le \frac{(A-B)\,(1-c)}{n^k\,(n\,(1-B)+(1-A))}, \qquad n \ge 2. \tag{18}$$

Thus, for $0<|\mathrm{z}|=r<1$

$$|f(z)| \le \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))}z + \sum_{n=2}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n z^n$$

$|\mathrm{z}|=r$

$$\le \frac{1}{r} + \frac{(A-B)\,c}{(2-(A+B))}r + r^2\sum_{n=2}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n$$

$$\le \frac{1}{r} + \frac{(A-B)\,c}{(2-(A+B))}r + \frac{(A-B)\,(1-c)}{(3-(2B+A))}r^2$$

and

$$|f(z)| \ge \frac{1}{z} - \frac{(A-B)\,c}{(2-(A+B))}z - \sum_{n=2}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n z^n, \quad (|z|=r)$$

$$\ge \frac{1}{r} - \frac{(A-B)\,c}{(2-(A+B))}r - r^2\sum_{n=2}^{\infty}\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n$$

$$\ge \frac{1}{r} - \frac{(A-B)\,c}{(2-(A+B))}r - \frac{(A-B)\,(1-c)}{(3-(2B+A))}r^2$$

Thus the proof of the theorem is complete.

Theorem 4 *If the function $f(z)$ defined by (13) is in the class $\Sigma^*(A,B,k,c)$ for $0<|\mathrm{z}|=r<1$, then we have*

$$\frac{1}{r^2} - \frac{(A-B)\,c}{(2-(A+B))} - \frac{(A-B)\,(1-c)}{(3-(2B+A))}r \le \left|f'(z)\right|$$

$$\leq \frac{1}{r^2} + \frac{(A-B)\,c}{(2-(A+B))} + \frac{(A-B)\,(1-c)}{(3-(2B+A))}r.$$

with equality for

$$f_2(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))}z + \frac{(A-B)\,(1-c)}{(3-(2B+A))}z^2.$$

Proof. From Theorem 2, it follows that

$$n\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n \leq \frac{(A-B)\,(1-c)}{n^{k-1}\,(n\,(1-B)+(1-A))}, \qquad n \geq 2. \tag{19}$$

Thus, for $0 < |\mathrm{z}| = r < 1$, and making use of (19), we obtain

$$|f'(z)| \leq \left|\frac{-1}{z^2}\right| + \frac{(A-B)\,c}{(2-(A+B))} + \sum_{n=2}^{\infty} n\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n\,|z|^{n-1}, \qquad (|z| = r)$$

$$\leq \frac{1}{r^2} + \frac{(A-B)\,c}{(2-(A+B))} + r\sum_{n=2}^{\infty} n\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n$$

$$\leq \frac{1}{r^2} + \frac{(A-B)\,c}{(2-(A+B))} + \frac{(A-B)\,(1-c)}{(3-(2B+A))}r.$$

and

$$|f'(z)| \geq \left|\frac{-1}{z^2}\right| - \frac{(A-B)\,c}{(2-(A+B))} - \sum_{n=2}^{\infty} n\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n\,|z|^{n-1}, \qquad (|z| = r)$$

$$\geq \frac{1}{r^2} - \frac{(A-B)\,c}{(2-(A+B))} - r\sum_{n=2}^{\infty} n\frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|}a_n$$

$$\geq \frac{1}{r^2} - \frac{(A-B)\,c}{(2-(A+B))} - \frac{(A-B)\,(1-c)}{(3-(2B+A))}r.$$

The proof is complete.

5 Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class $\Sigma^*(A, B, k, c)$ is given by the following theorem:

Theorem 5 *If the function f given by (13) is in the class $\Sigma^*(A, B, k, c)$, then f is starlike of order $\delta(0 \leq \delta \leq 1)$ in the disk $|z| < r_1(A, B, k, c, \delta)$ where $r_1(A, B, k, c, \delta)$ is the largest value for which*

$$\frac{(3-\delta)(A-B)c}{(2-(A+B))}r^2 + \frac{(n+2-\delta)(A-B)(1-c)}{n^k(n(1-B)+(1-A))}r^{n+1} \leq (1-\delta).$$

for $n \geq 2$. The result is sharp for function $f_n(z)$ given by (16).

Proof. It is enough to highlight that

$$\left|\frac{(z)f'(z)}{f(z)} + 1\right| \leq 1-\delta$$

for $|z| < r_1$. we have

$$\left|\frac{(z)f'(z)}{f(z)} + 1\right| = \left|\frac{\frac{2(A-B)c}{(2-(A+B))}z + \sum_{n=2}^{\infty}(n+1)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n z^n}{\frac{1}{z} - \frac{(A-B)c}{(2-(A+B))}z - \sum_{n=2}^{\infty}\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n z^n}\right| \leq 1-\delta. \tag{20}$$

Hence (20) holds true if

$$\frac{2(A-B)c}{(2-(A+B))}r^2 + \sum_{n=2}^{\infty}(n+1)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}|a_n|r^{n+1}$$

$$\leq (1-\delta)\left(1 - \frac{(A-B)c}{(2-(A+B))}r^2 - \sum_{n=2}^{\infty}\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n r^{n+1}\right). \tag{21}$$

or

$$\frac{(3-\delta)(A-B)c}{(2-(A+B))}r^2 + \sum_{n=2}^{\infty}(n+2-\delta)\frac{|(a)_{n+1}|}{|(c)_{n+1}|}a_n r^{n+1} \leq (1-\delta) \tag{22}$$

and it follows that from (14), we may take

$$a_n \leq \frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|} \frac{(A-B)(1-c)}{n^k\left(n(1-B)+(1-A)\right)} \lambda_n, \qquad (n \geq 2). \tag{23}$$

where $\lambda_n \geq 0$ and $\sum_{n=2}^{\infty} \lambda_n \leq 1$.
For each fixed r, we choose the positive integer $n_o = n_o(r)$ for which

$$\frac{n+2-\delta}{n^k\left(n(1-B)+(1-A)\right)} \left|\frac{(a)_{n+1}}{(c)_{n+1}}\right| r^{n+1}$$

is maximal. Then it follows that

$$\sum_{n=2}^{\infty} (n+2-\delta) \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} a_n r^{n+1} \tag{24}$$

$$\leq \frac{(n_o+2-\delta)(A-B)(1-c)}{n_o^k\left(n_o(1-B)+(1-A)\right)} r^{n_o+1}.$$

Then f is starlike of order δ in $0 < |z| < r_1(A, B, k, c, \delta)$ provided that

$$\frac{(3-\delta)(A-B)c}{(2-(A+B))} r^2 + \frac{(n_o+2-\delta)(A-B)(1-c)}{n_o^k\left(n_o(1-B)+(1-A)\right)} r^{n_o+1} \tag{25}$$

$$\leq (1-\delta)$$

we find the value $r_o = r_o(k, \beta, c, \delta, n)$ and the corresponding integer $n_o(r_o)$ so that

$$\frac{(3-\delta)(A-B)c}{(2-(A+B))} r^2 + \frac{(n_o+2-\delta)(A-B)(1-c)}{n_o^k\left(n_o(1-B)+(1-A)\right)} r^{n_o+1} \tag{26}$$

$$= (1-\delta)$$

Then this value is the radius of starlikeness of order δ for function f belonging to the class $\Sigma^*(A, B, k, c)$.

Theorem 6 *If the function f given by (13) is in the class $\Sigma^*(A, B, k, c)$, then f is convex of order $\delta(0 \leq \delta \leq 1)$ in the disk $|\mathrm{z}| < r_2(A, B, k, c, \delta)$ where $r_2(A, B, k, c, \delta)$ is the largest value for which*

$$\frac{(3-\delta)(A-B)c}{(2-(A+B))}r^2 + \frac{(n+2-\delta)(A-B)(1-c)}{n^{k-1}(n(1-B)+(1-A))}r^{n+1} \leq (1-\delta).$$

The result is sharp for function f_n given by (16).

Proof. By using the same technique in the proof of theorem (5) we can show that

$$\left|\frac{(z)f''(z)}{f'(z)} + 2\right| \leq (1-\delta).$$

for $|\mathrm{z}| < r_2$ with the aid of Theorem 2. Thus , we have the assertion of Theorem 6.

6 Convex Linear Combination

Our next result involves a linear combination of function of the type (13).

Theorem 7 *If*

$$f_1(z) = \frac{1}{z} + \frac{(A-B)c}{(2-(A+B))}z \tag{27}$$

and

$$f_n = \frac{1}{z} + \frac{(A-B)c}{(2-(A+B))}z + \tag{28}$$

$$\sum_{n=2}^{\infty}\frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\frac{(A-B)(1-c)}{(n(1-B)+(1-A))}z^n, \qquad n \geq 2.$$

Then $f \in \Sigma^*(A,B,k,c)$ if and only if it can expressed in the form

$$f(z)=\sum_{n=2}^{\infty}\lambda_n f_n(z) \tag{29}$$

where $\lambda_n \geq 0$ and $\sum\limits_{n=2}^{\infty}\lambda_n \leq 1$.

Proof. From (27),(28) and (29), we have

$$f(z)=\sum_{n=2}^{\infty}\lambda_n f_n(z)=\frac{1}{z}+\frac{(A-B)\,c}{(2-(A+B))}z+$$

$$\sum_{n=2}^{\infty}\frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\frac{(A-B)\,(1-c)\,\lambda_n}{(n\,(1-B)+(1-A))}z^n.$$

Since

$$\sum_{n=2}^{\infty}\frac{\left|(c)_{n+1}\right|(A-B)\,(1-c)\,\lambda_n}{\left|(a)_{n+1}\right|(n\,(1-B)+(1-A))}.\frac{\left|(a)_{n+1}\right|(n\,(1-B)+(1-A))}{\left|(c)_{n+1}\right|(A-B)\,(1-c)}$$

$$=\sum_{n=2}^{\infty}\lambda_n=1-\lambda_1\ \leq 1$$

it follows from Theorem 2 that the function $f \in \Sigma^*(A,B,k,c)$.

Conversely, let us suppose that $f \in \Sigma^*(A,B,k,c)$. Since

$$a_n \leq \frac{\left|(c)_{n+1}\right|}{\left|(a)_{n+1}\right|}\frac{(A-B)\,(1-c)}{n^k\,(n\,(1-B)+(1-A))},\qquad (n\geq 2)\,.$$

Setting

$$\lambda_n=\frac{n^k\left|(a)_{n+1}\right|(n\,(1-B)+(1-A))}{\left|(c)_{n+1}\right|(A-B)\,(1-c)}a_n.$$

and

$$\lambda_1 = 1 - \sum_{n=2}^{\infty} \lambda_n$$

It follows that

$$f(z) = \sum_{n=2}^{\infty} \lambda_n f_n(z)$$

Thus complete the proof of the theorem.

Theorem 8 *The class $\Sigma^*(A, B, k, c)$ is closed under linear combination.*

Proof. Suppose that the function f be given by (13), and let the function g be given by

$$g(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))} z + \sum_{n=2}^{\infty} |b_n| z^n, \qquad (b_n \geq 2)\,.$$

Assuming that f and g are in the class $\Sigma^*(A, B, k, c)$, it is enough to prove that the function H defined by

$$H(z) = \lambda f(z) + (1-\lambda)\, g(z) \qquad (0 \leq \lambda \leq 1)$$

is also in the class $\Sigma^*(A, B, k, c)$.

Since

$$H(z) = \frac{1}{z} + \frac{(A-B)\,c}{(2-(A+B))} z + \sum_{n=2}^{\infty} |a_n \lambda + (1-\lambda)\, b_n| z^n,$$

we observe that

$$\sum_{n=2}^{\infty} \frac{\left|(a)_{n+1}\right|}{\left|(c)_{n+1}\right|} \left[n^k \left(n\,(1-B) + (1-A)\right)\right] |a_n \lambda + (1-\lambda)\, b_n| \leq (A-B)\,(1-c)\,.$$

with the aid of Theorem 2. Thus $H \in \Sigma^*(A, B, k, c)$. Hence the theorem.

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