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$\mu-$ Geodetic Iteration Number and $\mu-$ Geodetic Number of a Fuzzy Graph

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Abstract

In this paper the concepts of $\mu-$geodesic, $\mu-$eccentricity, $\mu-$radius, $\mu-$diameter, $\mu-$center, $\mu-$geodetic closure, $\mu-$geodetic iteration number are introduced. It is proved that if $G : (V, \sigma, \mu)$ is a connected fuzzy graph on n nodes such that each pair of nodes is joined by a strong arc then the $\mu-$distance between two nodes is the reciprocal of its arc length. Also the concepts of $\mu-$convex set, $\mu-$geodetic cover, $\mu-$geodetic basis, $\mu-$geodetic number, $\mu-$check node, $\mu-$convex hull, $\mu-$hull number are introduced. A sufficient condition for a fuzzy graph to have its node set as $\mu-$geodetic basis is obtained. $\mu-$peripheral vertex, $\mu-$peripheral path and $\mu-$eccentric vertex of fuzzy graph are analyzed.

Keywords: *$\mu-$geodesic, $\mu-$eccentricity, $\mu-$radius, $\mu-$geodetic closure, $\mu-$geodetic iteration number, $\mu-$convex set, $\mu-$geodetic cover, $\mu-$geodetic basis, $\mu-$geodetic number, $\mu-$check node, $\mu-$convex hull, $\mu-$hull number.*

1 Introduction

Fuzzy graphs are introduced by Rosenfeld [8]. Rosenfeld has obtained the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and established some of the properties [8]. Bhattacharya has introduced fuzzy groups and metric notion in fuzzy graphs. Bhutani and Rosenfeld have introduced the concept of strong arcs [1] and geodesic distance in fuzzy graphs [2]. The definition of a geodesic basis, median are also given by the same author. Several important works on fuzzy graphs can be found in [9]. Some metric aspects using the $\mu-$distance is defined by Rosenfeld [8] and further studied by Sunitha and Vijayakumar [11]. In this paper geodetic iteration number and geodetic number of fuzzy graphs based on $\mu-$distance is introduced.

2 Preliminaries

The following definitions are from [8], [1], [7], [6] and [10].

A fuzzy graph is denoted by $G:(V,\sigma,\mu)$ where V is a vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ. i.e., $\mu(x,y)\leq\sigma(x)\wedge\sigma(y)\ \forall\ x,y\in V$. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(x,x)=\sigma(x),\forall x$) and symmetric(i.e., $\mu(x,y)=\mu(y,x),\forall(x,y)$). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^*=\{u\in V:\sigma(u)>0\}$ and $\mu^*=\{(u,v)\in V\times V:\mu(u,v)>0\}$. The fuzzy graph $H:(\tau,\nu)$ is said to be a partial fuzzy subgraph of $G:(\sigma,\mu)$ if $\nu\subseteq\mu$ and $\tau\subseteq\sigma$. Let $P\subseteq V$, the fuzzy graph $H:(P,\tau,\nu)$ is called a fuzzy subgraph of $G:(V,\sigma,\mu)$ induced by P if $\tau(x)=\sigma(x)\ \forall\ x\in$ P and $\nu(x,y)=\mu(x,y)\ \forall\ x,y\in P$. $G:(V,\sigma,\mu)$ is called trivial if $|\sigma^*|=1$.

A path P of length n is a sequence of distinct nodes $u_0,u_1,...,u_n$ such that $\mu(u_{i-1},u_i)>0, i=1,2,...,n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0=u_n$ and $n\geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and is denoted by $CONN_G(x,y)$. A fuzzy graph $G:(\sigma,\mu)$ is connected if for every x,y in $\sigma^*, CONN_G(x,y)>0$. A fuzzy graph G is said to be complete if $\mu(u,v)=\sigma(u)\wedge\sigma(v),\forall u,v\in\sigma^*$. A connected fuzzy graph $G:(\sigma,\mu)$ is a fuzzy tree if it has a fuzzy spanning subgraph $F:(\sigma,\nu)$, which is a tree where for all arcs (x,y) not in F there exists a path from x to y in F whose strength is more than $\mu(x,y)$. An arc of a fuzzy graph is called strong if its weight is at least as great as the connectedness of its end nodes when it is deleted . Depending on $CONN_G(x,y)$ of an arc (x,y) in a fuzzy graph G, Sunil Mathew and M.S.Sunitha [10] defined three different types of arcs. Note that $CONN_{G-(x,y)}(x,y)$ is the the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x,y). An arc (x,y) in G is $\alpha-$ strong if $\mu(x,y)>CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is $\beta-$ strong if $\mu(x,y)=CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is $\delta-$ arc if $\mu(x,y)<CONN_{G-(x,y)}(x,y)$. A fuzzy cut node w is a node in G whose removal reduces the strength of connectedness between some pair of nodes in

G. If $\mu(u,v) > 0$, then u and v are called neighbors. Also v is called strong neighbor of u if arc (u,v) is strong. A node z is a fuzzy end node of G if it has exactly one strong neighbor in G.

For any path P: $u_0, u_1, ..., u_n$ the $\mu-$ length of P, $l(P)$ is defined as the sum of reciprocals of arc weights . That is $l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)}$. If $n = 0$ define $l(P) = 0$, and $\mu-$ distance $d_\mu(u,v)$ is the smallest $\mu-$length of any $u-v$ path.

3 $\mu-$ Geodesics in Fuzzy Graph

In crisp graph the concept of geodesic and geodesic iteration number are discussed in [3] and [4]. Here we are extending these ideas to fuzzy graphs. Depending on $\mu-$distance we define $\mu-$geodesic, $\mu-$eccentricity, $\mu-$radius, $\mu-$diameter, $\mu-$center, $\mu-$geodetic closure and $\mu-$geodetic iteration number as follows.

Definition 3.1 *Any path P from x to y with smallest $\mu-$length is called $\mu-$geodesic from x to y. ı.e., Any path P from x to y whose $\mu-$length is $d_\mu(u,v)$ is called $\mu-$geodesic from x to y.*

Definition 3.2 *The $\mu-$eccentricity $e_\mu(u)$ of a node u in G is given by*

$$e_\mu(u) = Max_{v \in V} d_\mu(u,v)$$

The minimum $\mu-$eccentricity among the vertices of G is its $\mu-$radius denoted by $r_\mu(G)$.

$$r_\mu(G) = Min_{v \in V} e_\mu(u)$$

A node v is a $\mu-$cental node if,

$$e_\mu(v) = r_\mu(G)$$

Let $C_\mu(G)$ be the set of all $\mu-$central nodes of G. Then the fuzzy subgraph induced by $C_\mu(G)$ denoted by $< C_\mu(G) >$ is called $\mu-$ center of G.

The maximum $\mu-$eccentricity among the vertices of G is its $\mu-$diameter denoted by $d_\mu(G)$.

$$d_\mu(G) = Max_{v \in V} e_\mu(u)$$

A node v is a $\mu-$peripheral node or $\mu-$diametral node if ,

$$e_\mu(v){=}d_\mu(G)$$

Example 3.3 *Consider the fuzzy graph given in Fig.1.*

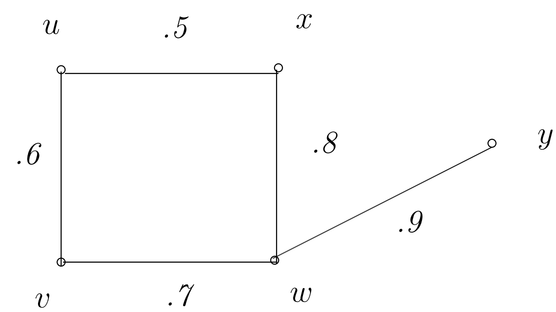

Fig-1

Here $\mu-$peripheral nodes are u and y.
$\mu-$central nodes are x and v.
$r_\mu(G) = 2.68$.
$d_\mu(G) = 4.21$.

Definition 3.4 *Let S be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Then the $\mu-$geodetic closure of S is the set of all nodes that lie on $\mu-$geodesics between nodes of S denoted by (S_μ).*

Example 3.5 *Consider the fuzzy graph given in Fig.1.*

If $S = \{u, w\}$.
Then $(S_\mu) = \{u, v, w\}$.
Similarly if $S = \{u, x, y\}$.
Then $(S_\mu) = \{u, v, w, x, y\}$.

4 $\mu-$ Geodetic Iteration Number for a Fuzzy Graph [$\mu-$gin(G)]

Let S be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Let $S_\mu^1, S_\mu^2, ...,$ are $\mu-$closures where $S_\mu^1 = (S_\mu), S_\mu^2 = (S_\mu^1) = ((S_\mu))$ etc. Since we consider only finite fuzzy graphs, the process of taking closures must terminate with some smallest n such that $S_\mu^n = S_\mu^{n-1}$. That is repeat the closure operation until the stability occurs.

Definition 4.1 *The smallest value of n so that $S_\mu^n = S_\mu^{n-1}$ is called $\mu-$geodetic iteration number of S denoted by $\mu-$ gin(S). Now $\mu-$gin(G) is the maximum value of $\mu-$gin(S), for all $S \subset V(G)$.*

Remark 4.2 *For a trivial fuzzy graph G, $\mu-$gin(G)=0.*

Example 4.3 *Consider the fuzzy graph given in Fig.1.*

Taking $S = \{u, x, y\}$
$S^1_\mu = (S_\mu) = \{u, x, v, w, y\}$
$S^2_\mu = S^1_\mu$
Therefore
$\mu-$gin(S)=2.
It can be verified that maximum value of $\mu-$gin(S)=2 for all $S \subset V(G)$.
Therefore
$\mu-$gin(G)=2.

Theorem 4.4 *Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph on n nodes such that each pair of nodes is joined by a strong arc. Then*

$$d_\mu(u, v) = \tfrac{1}{\mu(u,v)}.$$

Also

$$d_\mu(u, v) = \tfrac{1}{CONN_G(u,v)}.$$

Proof
Given that all arcs in G are strong. Thus G contain only $\alpha-$ strong and $\beta-$ strong arcs. Therefore we have two cases.
Case.1
Let (u, v) be an arc in G which is $\beta-$ strong. Consider all other $u - v$ paths in G. Then the weight of the weakest arc in any $u - v$ path is $\mu(u, v)$.
Therefore
$CONN_G(u, v) = \mu(u, v)$.(By definition of $\beta-$ strong)
Now let $P : u = u_0, u_1, ..., u_n = v$ be such a $u-v$ path. Then the $\mu-$ length of the path P is

$$l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)} > \frac{1}{\mu(u, v)}$$

Also $\mu-$distance $d_\mu(u, v)$ is the smallest $\mu-$length of any $u-v$ path. Therefore

$$d_\mu(u, v) = \tfrac{1}{\mu(u,v)}.$$

Case.2
Let (u, v) be an arc in G which is $\alpha-$ strong. Then
$CONN_G(u, v) = \mu(u, v)$ (By definition of $\alpha-$ strong).
Consider all other u-v paths in P. Let $P : u = u_0, u_1, ..., u_n = v$ be such a $u - v$ path and (x, y) be an arc in G. Then

$$\mu(x,y) < \mu(u,v) \text{ (By definition of } \alpha- \text{ strong)}$$

i.e.,

$$\frac{1}{\mu(x,y)} > \frac{1}{\mu(u,v)}$$

Hence

$$l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)} > \frac{1}{\mu(u,v)}$$

Also $\mu-$ distance $d_\mu(u,v)$ is the smallest $\mu-$length of any $u-v$ path. Therefore

$$d_\mu(u,v) = \frac{1}{\mu(u,v)}$$

If the arc is $\alpha-$ strong or $\beta-$ strong, then

$$\mu(u,v) = CONN_G(u,v) \text{ [10]}$$

Therefore

$$d_\mu(u,v) = \frac{1}{CONN_G(u,v)}.$$

'

Hence the proof.

Corollary 4.5 *For a complete fuzzy graph $G : (V, \sigma, \mu)$ on n nodes*

$$d_\mu(u,v) = \frac{1}{\mu(u,v)}.$$

Remark 4.6 *For a complete fuzzy graph G ,each arc is a $\mu-$geodesic between its end nodes. So when we consider any $S \subseteq V(G)$, any pair of nodes in S is connected by a $\mu-$geodesic,i.e., no $\mu-$geodesic between a pair of nodes of S contains another node. So $S_\mu^1 = (S_\mu) = S$. This is true for any $S \subseteq V(G)$. Hence $\mu-$gin(G)=1 for a complete fuzzy graph G.*

Remark 4.7 *The converse of Theorem 4.4 need not be true. That is if G :(V,σ,μ) is a connected fuzzy graph with $d_\mu(u,v) = \frac{1}{\mu(u,v)}$ for each arc (u,v) $\forall u, v \in V(G)$, it does not imply that each pair of nodes in G is joined by a strong arc.*

Example 4.8 *Consider the fuzzy graph given in Fig.2.*

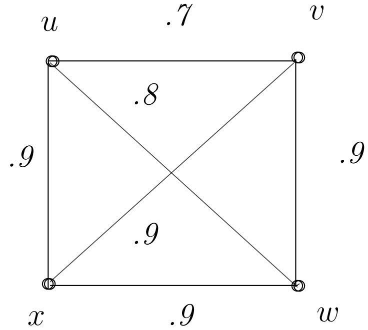

Fig.2

Here for each arc (u, v) *we have* $d_\mu(u, v) = \frac{1}{\mu(u,v)}$. *But arc* (u, v) *and arc* (u, w) *are not strong arcs.*

5 $\mu-$ Geodetic Number of a Fuzzy Graph [$\mu-$gn(G)]

Depending on $\mu-$distance we define $\mu-$convex set, $\mu-$geodetic cover, $\mu-$geodetic basis, and $\mu-$geodetic number of a fuzzy graph as follows. Then a sufficient condition for a fuzzy graph to have its node set as $\mu-$geodetic basis is obtained.

Definition 5.1 *A set* S *is* $\mu-$*convex if all nodes on any* $\mu-$*geodesic between two of its nodes are contained in* S. *Thus* S *is convex if* (S_μ)=S.

Example 5.2 *Consider the fuzzy graph given in Fig.1.*

If $S = \{u, v, w\}$, *then* (S_μ)=S. *Therefore* S *is a* $\mu-$*convex set* .

Definition 5.3 *A* $\mu-$*geodetic cover of* G *is a set* $S \subseteq V(G)$ *such that every node of* G *is contained in a* $\mu-$ *geodesic joining some pair of nodes in* S.

Example 5.4 *Consider the fuzzy graph given in Fig.1.*
If $S = \{u, x, y\}$.
Then $(S_\mu) = \{u, w, x, v, y\} = V(G)$.
Therefor S *is a* $\mu-$*geodetic cover.*

Consider the fuzzy graph given in Fig.2.
If $S = \{u, v, x, w\}$.
Then $(S_\mu) = \{u, v, , x, w\} = V(G)$.
Therefor S *is a* $\mu-$*geodetic cover.*

Proposition 5.5 *A connected fuzzy graph has at least one* $\mu-$*geodetic cover.*

Definition 5.6 *The* $\mu-$*geodetic number of* G *denoted by* $\mu-gn(G)$, *is the minimum order of its* $\mu-$*geodetic covers and any cover of order* $\mu-gn(G)$ *is a* $\mu-$*geodetic basis.*

Example 5.7 *Consider the fuzzy graph given in Fig.1.*

Here $\{u, x, y\}$ *is a* $\mu-$*geodetic basis and* $\mu-$*gn(G)= 3.*

Definition 5.8 *For a* $\mu-$*geodetic cover* S*, a node in* $G \setminus S$ *is called a* $\mu-$*check node.*

Remark 5.9 *In crisp graphs [3] the unique geodetic basis of a tree consists of all its end nodes. But for a fuzzy tree* $\mu-$*geodetic basis need not be the set of fuzzy end nodes of* G*.*

Example 5.10 *Consider the fuzzy graph given in Fig.3.*

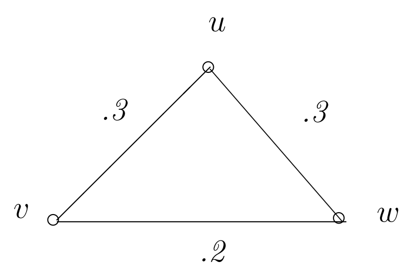

Fig.3

Here fuzzy end nodes are v *and* w*. But* $\{v,w\}$ *is not a* $\mu-$*geodetic cover, and* $\mu-$*geodetic basis is* $\{v,w,u\}$ *.*

Theorem 5.11 *Let* $G :(V,\sigma,\mu)$ *be a connected fuzzy graph on* n *nodes such that each pair of nodes in* G *is joined by a strong arc. Then* $\mu-$*geodetic number,* $\mu-$*gn(G)=* n*.*

Proof

Given $G :(V,\sigma,\mu)$ be a connected fuzzy graph on n nodes such that each pair of nodes in G is joined by a strong arc. Then

$d_{\mu}(u, v) = \frac{1}{CONN_G(u,v)}$ for each arc (u,v).[by Theorem 4.4]

Therefore no node lie on a $\mu-$geodesic between any two other nodes. Hence $\mu-$geodetic basis consists of all nodes of G. Thus $\mu-$gn(G)= n.

Corollary 5.12 *For a complete fuzzy graph* G*,* $\mu-$*gn(G)=* n*.*

Remark 5.13 *Converse of Theorem 5.11 need not be true. If* $G :(V,\sigma,\mu)$ *is a connected fuzzy graph on* n *nodes with* $\mu-$*gn(G)=* n*, it does not imply that each pair of nodes in* G *is joined by a strong arc.*

Consider the fuzzy graph given in Fig.2.

$\mu-$*gn(G)=* 4*, But arc* (u, v) *and arc* (u, w) *are not strong arcs.*

Theorem 5.14 *For any connected fuzzy graph G, $\mu-gn(G)=2$ if and only if there exists $\mu-$peripheral nodes u and v such that every node of G is on a $\mu-$peripheral path joining u and v. Also let P:$u = u_0$,u_1 ,u_2 ,...,$u_n = v$ be a $\mu-$peripheral path then*

$$d_\mu\ (u,v) = d_\mu\ (u_0, u_1) + d_\mu\ (u_1, u_2) + d_\mu\ (u_2, u_3) + \ldots + d_\mu\ (u_{n-1}, u_n).$$

Proof

Let u and v be such that each node of G is on $\mu-$peripheral path P joining u and v. Since G is nontrivial, $\mu-$gn(G) ≥ 2. Since P is a $\mu-$geodesic joining u and v, each node of G is on a $\mu-$geodesic between u and v. So S=$\{u,v\}$ is a $\mu-$geodetic basis and $\mu-$gn(G)= 2.

Conversely let $\mu-$gn(G) = 2 and $S = \{u,v\}$ be a $\mu-$geodetic basis for G.
To Prove that d_μ(G) = d_μ (u,v).
Assume d_μ $(u,v) < d_\mu$(G).
Then $\exists$ $\mu-$peripheral nodes s and t such that s and t belong to distinct $\mu-$geodesics joining u and v and d_μ $(s,t) = d_\mu$(G).
Then, d_μ $(u,v) = d_\mu$ (u,s) + d_μ (s,v) ..... (1)
d_μ $(u,v) = d_\mu$ (u,t) + d_μ (t,v) ..... (2)
d_μ $(s,t) \leq d_\mu$ (s,u) + d_μ (u,t) .... (3)
d_μ $(s,t) \leq d_\mu$ (s,v) + d_μ (v,t) .... (4)
Since d_μ $(u,v) < d_\mu$ (s,t)
(3) $\Longrightarrow d_\mu$ $(u,v) < d_\mu$ (s,u) + d_μ (u,t) and by (1)
d_μ $(s,v) < d_\mu$ (u,t) and from (4)
d_μ $(s,t) < d_\mu$ (u,t) + d_μ (v,t) = d_μ (u,v) by (1), which is a contradiction. Thus u and v must be $\mu-$peripheral nodes.

Next Given P:$u = u_0$,u_1 ,u_2 ,...,$u_n = v$ be a $\mu-$peripheral path. Since every node of G is on $\mu-$peripheral path,

$$d_\mu(u_{i-1}, u_i) = \tfrac{1}{\mu(u_{i-1},u_i)}.$$

$$\text{Therefore } d_\mu(u,v) = \min \{\sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)}\}.$$
$$= \sum_{i=1}^{n} d_\mu(u_{i-1}, u_i).$$

Therefore

$$d_\mu\ (u,v) = d_\mu\ (u_0, u_1) + d_\mu\ (u_1, u_2) + d_\mu\ (u_2, u_3) + \ldots + d_\mu\ (u_{n-1}, u_n)$$

Hence the proof.

Remark 5.15 *In crisp graph G, if v is a node that is farthest from u, then v is not a cut node of G [5]. But in fuzzy graphs, if v is a node that is farthest from u in G, then v can be a fuzzy cut node of G. That is in fuzzy graphs fuzzy cut node can be $\mu-$eccentric node and $\mu-$peripheral node .*

Example 5.16 *Consider the fuzzy graph given in Fig.4.*

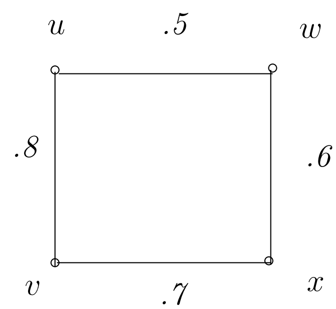

Fig.4

In Fig.4 v is a fuzzy cut node and v is an $\mu-$eccentric node of w as well. Also v is a $\mu-$peripheral node.

6 $\mu-$ Convex Hull of a Fuzzy Graph

In this section $\mu-$convex hull and $\mu-$hull number of a fuzzy graph with respect to $\mu-$distance is defined.

Definition 6.1 *Let $S \subseteq V(G)$ and repeatedly take its closures $S_\mu^1 = (S_\mu), S_\mu^2 = (S_\mu^1) = ((S_\mu))$ etc. Since we consider only fuzzy graphs with finite number of nodes, this process of taking closures must terminate with some smallest n such that $S_\mu^n = S_\mu^{n-1}$. The resulting set is called $\mu-$convex hull of S in G, and is denoted by $[S_\mu]$.*

Example 6.2 *Consider the fuzzy graph given in Fig.1.*

Here let $S = \{u, x, y\}$, which is not $\mu-$convex, and $\mu-$convex hull of S in G, $[S_\mu] = \{u, v, w, x, y\}$.

Remark 6.3 *It is clear from the definition that a subset $S \subseteq V(G)$ is $\mu-$convex if and only if $[S_\mu] = S$. Also $[S_\mu]$ is the smallest $\mu-$convex set containing S.*

Definition 6.4 *The minimum order of the set $S \subseteq V(G)$ such that $[S_\mu] = V(G)$ is called the $\mu-$hull number of G denoted by $h_\mu(G)$ and such a set is called minimum $\mu-$hull set of G.*

Example 6.5 *Consider the fuzzy graph given in Fig.1.*

$S = \{u, x, y\}$.

$[S_\mu] = \{u, v, w, x, y\} = V(G)$.

$S = \{u, x, y\}$ *is a minimum* $\mu-$*hull set.*

$h_\mu(G) = 3$.

Remark 6.6 *Let* $G : (V, \sigma, \mu)$ *be a connected fuzzy graph with* G^* *complete and all arcs in* G *are strong. Then* $h_\mu(G) = n$.

Proposition 6.7 *For a connected fuzzy graph* $G : (V, \sigma, \mu)$, $2 \leq h_\mu(G) \leq n$, *where* n *is the number of nodes in* G.

7 Conclusion

In this paper, we introduced $\mu-$geodesic, $\mu-$eccentricity, $\mu-$radius, $\mu-$diameter, $\mu-$center, $\mu-$geodetic closure, $\mu-$geodetic iteration number, $\mu-$convex set, $\mu-$geodetic cover, $\mu-$geodetic basis, $\mu-$geodetic number, and $\mu-$convex hull of a fuzzy graph and studied some properties.

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