Gen. Math. Notes, Vol. 2, No. 2, February 2011, pp.102-110
ISSN 2219-7184; Copyright ©ICSRS Publication, 2011
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Truncated Hypergeometric Series Motivated by the Works of Slater and Verma

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(Received 27-11-10/ Accepted 31-12-10)

Abstract

Motivated by the works of L.J. Slater and A. Verma, we have derived some results on truncated unilateral generalized hypergeometric series of positive unit argument subject to certain conditions in numerator and denominator parameters. The results presented here are presumably new.

Keywords: *Pochhammer symbol, Gaussian ordinary hypergeometric function, Truncated unilateral series.*

1 Introduction

Terminating Generalized Hypergeometric Series

$$ {}_AF_B\left[\begin{array}{ll} -N,\, a_2,\, a_3,\, \ldots,\, a_A & ; \\ & \quad z \\ b_1,\, b_2,\, \ldots,\, b_B & ; \end{array}\right] = \sum_{k=0}^{N} \frac{(-N)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k\, k!} \qquad (1.1) $$

where $N \in \{1, 2, 3, \dots\}$, remaining numerator and denominator parameters $a_2, a_3, \cdots, a_A, b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and Pochhammer's symbol $(c)_k$ is given by $(c)_k = \prod_{j=0}^{k-1}(c+j)$.

Similarly

$$ {}_AF_B\left[\begin{array}{lr} -M,\, -N,\, a_3,\, a_4,\, \ldots,\, a_A \;; & \\ & z \\ b_1,\, b_2,\, \ldots,\, b_B \;; & \end{array}\right] = \sum_{k=0}^{\min\{M,N\}} \frac{(-M)_k(-N)_k(a_3)_k(a_4)_k\cdots(a_A)_k\, z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k\, k!} \tag{1.2} $$

where $M, N \in \{1, 2, 3, \ldots\}$ and remaining numerator and denominator parameters $a_3, a_4, \cdots, a_A,\ b_1, b_2, \cdots, b_B$ are neither zero nor negative integers.

Truncated Generalized Hypergeometric Series[3,p.83(2.6.1.1)]

$$ {}_AF_B\left[\begin{array}{lr} (a_A) \;; & \\ & z \\ (b_B) \;; & \end{array}\right]_N = {}_AF_B\left[\begin{array}{lr} a_1,\, a_2,\, \ldots,\, a_A \;; & \\ & z \\ b_1,\, b_2,\, \ldots,\, b_B \;; & \end{array}\right] \text{to } (N+1) \text{ terms} $$

$$ = \sum_{k=0}^{N} \frac{\prod_{j=1}^{A}(a_j)_k\, z^k}{\prod_{j=1}^{B}(b_j)_k k!} \tag{1.3} $$

where numerator and denominator parameters are neither zero nor negative integers and $A,\ B$ are non-negative integers. When $N \to \infty$ then (1.3) reduces to a non terminating generalized hypergeometric series.

Suppose M and N are positive integers such that $M < N$, then

$$ {}_AF_B\left[\begin{array}{lr} -N,\, a_2,\, \ldots,\, a_A \;; & \\ & z \\ b_1,\, b_2,\, \ldots,\, b_B \;; & \end{array}\right] \text{to } (M+1) \text{ terms} $$

$$ = {}_AF_B\left[\begin{array}{lr} -N,\, a_2,\, \ldots,\, a_A \;; & \\ & z \\ b_1,\, b_2,\, \ldots,\, b_B \;; & \end{array}\right]_M = \sum_{k=0}^{M} \frac{(-N)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k\, k!} \tag{1.4} $$

The symbol $S_r(c_1, c_2, c_3, \ldots, c_B)$ represents the sum of all possible combinations of the products of parameters taken "r" at a time from the set of "B" parameters $\{c_1, c_2, c_3, \ldots, c_B\}$.

For the sake of convenience $S_r(a_0, a_1, a_2, \ldots, a_A)$ and $S_r(b_1, b_2, \ldots, b_A)$ are abbreviated by $S_r(a_0, (a_A))$ and $S_r((b_A))$ respectively, in (1.6), (1.7), (1.9), (1.10) and (1.11).

Summation Theorem of Slater[3,pp.83-84(2.6.1.1, 2.6.1.7); see also 2].

$$ {}_{A+1}F_A\left[\begin{array}{lr} a_0,\, a_1,\, a_2,\, \ldots,\, a_A \;; & \\ & 1 \\ 1+b_1,\, 1+b_2,\, \ldots,\, 1+b_A \;; & \end{array}\right]_N $$

$$= \frac{(1+a_0)_N\ (1+a_1)_N\ (1+a_2)_N \cdots (1+a_A)_N}{N!\ (1+b_1)_N\ (1+b_2)_N\ \cdots (1+b_A)_N} \tag{1.5}$$

subject to "A" number of conditions [3,p.84(2.6.1.4),(2.6.1.5),(2.6.1.8)] in compact notation, given by (1.6)

$$S_r(a_0,(a_A)) = S_r((b_A)),\ A, N \in \{1,2,3,\dots\} \text{ and } r = 1,2,3,\dots,A \tag{1.6}$$

and

$$S_{A+1}(a_0,(a_A)) \neq 0 \tag{1.7}$$

Summation Theorem of Verma[1,p.233(3.3)].

$${}_{A+2}F_{A+1}\left[\begin{array}{lr} a_0,\ a_1,\ a_2,\ \dots,\ a_A,\ 1-\alpha^* & ; \\ & 1 \\ 1+b_1,\ 1+b_2,\ \dots,\ 1+b_A,\ -\alpha^* & ; \end{array}\right]_N$$

$$= \frac{(1+a_0)_N\ (1+a_1)_N\ (1+a_2)_N \cdots (1+a_A)_N}{N!\ (1+b_1)_N\ (1+b_2)_N \cdots (1+b_A)_N} \tag{1.8}$$

provided with "$(A-1)$" number of conditions in compact notation, given by (1.9)

$$S_r(a_0,(a_A)) = S_r((b_A)),\ A \in \{2,3,4,\dots\} \text{ and } r = 1,2,3,\dots,(A-1) \tag{1.9}$$

and

$$S_A(a_0,(a_A)) \neq S_A((b_A)),\ S_{A+1}(a_0,(a_A)) \neq 0,\ N \in \{1,2,3,\dots\} \tag{1.10}$$

where

$$\alpha^* = -\ \frac{S_{A+1}(a_0,(a_A))}{\{S_A(a_0,(a_A)) - S_A((b_A))\}} \tag{1.11}$$

In next sections motivated by the above works (1.5) of Slater and (1.8) of Verma, we have established some summation theorems for truncated unilateral generalized hypergeometric series, by using series iteration techniques and theory of polynomial equations.

2 Main Result

Let S_{m+1} represent the sum of first $(m+1)$ terms of a series and T_{m+1} represent $(m+1)$th term of the same series from the beginning.
Suppose

$$S_{m+1} = \frac{(2+c)_{2m}(2+g_1)_{2m}(2+g_2)_{2m}\cdots(2+g_B)_{2m}}{(2+s)_{2m}(2+h_1)_{2m}(2+h_2)_{2m}\cdots(2+h_B)_{2m}} \tag{2.1}$$

when $m = 0$, we get $S_1 = T_1 = 1$

For $m = 1, 2, 3, \ldots$, we have

$$T_{m+1} = S_{m+1} - S_m = \frac{(2+c)_{2m-2}(2+g_1)_{2m-2}(2+g_2)_{2m-2}\cdots(2+g_B)_{2m-2}}{(2+s)_{2m}(2+h_1)_{2m}(2+h_2)_{2m}\cdots(2+h_B)_{2m}}\times$$
$$\times[(c+2m)(1+c+2m)(g_1+2m)(1+g_1+2m)\cdots(g_B+2m)(1+g_B+2m)-$$
$$-(s+2m)(1+s+2m)(h_1+2m)(1+h_1+2m)\cdots(h_B+2m)(1+h_B+2m)] \quad (2.2)$$

$$T_{m+1} = \frac{(2+c)_{2m-2}(2+g_1)_{2m-2}(2+g_2)_{2m-2}\cdots(2+g_B)_{2m-2}}{(2+s)_{2m}(2+h_1)_{2m}\ (2+h_2)_{2m}\cdots(2+h_B)_{2m}}\times[\text{A Polynomial } W^* \text{ in } m] \quad (2.3)$$

where

$$W^* = [\{(J_1+K_1)-(P_1+Q_1)\}(2m)^{2B+1}+\{(J_2+K_2+J_1K_1)-(P_2+Q_2+P_1Q_1)\}(2m)^{2B}+$$
$$+\{(J_3+K_3+J_1K_2+J_2K_1)-(P_3+Q_3+P_1Q_2+P_2Q_1)\}(2m)^{2B-1}+\cdots+$$
$$+\{(J_{B+1}+K_{B+1}+J_1K_B+J_2K_{B-1}+\cdots+J_BK_1)-(P_{B+1}+Q_{B+1}+P_1Q_B+P_2Q_{B-1}+\cdots+P_BQ_1)\}(2m)^{B+1}+$$
$$+\{(J_{B+1}K_1+J_BK_2+J_{B-1}K_3+\cdots+J_1K_{B+1})-(P_{B+1}Q_1+P_BQ_2+P_{B-1}Q_3+\cdots+P_1Q_{B+1})\}(2m)^{B}+$$
$$+\{(J_{B+1}K_2+J_BK_3+J_{B-1}K_4+\cdots+J_2K_{B+1})-(P_{B+1}Q_2+P_BQ_3+P_{B-1}Q_4+\cdots+P_2Q_{B+1})\}(2m)^{B-1}+\cdots$$
$$+\{(J_{B+1}K_{B-1}+J_BK_B+J_{B-1}K_{B+1})-(P_{B+1}Q_{B-1}+P_BQ_B+P_{B-1}Q_{B+1})\}(2m)^2+$$
$$+\{(J_{B+1}K_B+J_BK_{B+1})-(P_{B+1}Q_B+P_BQ_{B+1})\}(2m)+\{(J_{B+1}K_{B+1})-(P_{B+1}Q_{B+1})\}] \quad (2.4)$$

For the sake of convenience in W^*, $J_r(c, g_1, g_2, \ldots, g_B)$, $K_r(1+c, 1+g_1, 1+g_2, \ldots, 1+g_B)$, $P_r(s, h_1, h_2, \ldots, h_B)$ and $Q_r(1+s, 1+h_1, 1+h_2, \ldots, 1+h_B)$ are abbreviated by J_r, K_r, P_r and Q_r respectively, $r \in \{1, 2, 3, \ldots, (B+1)\}$.

The symbol $J_r(c, g_1, g_2, \ldots, g_B)$ represents the sum of all possible combinations of the products of parameters taken "r" at a time from the set of $(B+1)$ parameters $\{c, g_1, g_2, \ldots, g_B\}$ with similar interpretation for others. From (2.1)

we can obtain

$$S_{N+1} = \sum_{m=0}^{N} T_{m+1} = 1 + \sum_{m=1}^{N} T_{m+1} = \frac{(2+c)_{2N}(2+g_1)_{2N}(2+g_2)_{2N}\cdots(2+g_B)_{2N}}{(2+s)_{2N}(2+h_1)_{2N}(2+h_2)_{2N}\cdots(2+h_B)_{2N}} \quad (2.5)$$

where $N \in \{2, 3, 4, \ldots\}$, $B \in \{1, 2, 3, \ldots\}$ Since Pochhammer's symbol is

associated with Gamma function and Gamma function is undefined for zero and negative integers therefore numerator and denominator parameters are adjusted in such a way that each term of following results is completely well defined and meaningful then without any loss of convergence, we have the applications of (2.2) and (2.5), in following sections.

3 When W^* is a Polynomial of Degree Zero

Assume that for $k = 1, 2, 3, \ldots, (2B + 1)$, coefficients of $(2m)^k$ in W^* of (2.4), are zero and $J_{B+1}K_{B+1} \neq P_{B+1}Q_{B+1}$, then

$$ {}_{2B+3}F_{2B+2}\left[\begin{array}{lr} 1,\, \Delta(2;c),\, \Delta(2;g_1),\, \Delta(2;g_2),\, \ldots,\, \Delta(2;g_B) & ; \\ & 1 \\ \Delta(2;2+s),\, \Delta(2;2+h_1),\, \Delta(2;2+h_2),\, \ldots,\, \Delta(2;2+h_B) & ; \end{array}\right]_N - $$

$$ - \frac{s(1+s)\prod_{j=1}^{B}(h_j)_2}{(2+s)(3+s)\prod_{j=1}^{B}(2+h_j)_2} \times $$

$$ \times {}_{2B+3}F_{2B+2}\left[\begin{array}{lr} 1,\, \Delta(2;2+c),\, \Delta(2;2+g_1),\, \Delta(2;2+g_2),\, \ldots,\, \Delta(2;2+g_B) & ; \\ & 1 \\ \Delta(2;4+s),\, \Delta(2;4+h_1),\, \Delta(2;4+h_2),\, \ldots,\, ,\, \Delta(2;4+h_B) & ; \end{array}\right]_{N-1} $$

$$ = \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots (2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots (2+h_B)_{2N}} \tag{3.1} $$

where the symbol $\Delta(2; b)$ denote the array of 2 parameters given by $\frac{b}{2}$, $\frac{b+1}{2}$.

4 When W^* is a Linear Polynomial in m

Suppose that for $k = 2, 3, 4, \ldots, (2B + 1)$, coefficients of $(2m)^k$ in W^* of (2.4), are zero; $(J_{B+1}K_B + J_BK_{B+1}) \neq (P_{B+1}Q_B + P_BQ_{B+1})$ and $J_{B+1}K_{B+1} \neq P_{B+1}Q_{B+1}$, then

$$ {}_{2B+4}F_{2B+3}\left[\begin{array}{lr} 1,\, \Delta(2;c),\, \Delta(2;g_1),\, \Delta(2;g_2),\, \ldots,\, \Delta(2;g_B),\, 1-\alpha & ; \\ & 1 \\ \Delta(2;2+s),\, \Delta(2;2+h_1),\, \Delta(2;2+h_2),\, \ldots,\, \Delta(2;2+h_B),\, -\alpha & ; \end{array}\right]_N - $$

$$ - \frac{s(1+s)(\alpha-1)\prod_{j=1}^{B}(h_j)_2}{\alpha(2+s)(3+s)\prod_{j=1}^{B}(2+h_j)_2} \times $$

$$ \times {}_{2B+4}F_{2B+3}\left[\begin{array}{lr} 1,\, \Delta(2;2+c),\, \Delta(2;2+g_1),\, \Delta(2;2+g_2),\, \ldots,\, \Delta(2;2+g_B),\, 2-\alpha & ; \\ & 1 \\ \Delta(2;4+s),\, \Delta(2;4+h_1),\, \Delta(2;4+h_2),\, \ldots,\, \Delta(2;4+h_B),\, 1-\alpha & ; \end{array}\right]_{N-1} $$

$$= \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots(2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots(2+h_B)_{2N}} \tag{4.1}$$

where α is given by

$$\alpha = -\ \frac{\{(J_{B+1}K_{B+1}) - (P_{B+1}Q_{B+1})\}}{\{(2J_{B+1}K_B + 2J_BK_{B+1}) - (2P_{B+1}Q_B + 2P_BQ_{B+1})\}} \tag{4.2}$$

5 When W^* is a Quadratic Polynomial in m

Let for $k = 3, 4, 5, \ldots, (2B+1)$, coefficients of $(2m)^k$ in W^* of (2.4), are zero; $(J_{B+1}K_{B-1} + J_BK_B + J_{B-1}K_{B+1}) \neq (P_{B+1}Q_{B-1} + P_BQ_B + P_{B-1}Q_{B+1})$ and $J_{B+1}K_{B+1} \neq P_{B+1}Q_{B+1}$, then

$${}_{2B+5}F_{2B+4}\left[\begin{matrix} 1,\ \Delta(2;c),\ \Delta(2;g_1),\ \Delta(2;g_2),\ \ldots,\ \Delta(2;g_B),\ 1-\beta,\ 1-\gamma & ; \\ & & 1 \\ \Delta(2;2+s),\ \Delta(2;2+h_1),\ \Delta(2;2+h_2),\ \ldots,\ \Delta(2;2+h_B),\ -\beta,\ -\gamma & ; \end{matrix}\right]_N -$$

$$-\ \frac{s(1+s)(\beta-1)(\gamma-1)\prod_{j=1}^{B}(h_j)_2}{\beta\gamma(2+s)(3+s)\prod_{j=1}^{B}(2+h_j)_2}\times$$

$$\times{}_{2B+5}F_{2B+4}\left[\begin{matrix} 1,\ \Delta(2;2+c),\ \Delta(2;2+g_1),\ \Delta(2;2+g_2),\ \ldots,\ \Delta(2;2+g_B),\ 2-\beta,\ 2-\gamma & ; \\ & & 1 \\ \Delta(2;4+s),\ \Delta(2;4+h_1),\ \Delta(2;4+h_2),\ \ldots,\ \Delta(2;4+h_B),\ 1-\beta,\ 1-\gamma & ; \end{matrix}\right]_{N-1}$$

$$= \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots(2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots(2+h_B)_{2N}} \tag{5.1}$$

where β and γ are given by

$$\beta = \frac{(-2J_{B+1}K_B - 2J_BK_{B+1} + 2P_{B+1}Q_B + 2P_BQ_{B+1}) + \sqrt{D^*}}{8(J_{B+1}K_{B-1} + J_BK_B + J_{B-1}K_{B+1} - P_{B+1}Q_{B-1} - P_BQ_B - P_{B-1}Q_{B+1})} \tag{5.2}$$

$$\gamma = \frac{(-2J_{B+1}K_B - 2J_BK_{B+1} + 2P_{B+1}Q_B + 2P_BQ_{B+1}) - \sqrt{D^*}}{8(J_{B+1}K_{B-1} + J_BK_B + J_{B-1}K_{B+1} - P_{B+1}Q_{B-1} - P_BQ_B - P_{B-1}Q_{B+1})} \tag{5.3}$$

and

$$D^* = (2J_{B+1}K_B + 2J_BK_{B+1} - 2P_{B+1}Q_B - 2P_BQ_{B+1})^2 -$$

$$-16(J_{B+1}K_{B-1}+J_BK_B+J_{B-1}K_{B+1}-P_{B+1}Q_{B-1}-P_BQ_B-P_{B-1}Q_{B+1})(J_{B+1}K_{B+1}-P_{B+1}Q_{B+1}) \tag{5.4}$$

6 When W^* is a Polynomial of Highest Degree "$(2B+1)$" in m

When $(J_1+K_1) \neq (P_1+Q_1)$ and $J_{B+1}K_{B+1} \neq P_{B+1}Q_{B+1}$ in W^* of (2.4), then

$$
{}_{4B+4}F_{4B+3}\left[\begin{array}{lc} 1,\, \Delta(2;c),\, \Delta(2;g_1),\, \dots,\, \Delta(2;g_B),\, 1-\beta_1,\, \dots,\, 1-\beta_{2B+1} \quad ; & \\ & 1 \\ \Delta(2;2+s),\, \Delta(2;2+h_1),\, \dots,\, \Delta(2;2+h_B),\, -\beta_1,\, \dots,\, -\beta_{2B+1} \quad ; & \end{array}\right]_N -
$$

$$
- \frac{s(1+s)\prod_{j=1}^{2B+1}(\beta_j-1)\prod_{j=1}^{B}(h_j)_2}{(2+s)(3+s)\prod_{j=1}^{2B+1}(\beta_j)\prod_{j=1}^{B}(2+h_j)_2}\times
$$

$$
\times {}_{4B+4}F_{4B+3}\left[\begin{array}{lc} 1,\, \Delta(2;2+c),\, \Delta(2;2+g_1),\, \dots,\, \Delta(2;2+g_B),\, 2-\beta_1,\, \dots,\, 2-\beta_{2B+1} \quad ; & \\ & 1 \\ \Delta(2;4+s),\, \Delta(2;4+h_1),\, \dots,\, \Delta(2;4+h_B),\, 1-\beta_1,\, \dots,\, 1-\beta_{2B+1} \quad ; & \end{array}\right]_{N-1}
$$

$$
= \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots(2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots(2+h_B)_{2N}} \tag{6.1}
$$

where $\beta_1,\, \beta_2,\, \dots,\, \beta_{2B+1}$ are the roots(neither zero nor positive integers) of the equation $W^*=0$ and W^* is given by (2.4).

7 When W^* is a Polynomial of Degree "A" in m (where $B+1 \leq A \leq 2B,\ B \in \{2,3,4,\dots\}$)

Assume that for $k = A+1, A+2, A+3, \dots, 2B, 2B+1$, coefficients of $(2m)^k$ in W^* of (2.4), are zero; $(J_{2B-A+2}+K_{2B-A+2}+J_1K_{2B-A+1}+J_2K_{2B-A}+\cdots+J_{2B-A+1}K_1) \neq (P_{2B-A+2}+Q_{2B-A+2}+P_1Q_{2B-A+1}+P_2Q_{2B-A}+\cdots+P_{2B-A+1}Q_1)$ and $J_{B+1}K_{B+1} \neq P_{B+1}Q_{B+1}$, then

$$
{}_{2B+A+3}F_{2B+A+2}\left[\begin{array}{lc} 1,\, \Delta(2;c),\, \Delta(2;g_1),\, \dots,\, \Delta(2;g_B),\, 1-\delta_1,\, \dots,\, 1-\delta_A \quad ; & \\ & 1 \\ \Delta(2;2+s),\, \Delta(2;2+h_1),\, \dots,\, \Delta(2;2+h_B),\, -\delta_1,\, \dots,\, -\delta_A \quad ; & \end{array}\right]_N -
$$

$$
- \frac{s(1+s)\prod_{j=1}^{A}(\delta_j-1)\prod_{j=1}^{B}(h_j)_2}{(2+s)(3+s)\prod_{j=1}^{A}(\delta_j)\prod_{j=1}^{B}(2+h_j)_2}\times
$$

$$
\times {}_{2B+A+3}F_{2B+A+2}\left[\begin{array}{lc} 1,\, \Delta(2;2+c),\, \Delta(2;2+g_1),\, \dots,\, \Delta(2;2+g_B),\, 2-\delta_1,\, \dots,\, 2-\delta_A \quad ; & \\ & 1 \\ \Delta(2;4+s),\, \Delta(2;4+h_1),\, \dots,\, \Delta(2;4+h_B),\, 1-\delta_1,\, \dots,\, 1-\delta_A \quad ; & \end{array}\right]_{N-1}
$$

$$= \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots(2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots(2+h_B)_{2N}} \tag{7.1}$$

where $\delta_1,\ \delta_2,\ \ldots,\ \delta_A$ are the roots(neither zero nor positive integers) of the equation

$$\begin{aligned}&[\{(J_{2B-A+2}+K_{2B-A+2}+J_1K_{2B-A+1}+J_2K_{2B-A}+\cdots+J_{2B-A+1}K_1)-\\&-(P_{2B-A+2}+Q_{2B-A+2}+P_1Q_{2B-A+1}+P_2Q_{2B-A}+\cdots+P_{2B-A+1}Q_1)\}(2m)^A+\cdots+\\&+\{(J_{B+1}+K_{B+1}+J_1K_B+J_2K_{B-1}+\cdots+J_BK_1)-(P_{B+1}+Q_{B+1}+P_1Q_B+P_2Q_{B-1}+\cdots+P_BQ_1)\}(2m)^{B+1}+\\&+\{(J_{B+1}K_1+J_BK_2+J_{B-1}K_3+\cdots+J_1K_{B+1})-(P_{B+1}Q_1+P_BQ_2+P_{B-1}Q_3+\cdots+P_1Q_{B+1})\}(2m)^B+\\&+\{(J_{B+1}K_2+J_BK_3+J_{B-1}K_4+\cdots+J_2K_{B+1})-(P_{B+1}Q_2+P_BQ_3+P_{B-1}Q_4+\cdots+P_2Q_{B+1})\}(2m)^{B-1}+\cdots\\&+\{(J_{B+1}K_{B-1}+J_BK_B+J_{B-1}K_{B+1})-(P_{B+1}Q_{B-1}+P_BQ_B+P_{B-1}Q_{B+1})\}(2m)^2+\\&+\{(J_{B+1}K_B+J_BK_{B+1})-(P_{B+1}Q_B+P_BQ_{B+1})\}(2m)+\{(J_{B+1}K_{B+1})-(P_{B+1}Q_{B+1})\}]=0\end{aligned} \tag{7.2}$$

8 When W^* is a Polynomial of Degree "R" in m (where $0\le R\le B,\ \ B\in\{2,3,4,\ldots\}$)

Suppose that for $k = R+1, R+2, R+3, \ldots, 2B, 2B+1$, coefficients of $(2m)^k$ in W^* of (2.4), are zero; $(J_{2B-R+1}K_1+J_{2B-R}K_2+\cdots+J_1K_{2B-R+1}) \neq (P_{2B-R+1}Q_1+P_{2B-R}Q_2+\cdots+P_1Q_{2B-R+1})$ and $J_{B+1}K_{B+1}\neq P_{B+1}Q_{B+1}$, then

$${}_{2B+R+3}F_{2B+R+2}\left[\begin{array}{lc}1,\ \Delta(2;c),\ \Delta(2;g_1),\ \ldots,\ \Delta(2;g_B),\ 1-\zeta_1,\ \ldots,\ 1-\zeta_R & ; \\ & \quad 1\\ \Delta(2;2+s),\ \Delta(2;2+h_1),\ \ldots,\ \Delta(2;2+h_B),\ -\zeta_1,\ \ldots,\ -\zeta_R & ;\end{array}\right]_N -$$

$$-\frac{s(1+s)\prod_{j=1}^{R}(\zeta_j-1)\prod_{j=1}^{B}(h_j)_2}{(2+s)(3+s)\prod_{j=1}^{R}(\zeta_j)\prod_{j=1}^{B}(2+h_j)_2}\times$$

$$\times{}_{2B+R+3}F_{2B+R+2}\left[\begin{array}{lc}1,\ \Delta(2;2+c),\ \Delta(2;2+g_1),\ \ldots,\ \Delta(2;2+g_B),\ 2-\zeta_1,\ \ldots,\ 2-\zeta_R & ; \\ & \quad 1\\ \Delta(2;4+s),\ \Delta(2;4+h_1),\ \ldots,\ \Delta(2;4+h_B),\ 1-\zeta_1,\ \ldots,\ 1-\zeta_R & ;\end{array}\right]_{N-1}$$

$$= \frac{(2+c)_{2N}\ (2+g_1)_{2N}\ (2+g_2)_{2N}\ \cdots(2+g_B)_{2N}}{(2+s)_{2N}\ (2+h_1)_{2N}\ (2+h_2)_{2N}\ \cdots(2+h_B)_{2N}} \tag{8.1}$$

where $\zeta_1,\ \zeta_2,\ \ldots,\ \zeta_R$ are the roots(neither zero nor positive integers) of the equation

$$\begin{aligned}&[\{(J_{2B-R+1}K_1+J_{2B-R}K_2+\cdots+J_1K_{2B-R+1})-(P_{2B-R+1}Q_1+P_{2B-R}Q_2+\cdots+P_1Q_{2B-R+1})\}(2m)^R+\cdots+\\&+\{(J_{B+1}K_B+J_BK_{B+1})-(P_{B+1}Q_B+P_BQ_{B+1})\}(2m)+\{(J_{B+1}K_{B+1})-(P_{B+1}Q_{B+1})\}]=0\end{aligned} \tag{8.2}$$

References

[1] A. Verma, A note on the summation of the generalised Hypergeometric functions, *Math. Comp.*, 21 (1967), 232-236.

[2] L.J. Slater, A note on the partial sum of a certain Hypergeometric series, *Math. Gaz.*, 39 (1955), 217-218.

[3] L.J. Slater, *Generalized Hypergeometric Functions*, Cambridge Univ. Press, Cambridge, London and New York, (1966).