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Fractional Integrals Involving General Class of Polynomials, H–Function and Multivariable I–Function

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Abstract

In this paper we obtain a Eulerian integral and a main theorem based on the functional operator associated with H–Function [2], general class of polynomial [8] and multivariable I–Function [14] having general arguments. The special class of the main theorem has also been given.

Keywords: *Reimann-Liouville operator, General polynomial, Fractional integral, H-function and multivariable I- function.*

1 Introduction

The Riemann–Liouville operator of fractional integration $R^m f$ of order *m* is defined by

$$ {}_xD_y^{-m}[f(y)] = \frac{1}{\Gamma(m)} \int_x^y (y-u)^{m-1} f(u)\, du \quad (1.1)$$

for Re(*m*) > 0 and a constant *x*

An equivalent for of Beta function is [2,p.10, eq.(13)]

$$\int_m^n (u-m)^{a-1}(n-u)^{b-1}du = (n-m)^{a+b-1}\,B(a,b) \qquad (1.2)$$

Whre $m, n \in$ R($x < y$), Re (a) > 0, Re (b) > 0.

Making use of [2, p.62, eq. (15)], we have

$$(pu+q) = (xp+q)^{\alpha}\left[1+\frac{p(u-x)}{xp+q}\right]^{\alpha}$$

$$\frac{(xp+q)^{\alpha}}{\Gamma(-\alpha)}\ \frac{1}{(2\pi i)}\int_{-i\infty}^{+i\infty}\Gamma(-\beta)\,\Gamma(-\beta-\alpha)\left[\frac{p(u-x)}{xp+q}\right]^{\beta}d\beta \qquad (1.3)$$

Where i = $\sqrt{-1}$; $p, q, \alpha \in$ c ; $x, u \in$ R ; | arg ($\frac{p}{xp+q}$) | < π and the path of integration is necessary in such a manner so as to separate the poles of Γ (–β) from those of Γ(β–α).

The multivariable I–function is defined and represented in the following manner [4]:

$$I[z_1,\ldots,z_r] = I^{0,n_2;0,n_3;\ldots;0,n_r:(m^1,n^1);\ldots;(m^{(r)},n^{(r)})}_{p_2,q_2;p_3,q_3;\ldots;p_r,q_r:(p^1,q^1);\ldots;(p^{(r)},q^{(r)})}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix} (a_{2j};\alpha'_{2j};\alpha''_{2j});(a_{3j};\alpha'_{3j};\alpha''_{3j});\ldots;(a_{rj};\alpha'_{rj};\alpha^{(r)}_{rj}): \\ (b_{2j};\beta'_{2j};\beta''_{2j});(b_{3j};\beta'_{3j};\beta''_{3j});\ldots;(b_{rj};\beta'_{rj};\beta^{(r)}_{rj}): \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (a'_j;\alpha'_j)_{1,p^1};\ldots;(a^{(r)}_j;\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b'_j;\beta'_j)_{1,q^1};\ldots;(b^{(r)}_j;\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix}\right] \qquad (1.4)$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\ldots\int_{L_r}\varphi_1(\xi_1)\ldots\varphi_r(\xi_r)\,\psi(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1}\ldots z_r^{\xi_r},\ d\xi_1,\ldots,d\xi_r$$

Where i = ($\sqrt{-1}$).

For convergence conditions and other details of multivariable I–function, see Prasad [4]. The Lauricella function $F_D^{(h)}$ is defined in the following integral from

$$\frac{\Gamma(a)\,\Gamma(b_1)\ldots\Gamma(b_h)}{\Gamma(c)}\ F_D^{(h)}[a,b_1,\ldots,b_h;\ c,\ x_1,\ldots,x_h]$$

$$= \frac{1}{(2\pi i)^h} \int_{-i\infty}^{+i\infty} \cdots \int_{-i\infty}^{+i\infty} \frac{\Gamma(a + \xi_1 + \ldots + \xi_h)\, \Gamma(b_1 + \xi_1) \ldots \Gamma(b_h + \xi_h)}{\Gamma(c + \xi_1 + \ldots + \xi_h)}$$

$$. \Gamma(-\xi_1) \ldots \Gamma(-\xi_h)\, (-x_1)^{\xi_1} \ldots (-x_h)^{\xi_r}\, d\xi_1 \ldots d\xi_h$$

(1.5)

Where max $[|\arg(-x_1)|, \ldots, |\arg(-x_h)|] < \pi;\ c = 0, -1, -2, \ldots$

To prove the Eulerien integrals, we use the following formula:

$$\int_x^y (u - x)^{a-1} (y - u)^{b-1} (p_1 u + q_1)^{\rho_1} \ldots (p_h u + q_h)^{\rho_h}\, du$$

$$= (y - x)^{a+b-1}\, B(a, b)\, (p_1, x + q_1)^{\rho_1} \ldots (p_h, x + q_h)^{\rho_h}$$

$$F_D^{(h)}\left[a, -\rho_1, \ldots, -\rho_h;\ a + b;\ -\frac{(y - x)p_1}{p_1 x + q_1}, \ldots, \frac{(y - x)p_h}{p_h x + q_h}\right]$$

(1.6)

Where $x, y \in R(x, y)$; $p_j, q_j, \rho_j, \in c\ (j - 1, \ldots, h)$;

Min[re(m), re(n)] >0 and max $\left[\left|\frac{(y - x)p_1}{p_1 x + q_1}\right|, \ldots, \left|\frac{(y - x)p_h}{p_h x + q_h}\right| < 1\right].$

Making use of the results (1.2), (1.3) and (1.5), we can prove the formula given in (1.6) . for $h = 1$ and $h = 2$, we get the known results [5, p.301 entry (2.2.6.1)] and [11, p.81, eq. (3.6)] respectively.

In what follows h is a positive integer and 0, …, 0 would mean h zero.

The series representation of H–function [2] is as follows

$$\bar{H}^{M,N}_{P,Q}[z] = \bar{H}^{M,N}_{P,Q}\left[z \left| \begin{matrix} (e_j, E_j; \alpha_j)_{1,N}, (e_j, E_j)_{N+1,P} \\ (f_j, F_j)_{1,M}, (f_j, F_j; \beta_j)_{M+1,Q} \end{matrix} \right.\right]$$

(1.7)

$$= \sum_{g=1}^{M} \sum_{k=0}^{\infty} \frac{(-1)^k\, \phi(\eta g, k)}{k!\, Fg}\, z^{\eta_{g,k}}$$

where

$$\frac{\prod_{\substack{j=1\\ j\neq g}}^{M}\Gamma(f_j - F_j\,\eta_{g,k})\prod_{j=1}^{M}\{\Gamma(1-e_j+E_j\,\eta_{g,k})\}^{\alpha_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-f_j+F_j\,\eta_{g,k})\}^{\beta_j}\prod_{j=N+1}^{P}\Gamma(e_j+E_j\,\eta_{g,k})\}}$$

and $\eta_{g,k} = \dfrac{fg+k}{Fg}$

for convergence conditions and other details of the H – function see Inayat–Hussain [2]

Srivastava [13,p.75, eqn.(1.1)] introduced the general class of polynomials defined in the following manner:

$$S_N^M\,[x] = \sum_{K=0}^{N/M}\frac{(-N)_{Mk}}{k\,!}\,A_{N,k},\ x^k,\ N = 0,1,2,\ \ldots \qquad (1.8)$$

Where M is an arbitrary positive integer and the coefficients $A_{N,k}\,(N, k > 0)$ are arbitrary constants, real or complex.

2 Integral

The main integral to be established here is

$$\int_m^n (u-m)^{\alpha-1}(n-u)^{b-1}\left\{\prod_{j=1}^{h}(p_j u + q_j)^{\rho_j}\right\}$$

$$\cdot\bar{H}_{P,Q}^{M,N}\left[x\,(u-m)^{\lambda}(n-u)^{\mu}\prod_{j=1}^{h}(p_j u + q_j)^{\sigma_j}\right]$$

$$\cdot S_{N'}^{M'}\left[z\,(u-m)^{c}(n-u)^{d}\prod_{j=1}^{h}(p_j u + q_j)^{\nu_j}\right]$$

$$\cdot I\begin{bmatrix} z_1\,(u-m)^{\gamma_1}(n-u)^{\tau_1}\prod_{j=1}^{h}(p_j u + q_j)^{-c'_j} \\ \vdots \\ z_r\,(u-m)^{\gamma_r}(n-u)^{\tau_r}\prod_{j=1}^{h}(p_j u + q_j)^{-c_j^{(r)}} \end{bmatrix} du$$

$$= G_1\sum_{g=0}^{M}\sum_{k=0}^{\infty}\sum_{r=0}^{N^1/M^1}\frac{(-1)^k\varphi(\eta g,\ k)}{k\,!\,Fg}\,\frac{(-N^1)_{m^1 r}}{r\,!}\,A_{N'},_r\ x^{\eta_{g,k}}\,z^r\,G_2\cdot G_3$$

$$I\begin{matrix} 0,\ n_2\ ;\ 0,\ n_3\ ;\ \ldots;\ 0,\ n_r\ +h+2 : (m^1,\ n^1);\ \ldots;\ (m^{(r)},\ n^{(r)});\ (1,0);\ \ldots\ ;\ (1,0) \\ p_2,\ q_2\ ;\ p_3,\ q_3\ ;\ \ldots;\ p_r+h+2,\ q_r+h+1 : (p^1,\ q^1);\ \ldots;\ (p^{(r)},\ q^{(r)});\ (0,1);\ \ldots\ ;\ (0,1) \end{matrix}$$

$$\begin{bmatrix} R_1 \\ \vdots \\ R_r \end{bmatrix} \begin{matrix} A_1, A_2, A_3, (a_{2j}\,;\,\alpha'_{2j}\,;\,\alpha''_{2j})_{1,\,p_2};\,(a_{3j}\,;\,\alpha'_{3j}\,;\,\alpha''_{3j})_{1,\,p_2};\,\ldots\,; \\ (b_{2j}\,;\,\beta'_{2j}\,;\,\beta'_{2j})_{1,\,q_2};\,(b_{3j}\,;\,\beta'_{3j}\,;\,\beta''_{3j}\,;\,\beta''_{3j})_{1,\,q_2};\,\ldots\,;\,(b_{rj},\,\beta'_{rj},\ldots,\,\beta^{(r)}_{rj},0,\ldots,0)_{1,\,q_r} \end{matrix}$$

$$\begin{matrix} (a_{rj},\,\alpha'_{rj},\ldots,\,\alpha^{(r)}_{rj},0,\ldots,0);\,(a'_{rj},\,\alpha'_{rj})_{1,\,p_1};\,\ldots\,;\,(a^{(r)}_{rj},\,\alpha^{(r)}_{rj})_{1,\,p^{(r)}};\ldots;\ldots;\ldots \\ A_4, A_5, (b'_j\,;\,\beta'_j)_{1,\,q^1};\,(b^{(r)}_j\,;\,\beta^{(r)}_j)_{1,\,q^{(r)}};\,(0,1);\,\ldots\,;\,(0,1) \end{matrix} \Bigg] \tag{2.1}$$

The following are the conditions of the validity of (2.1):

(i) $m,\ n \in R(m < n);\ \gamma_i\,;\,\tau_i\,;\ c^{(i)}_j,\ \ \gamma_j\,,\lambda\,,\ \mu\ ,\ \sigma_j\,,\ c,\ d \in R^+,\ \rho_j\ \in R,$
$p_j\,,\ q_j \in c\ ,\ z_i \in\ c\ (\ I = 1,\ \ldots,\ r\,;\ \ j = 1,\ \ldots,\ h);$

(ii) $\max\limits_{1 \le j \le h} \left[\left| \dfrac{(n-m)p_j}{p_j\,m + q_j} \right| \right] < 1;$

(iii) $\mathrm{Re}\left[a + \lambda\ \dfrac{f_j}{F_j} \ \sum\limits_{i=1}^{r}\ \gamma_i\ \dfrac{b^{(i)}_j}{\beta^{(i)}_j} \right] > 0\ \ (\ j = 1,\ \ldots,\ m^{(i)}\ ),$

$\mathrm{Re}\left[a + \mu\ \dfrac{f_j}{F_j} \ \sum\limits_{i=1}^{r}\ \tau_i\ \dfrac{b^{(i)}_j}{\beta^{(i)}_j} \right] > 0\ \ (\ j = 1,\ \ldots,\ m^{(i)}\ ),$

(iv) $\left| \arg\,(\,z_i\,)\ \prod\limits_{j=1}^{h}\ (\,p_j u + q_j)^{-c^{(i)}_j} \right| < \dfrac{\Gamma_i\,\pi}{2}\ \ (\ m \le u \le n\ ;\ i = 1,\ \ldots\ ,\ r\,),$

Where

$$U_i = \sum_{j=1}^{n^{(i)}} \alpha^{(i)}_j - \sum_{j=n^{(i)}+1}^{p^{(i)}} \alpha^{(i)}_j + \sum_{j=1}^{m^{(i)}} \beta^{(i)}_j - \sum_{j=m^{(i)}+1}^{q^{(i)}} \beta^{(i)}_j$$

$$+ \left(\sum_{j=1}^{n_2} \alpha^{(i)}_{2j} - \sum_{j=n_2+1}^{p_2} \alpha^{(i)}_{2j} \right) + \left(\sum_{j=1}^{n_3} \alpha^{(i)}_{2j} - \sum_{j=n_3+1}^{p_3} \alpha^{(i)}_{2j} \right) + \ldots$$

$$\left(\sum_{j=1}^{n_r} {}^{(i)} \quad \sum_{j=n_r+1}^{p_r} {}^{(i)} \right) \left(\sum_{j=1}^{q_2} {}^{(i)} \quad \sum_{j=1}^{q_3} {}^{(i)} \quad \sum_{j=1}^{q_r} {}^{(i)} \right)$$

$$+ \sum \alpha_{rj} - \sum \alpha_{rj} + \sum \beta_{2j} - \sum \beta_{3j} + \ldots + \sum \beta_{3j}$$

Here

$$G_1 = (n-m)^{a+b-1} \left\{ \prod_{j=1}^{h} (b_j m + q_j)^{\rho_j} \right\},$$

$$G_2 = (n-m)^{(\lambda+\mu)^{\eta_{g,k}}} \left\{ \prod_{j=1}^{h} (p_j m + q_j)^{\sigma_j \eta_{g,k}} \right\},$$

$$G_3 = (n-m)^{(c+d)\lambda} \left\{ \prod_{j=1}^{h} (p_j m + q_j)^{\nu_j \zeta} \right\}$$

$$A_1 = (1 - a - \lambda\eta_{g,k} - c\,\zeta : \gamma_1 ,\ldots, \gamma_r ,1,\ldots,1),$$

$$A_2 = (1 - b - \mu\eta_{g,k} - d\zeta : \tau_1 ,\ldots, \tau_r ,0,\ldots,0),$$

$$A_3 = (1 + \rho_j + \sigma_j\eta_{g,k} + \nu_j \zeta : c'_j ,\ldots, c_i^{(r)},0,\ldots, 1,\ldots,0)1,h$$

$$A_4 = (1 + \rho_j + \sigma_j\eta_{g,k} + \nu_j \zeta : c'_j ,\ldots, c_i^{(r)},0,\ldots,0)1,h$$

$$A_5 = [1-a - b - (\lambda + \mu)^{\eta_{g,k}} - (c + d)\zeta : (\gamma_1 + \tau_1),\ldots, (\gamma_r + \tau_r),1 ,\ldots,1];$$

$$R_1 = \begin{cases} z_1 (n - m)^{\gamma_1 + \tau_1} / \prod_{j=1}^{h} (pjm + qj)^{c'_j} \\ \vdots \\ z_r (n - m)^{\gamma_r + \tau_r} / \prod_{j=1}^{h} (pjm + qj)^{c_j^{(r)}} \end{cases}$$

$$\text{and } R_2 = \begin{cases} (n - m)p_1 \;/\; (p_1 m + q_1) \\ \vdots \\ (n - m)p_h \;/\; (p_h m + q_h) \end{cases}$$

Proof : In order to prove (2.1), expand the multivariable I–function in terms of Mellin–Barnes type of contour integral by (1.4) and H–function of general class of polynomial $S_{N'}^{M'}$ given by (1.7) and (1.8) respectively now interchange the order of summation and integration (which is permissible under the conditions of validity stated above) making case of the results in (1.3), (1.5) and (1.6), we get the desired result.

3 Theorem

Let

$$f(u) = (u - m)^{a-1} \left\{ \prod_{j=1}^{h} (p_j u + q_j)^{\rho_j} \right\}$$

$$\cdot \bar{H}_{P,Q}^{M,N} \left[x (u - m)^{\lambda} \prod_{j=1}^{h} (p_j u + q_j)^{\sigma_j} \right]$$

$$\cdot S_{N'}^{M'} \left[z (u - m)^{c} \prod_{j=1}^{h} (p_j u + q_j)^{\nu_j} \right]$$

$$\cdot I\begin{bmatrix} W_1\,(u-m)^{\gamma_1}\prod_{j=1}^{h}(p_j u+q_j)^{-c'_j} \\ \vdots \\ W_r\,(u-m)^{\gamma_r}\prod_{j=1}^{h}(p_j u+q_j)^{-c_j^{(r)}} \end{bmatrix}$$

Then

$${}_m D_y^{-b}(f(y)) = I_1 \sum_{g=0}^{M}\sum_{k=0}^{\infty}\sum_{r=0}^{N^1/M^1} \frac{(-1)^k \varphi(\eta_{g,k})}{k!\,Fg}\,\frac{(-N^1)m^1\,r}{r!}\; A_{N'},r\; x^{\eta_{g,k}} z^r \cdot I_2 \cdot I_3$$

$$I^{\,0,\,n_2\,;\,0,\,n_3\,;\,\ldots;\,0,\,n_r\,+h+1\,:\,(m^1,\,n^1);\,\ldots;\,(m^{(r)},\,n^{(r)});\,(1,0);\,\ldots\,;\,(1,0)}_{\,p_2,\,q_2\,;\,p_3,\,q_3\,;\,\ldots;\,p_r+h+1,\,q_r+h+1:\,(p^1,\,q^1);\,\ldots;\,(p^{(r)},\,q^{(r)});\,(0,1);\,\ldots\,;\,(0,1)}$$

$$\left[\begin{matrix} Y_1 \\ \vdots \\ Y_r \end{matrix}\right|\begin{matrix} D_1, D_2, (a_{2j}\,;\,\alpha'_{2j}\,;\,\alpha''_{2j})_{1,\,p_2};\,(a_{3j}\,;\,\alpha'_{3j}\,;\,\alpha''_{3j})_{1,\,p_2};\,\ldots\,; \\ (b_{2j}\,;\,\beta'_{2j}\,;\,\beta'_{2j})_{1,\,q_2};\,(b_{3j}\,;\,\beta'_{2j}\,;\,\beta''_{3j}\,;\,\beta''_{3j})_{1,\,q_3};\,\ldots\,;\,(b_{rj},\,\beta'_{rj},\ldots,\,\beta^{(r)}_{rj}, \end{matrix}$$

$$\left.\begin{matrix} (a_{rj},\,\alpha'_{rj},\ldots,\,\alpha^{(r)}_{rj},0,\ldots,0);\,(a'_{rj},\,\alpha'_{rj})_{1,\,p_1};\,\ldots\,;\,(a^{(r)}_{rj},\,\alpha^{(r)}_{rj})_{1,\,p^{(r)}};\ldots;\ldots;\ldots \\ 0,\ldots,0)_{1,\,q_r}\;\; D_3, D_4, (b'_j\,;\,\beta'_j)_{1,\,q^1};\,(b^{(r)}_j\,;\,\beta^{(r)}_j)_{1,\,q^{(r)}};\,(0,1);\,\ldots\,;\,(0,1) \end{matrix}\right]$$

(3.1)

valid under the same conditions as needed for integral (2.1)

Where

$$I_1 = (y-m)^{a+b-1}\left\{\prod_{j=1}^{h}(\,b_j m+q_j\,)^{\rho_j}\right\},$$

$$I_2 = (y-m)^{\lambda\eta_{g,k}}\left\{\prod_{j=1}^{h}(p_j m+q_j\,)^{\sigma_j\eta_{g,k}}\right\},$$

$$I_3 = (y-m)^{(c\zeta}\left\{\prod_{j=1}^{h}(p_j m+q_j\,)^{\nu_j s}\right\}$$

$$D_1 = [1-a-\lambda\eta_{g,k}-c\,\zeta : \gamma_1,\ldots,\gamma_r,1,\ldots,1],$$

$$D_2 = [1+\rho_j+\sigma_j\eta_{g,k}+\nu_j\,\zeta : c'_j,\ldots,c_i^{(r)},0,\ldots,1,\ldots,0]_{1,h}$$

$$D_3 = [1+\rho_j+\sigma_j\eta_{g,k}+\nu_j\,\zeta : c'_j,\ldots,c_i^{(r)},0,\ldots,0]_{1,h}$$

$$D_4 = [1-a-b-\lambda\eta_{g,k}-c\,\zeta : \gamma_1,\ldots,\gamma_r,1,\ldots,1].$$

$$Y_1 = \begin{cases} W_1 (y-m)^{\gamma_1} / \prod_{j=1}^{h} (p_j m + q_j)^{c'_j} \\ \vdots \\ W_r (y-m)^{\gamma_r} / \prod_{j=1}^{h} (p_j m + q_j)^{c_j^{(r)}} \end{cases}$$

$$\text{and } Y_2 = \begin{cases} (y-m)p_1 / (p_1 m + q_1) \\ \vdots \\ (y-m)p_h / (p_h m + q_h) \end{cases}$$

4 Special Cases

(i) if we put $m' = 1$, $z = 1$ and

$$A_{N',r} = \frac{(\alpha+1)_{N'} (\alpha+\beta+N'+1)_s}{(\alpha+1)_\zeta \, N'!} \quad \text{in (3.1)}$$

We get the following theorem involving Jacobi Polynomial [4,p.677, eqn.(4.1)]

Corollary

$$\text{let } f(u) = (u-m)^{\alpha-1} \left\{ \prod_{j=1}^{h} (p_j u + q_j)^{\rho_j} \right\}$$

$$\cdot \bar{H}_{P,Q}^{M,N} \left[x (u-m)^{\lambda} \prod_{j=1}^{h} (p_j u + q_j)^{\sigma_j} \right]$$

$$\cdot P_{N'}^{(\alpha,\beta)} \left[1 - 2 (u-m)^{c} \prod_{j=1}^{h} (p_j u + q_j)^{v_j} \right]$$

$$\cdot I \begin{bmatrix} W_1 (u-m)^{\gamma_1} \prod_{j=1}^{h} (p_j u + q_j)^{-c'_j} \\ \vdots \\ W_r (u-m)^{\gamma_r} \prod_{j=1}^{h} (p_j u + q_j)^{-c_j^{(r)}} \end{bmatrix}$$

Then

$${}_m D_y^{-b} (f(y)) = I_1 \sum_{g=0}^{M} \sum_{k=0}^{\infty} \sum_{s=0}^{N^1} \frac{(-1)^k \varphi(\eta_{g,k})}{k! \, F_g} \frac{(-N^1)_{m^1 s}}{s!}$$

$$\cdot \frac{(\alpha+1)_{N'} (\alpha+\beta+N'+1)_\zeta}{(\alpha+1)_s \, N'!} \cdot x^{\eta_{g,k}} z^r . I_2 . I_3$$

$$I \begin{matrix} 0, n_2 ; 0, n_3 ; \ldots; 0, n_r + h+1 : (m^1, n^1); \ldots; (m^{(r)}, n^{(r)}); (1,0); \ldots ; (1,0) \\ p_2, q_2 ; p_3, q_3 ; \ldots; p_r + h+1, q_r + h+1 : (p^1, q^1); \ldots; (p^{(r)}, q^{(r)}); (0,1); \ldots ; (0,1) \end{matrix}$$

$$\left[\begin{matrix} Y_1 \\ \vdots \\ Y_r \end{matrix}\right| \begin{matrix} D_1, D_2, (a_{2j}; \alpha'_{2j}; \alpha''_{2j})_{1, p_2}; (a_{3j}; \alpha'_{3j}, \alpha''_{3j}, \alpha'''_{3j})_{1, p_3}; \ldots ; \\ (b_{2j}; \beta'_{2j}; \beta''_{2j})_{1, q_2}; (b_{3j}; \beta'_{3j}; \beta''_{3j}; \beta'''_{3j})_{1, q_3}; \ldots ; (b_{rj}, \beta'_{rj}, \ldots, \beta^{(r)}_{rj}, \end{matrix}$$

$$(a_{rj}, \alpha'_{rj}, \ldots, \alpha^{(r)}_{rj}, 0, \ldots, 0)_{1, p_r}; (a'_{rj}, \alpha'_{rj})_{1, p^1}; \ldots ; (a^{(r)}_{rj}, \alpha^{(r)}_{rj})_{1,}$$

$p^{(r)}; \ldots; \ldots; \ldots$

$$0, \ldots, 0)_{1, q_r} \; D_3, D_4, (b'_j; \beta'_j)_{1, q^1}; (b^{(r)}_j; \beta^{(r)}_j)_{1, q^{(r)}}; (0,1); \ldots ; (0,1) \Bigg]$$

which holds true under the same conditions as (4.1) given in (3.1) and where I_1, I_2, I_3, D_1, D_2, D_3, D_4, Y_1 and Y_2 are the same as in (3.1).

(ii) For $n_2 = n_3 = \ldots = n_{r-1} = 0 = p_2 = \ldots = p_{r-1}$, $q_1 = q_2 = \ldots = q_{r-1}$, $N' = 0$ and $\lambda = 0$, $\mu = 0$, $\sigma_j \to 0$ $\gamma_i = 0$ ($i = 1, \ldots, r$) and $h = 1$ in (3.1) then we arrive at the results given by Srivastava and Hussain [6].

(iii) $N' = 0$ in (3.1), we get the main theorem obtained by Chaurasia and Kumar (1.2).

(iv) If we get $n_2 = n_3 = \ldots = n_{r-1} = 0 = p_2 = p_3 = \ldots = p_{r-1} = q_1 = q_2 = \ldots = q_{r-1}$, $N' = 0$ and $\lambda = 0$, $\mu = 0$, $\sigma_j \to 0$

the results given in (2.1) and (3.1) reduces to the known results obtained by Saigo and Saxena (9)

(v) On specializing the parameters, we get the requests obtained by Chaurasia and Singhal [13].

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