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On Upper and Lower Weakly c-e-Continuous Multifunctions

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Abstract

In this paper we have introduce and study a new class of multifunction called weakly c-e-continuous multifunctions in topological spaces.

Keywords: *Topological spaces, e-open sets, e-closed sets, weakly c-e-continuous multifunctions.*

1 Introduction

It is well known that various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good number of them have been extended to the setting of multifunctions. This implies that both, functions and multifunctions are important tools for studying other properties of spaces and for constructing new spaces from previously existing ones. In this paper, we have introduce and study upper and lower weakly c-e-continuous multifunctions in topological spaces and to obtain some characterizations of these new continuous multifunctions and present several of their properties.

2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) means topological spaces on which no separation axioms are assumed unless explicitly stated. For any subset A of X, the closure and the interior of A are denoted by (A) and (A), respectively. A subset A of X is said to be regular open [14] (resp. semiopen [9], preopen [10], α-open [12], β-open [1], b-open [2](= γ-open [5])) if $A = ((A))$ (resp. $A \subset ((A))$, $A \subset ((A))$, $A \subset (((A)))$, $A \subset (((A)))$, $A \subset (((A)) \cup ((A)))$. The complement of semiopen (resp. regular open, preopen, semiopen, α-open, β-open, b-open) is called semiclosed [4] (resp. regular closed, preclosed [10], semiclosed [9], α-closed [11], β-closed [1], b-closed [2]). The intersection of all semiclosed (resp. preclosed, semiclosed, α-closed, β-closed, b-closed) sets containing A is called the semiclosure [3] (resp. preclosure [10], α-closure [11], β-closure [1], b-closure [2]) of A and is denoted by $s(A)$ (resp. $p(A)$, $\alpha(A)$, $\beta(A)$, $b(A)$). A set $A \subset X$ is said to be δ-open [15] if it is the union of regular open sets of X. The complement of a δ-open set is called δ-closed. The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [15] of A and is denoted by $_\delta(A)$. The union of all δ-open sets of (X, τ) contained in A is called the δ-interior [15] of A and is denoted by $_\delta(A)$. A subset A of (X, τ) is said to be e-open [5] if $A \subset (_\delta(A)) \cup (_\delta(A))$. The complement of an e-open set is called e-closed [5]. While, the family of all e-open (resp. e-closed) subsets of (X, τ) is denoted by $EO(X)$ (resp. $EC(X)$). The intersection (resp. union) of all e-closed (resp. e-open) sets of (X, τ) containing (resp. contained in) A is called the e-closure [5] (resp. e-interior [5]) of A and is denoted by $e(A)$ (resp.$e(A)$). By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, also we always assume that $F(x) \neq \varnothing$ for all $x \in X$. For a multifunction $F : X \to Y$, the upper and lower inverse of any subset A of Y by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A) = \{x \in X : F(x) \subseteq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. In particular, $F(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$.

3 Weakly c-e-Continuous Multifunctions

Definition 3.1 *A multifunction $F : X \to Y$ is said to be :*

(i) upper weakly c-e-continuous if for each $x \in X$ and each open set V of Y having connected complement such that $x \in F^+(V)$, there exists a $U \in EO(X, x)$ such that $U \subset F^+((V))$;

(ii) lower weakly c-e-continuous for each $x \in X$ and each open set V of Y having connected complement such that $x \in F^-(V)$, there exists a $U \in EO(X, x)$ such that $U \subset F^-((V))$.

Definition 3.2 *A multifunction $F : X \to Y$ is said to be [?]:*

(i) upper c-e-continuous if for each $x \in X$ and each open set V of Y having connected complement such that $x \in F^+(V)$, there exists a $U \in EO(X, x)$ such that $U \subset F^+(V)$;

(ii) lower c-e-continuous for each $x \in X$ and each open set V of Y having connected complement such that $x \in F^-(V)$, there exists a $U \in EO(X, x)$ such that $U \subset F^-(V)$.

Theorem 3.3 *For a multifunction $F : X \to Y$, the following statements are equivalent :*

(i) F is upper weakly c-e-continuous;

(ii) $F^+(V) \subset e(F^+((V)))$ for any open set V having connected complement;

(iii) $e(F^-((K))) \subset F^-(K)$ for any closed connected set K;

(iv) for each $x \in X$ and each open set V having connected complement and containing $F(x)$, there exists an e-open set U containing x such that $F(U) \subset (V)$.

Proof: (i)⇒(ii): Let V be any open set having connected complement and $x \in F^+(V)$. By (i), there exists an e-open set U containing x such that $U \subset F^+((V))$. Hence, $x \in e(F^+((V)))$.
(ii)⇒(i): Let V be any open set having connected complement and $x \in F^+(V)$. By (ii), $x \in F^+(V) \subset e(F^+(((V))) \subset F^+((V)))$. Take $U = e(F^+((V)))$. Thus, we obtain that F is upper weakly c-e-continuous.
(ii)⇔(iii): Let K be any closed connected set of Y. Then, $Y \backslash K$ is an open set having connected complement. By (ii), $F^+(Y \backslash K) = X \backslash F^-(K) \subset e(F^+((Y \backslash K)))$ $= e(F^+(Y \backslash (K))) =$
$X \backslash e(F^-((K)))$. Thus, $e(F^-((K))) \subset F^-(K)$. The converse is similar.
(i)⇔(iv): Obvious.

Remark 3.4 *It is clear that every upper c-e-continuous multifunction is upper weakly c-e-continuous. But the converse is not true in general, as the following example shows.*

Example 3.5 *Let $X = \{a, b, c, \}$ with topology $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$, $Y = \{a, b, c, \}$ with toplogy $\sigma = \{\varnothing,\ \{c\},\ X\}$ and the identity multifunction $F : (X, \tau) \to (Y, \sigma)$ given by $F(x) = \{x\}$ for each $x \in X$. Then clearly F is upper weakly c-e-continuous but not upper c-e-continuous.*

Theorem 3.6 *For a multifunction $F : X \to Y$, the following statements are equivalent :*

(i) F *is lower weakly c-e-continuous;*

(ii) $F^-(V) \subset e(F^-((V)))$ *for any open set* V *having connected complement;*

(iii) $e(F^-((K))) \subset F^+(K)$ *for any closed connected set* K*;*

(iv) for each $x \in X$ *and each open set* V *having connected complement such that* $f(x) \cap V \neq \emptyset$*, there exists an e-open set* U *containing* x *such that* $F(u) \cap (V) \neq \emptyset$ *for each* $u \in U$*.*

Proof: It is similar to the proof of the Theorem 3.3.

Theorem 3.7 *Let* $F : X \to Y$ *be a multifunction such that* $F(x)$ *is an open set of* Y *for each* $x \in X$*. Then* F *is lower c-e-continuous if and only if lower weakly c-e-continuous.*

Proof: Let $x \in X$ and V be an open set of Y having connected complement such that $F(x) \cap V \neq \emptyset$. Then there exists an e-open set U containing x such that $F(u) \cap (V) \neq \emptyset$ for each $u \in U$. Since $F(u)$ is open, $F(u) \cap V \neq \emptyset$ for each $u \in U$ and hence F is lower c-e-continuous. The converse follows by Remark 3.4.

Theorem 3.8 *If* $F : X \to Y$ *is lower weakly c-e-continuous and there exists an open basis* $\beta = \{V_i : i \in I\}$ *of the topology for* Y *such that* V_i *has connected complement and* $F^-((V_i)) \subset F^-(V_i)$ *for every* $i \in I$*, then* F *is lower c-e-continuous.*

Proof: Let $\beta = \{V_i : i \in I\}$ be an open basis of the topology for Y such that V_i has connected complement and $F^-((V_i)) \subset F^-(V_i)$ for every $i \in I$. For any open set V having connected complement, there exists a subset β_0 of β such that $V = \bigcup_{i \in \beta_0} V_i$. Therefore, by Theorem 3.6, we obtain that $F^-(V) = F^-(\bigcup_{i \in \beta_0} V_i) = \bigcup_{i \in \beta_0} F^-(V_i) \subset \bigcup_{i \in \beta_0} e(F^-((V_i))) \subset \bigcup_{i \in \beta_0} e(F^-(V_i)) \subset e(\bigcup_{i \in \beta_0} F^-(V_i)) =$
$e(F^-(\bigcup_{i \in \beta_0} V_i)) = e(F^-(V))$. This shows that F is lower c-e-continuous

Theorem 3.9 *If* $F : X \to Y$ *is upper weakly c-e-continuous and satisfies* $F^+((V)) \subset F^+(V)$ *for every open set* V *having connected complement in* Y*, then* F *is upper c-e-continuous.*

Proof: Let V be any open set having connected complement. Since F is weakly c-e-continuous, we have $F^+(V) \subset e(F^+((V)))$ and hence $F^+(V) \subset e(F^+((V))) \subset e(F^+(V))$. Thus, $F^+(V)$ is e-open and it follows that F is upper c-e-continuous.

Definition 3.10 *A topological space (X, τ) is said to be c-normal if for every disjoint closed sets V_1 and V_2 of X, there exist two disjoint open sets U_1 and U_2 having connected complement such that $V_1 \subset U_1$ and $V_2 \subset U_2$ and $U_1 \cap U_2$.*

Theorem 3.11 *Let $F : X \to Y$ be a multifunction such that $F(x)$ is closed in Y for each $x \in X$ and Y is c-normal. Then F is upper weakly c-e-continuous if and only if F is upper e-continuous.*

Proof: Suppose that F is upper weakly c-e-continuous. Let $x \in X$ and G be an open set having connected complement and containing $F(x)$. Since $F(x)$ is closed in Y, by the c-normality of Y, there exist open sets V and W having connected complements such that $F(x) \subset V$, $X\backslash G \subset W$ and $V \cap W = \emptyset$. We have $F(x) \subset V \subset (V) \subset (X\backslash W) = X\backslash W \subset G$. Since F is upper weakly c-e-continuous, there exists an e-open set U containing x such that $F(U) \subset (V) \subset G$. This shows that F is upper c-e-continuous. The converse follows by Remark 3.4.

Definition 3.12 *A subset A of a topological space (X, τ) is said to be:*

(i) α-regular [8] if for each $a \in A$ and each open set U containing a, there exists an open set G of X such that $a \in G \subset (G) \subset U$;

(ii) α-paracompact [8] if every X-open cover A has an X-open refinement which covers A and is locally finite for each point of X.

Lemma 3.13 *[8] If A is an α-paracompact and α-regular set of a topological space (X, τ) and U is an open neighbourhood of A, then there exists an open set G of X such that $A \subset G \subset (G) \subset U$.*

For a multifunction $F : X \to Y$, by $(F) : X \to Y$ we denote a multifunction as follows: $(F)(x) = (F(x))$ for each $x \in X$. Similarly, we denote sF and eF.

Lemma 3.14 *If $F : X \to Y$ is a multifunction such that $F(x)$ is α-paracompact and α-regular for each $x \in X$, then we have the following*

(i) $G^+(V) = F^+(V)$ for each open set V of Y,

(ii) $G^-(V) = F^-(V)$ for each closed set V of Y,
where G denotes F, sF, pF, αF, bF, βF or eF.

Lemma 3.15 *For a multifunction $F : X \to Y$, we have the following*

(i) $G^-(V) = F^-(V)$ for each open set V of Y,

(ii) $G^+(V) = F^+(V)$ *for each open set* V *of* Y,
where G *denotes* F, sF, pF, αF, bF, βF *or* eF.

Theorem 3.16 *Let* $F : X \to Y$ *be a multifunction such that* $F(x)$ *is* α-*regular and* α-*paracompact for every* $x \in X$. *Then the following properties are equivalent:*

(i) F *is upper weakly c-e-continuous;*

(ii) F *is upper weakly c-e-continuous;*

(iii) sF *is upper weakly c-e-continuous;*

(iv) pF *is upper weakly c-e-continuous;*

(v) αF *is upper weakly c-e-continuous;*

(vi) bF *is upper weakly c-e-continuous;*

(vii) βF *is upper weakly c-e-continuous;*

(viii) eF *is upper weakly c-e-continuous.*

Proof: We put $G = (F)$, sF, pF, αF, bF, βF or eF in the sequel.

Necessity: Suppose that F is upper weakly c-e-continuous. Then it follows by Theorem 3.3 and Lemmas 3.14 and 3.15 that for every open set V of Y containing $F(x)$ having connected complement, $G^+(V) = F^+(V) \subset e(F^+((V))) = e(G^+((V)))$. By Theorem 3.3, G is upper weakly c-e-continuous.

Sufficiency: Suppose that G is upper weakly c-e-continuous. Then it follows by Theorem 3.3 and Lemmas 3.14 and 3.15 that for every open set V of Y containing $G(x)$ having connected complement, $F^+(V) = G^+(V) \subset e(G^+((V))) = e(F^+((V)))$. It follows by Theorem 3.3 that F is upper weakly c-e-continuous.

Theorem 3.17 *Let* $F : X \to Y$ *be a multifunction such that* $F(x)$ *is* α-*regular and* α-*paracompact for every* $x \in X$. *Then the following properties are equivalent:*

(i) F *is lower weakly c-e-continuous;*

(ii) F *is lower weakly c-e-continuous;*

(iii) sF *is lower weakly c-e-continuous;*

(iv) pF *is lower weakly c-e-continuous;*

(v) αF *is lower weakly c-e-continuous;*

(vi) bF *is lower weakly c-e-continuous;*

(vii) βF *is lower weakly c-e-continuous;*

(viii) eF *is lower weakly c-e-continuous.*

Proof: Similar to the proof of Theorem 3.3.

Definition 3.18 *Let A be a subset of a topological space X. The e-frontier of A, denoted by $efr(A)$, is defined by $efr(A) = e(A) \cap e(X \backslash A) = e\text{-}(A) \backslash e\text{-}(A)$.*

Theorem 3.19 *Let $F : X \to Y$ be a multifunction. The set of all points x of X such that F is not upper weakly c-e-continuous (resp. lower weakly c-e-continuous) is identical with the union of the e-frontiers of the upper (resp. lower) inverse images of the closure of open sets containing (resp. meetings) $F(x)$ and having connected complement.*

Proof: Let x be a point of X at which F is not upper weakly c-e-continuous. Then there exists an open set V containing $F(x)$ and having connected complement such that $U \cap (X \backslash F^+((V))) \neq \emptyset$ for every e-open set U containing x. Therefore, $x \in e(X \backslash F^+((V)))$. Since $x \in F^+(V)$, we have $x \in e(F^+((V)))$ and hence $x \in efr(F^+((V)))$. Conversely, if F is upper weakly c-e-continuous at x, then for every open set V containing $F(x)$ and having connected complement there exists an e-open set U containing x such that $F(U) \subset (V)$; hence $U \subset F^+((V))$. Therefore, we obtain $x \in U \subset e(F^+((V)))$. This contradicts that $x \in e\text{-}Fr(F^+((V)))$.
The case when F is lower weakly c-e-continuous is similarly shown.

Definition 3.20 *A topological space (X, τ) is said to be strongly c-normal if for every disjoint closed sets V_1 and V_2 of X, there exist two disjoint open sets U_1 and U_2 having connected complement such that $V_1 \subset U_1$, $V_2 \subset U_2$ and $(U_1) \cap (U_2) = \emptyset$.*

Theorem 3.21 *If Y is a strongly c-normal space and $F_i : X_i \to Y$ is upper weakly c-e-continuous multifunction such that F_i is point closed for i = 1, 2, then a set $\{(x_1, x_2) \in X_1 \times X_2$: $F_1(x_1) \cap F_2(x_2) \neq \varnothing\}$ is e-closed in $X_1 \times X_2$.*

Proof: Let $A = \{(x_1, x_2) \in X_1 \times X_2$: $F_1(x_1) \cap F_2(x_2) \neq \varnothing\}$ and $(x_1, x_2) \in (X_1 \times X_2) \backslash A$. Then $F_1(x_1) \cap F_2(x_2) = \varnothing$. Since Y is strongly c-normal and F_i is point closed for $i = 1$, 2, there exist disjoint open sets V_1, V_2 having connected complement such that $F_i(x_i) \subset V_i$ for $i = 1$, 2. We have $(V_1) \cap (V_2) = \emptyset$. Since F_i is upper weakly c-e-continuous, there exist e-open sets U_1 and U_2 containing x_1 and x_2, respectively such that $F_i(U_i) \subset (V_i)$ for $i = 1$, 2. Put $U = U_1 \times U_2$, then U is an e-open set and $(x_1, x_2) \in U \subset (X_1 \times X_2) \backslash A$. This shows that $(X_1, \times X_2) \backslash A$ is e-open; hence A is e-closed in $X_1 \times X_2$.

Definition 3.22 *A multifunction $F : X \to Y$ is said to be:*

(i) lower weakly continuous [13] if for each $x \in X$ and each open set V of Y such that $x \in F^{-}(V)$, there exists an open set U containing x such that $U \subset F^{-}((V))$;

(ii) upper weakly continuous [13] for each $x \in X$ and each open set V of Y such that $x \in F^{+}(V)$, there exists an open set U containing x such that $U \subset F^{+}((V))$.

Theorem 3.23 *Let F and G be respectively upper weakly c-e-continuous and upper weakly continuous multifunctions from a topological space (X, τ) to a strongly c-normal space (Y, σ). Then the set $K = \{x : F(x) \cap G(x) \neq \varnothing\}$ is e-closed in X.*

Proof: Let $x \in X \backslash K$. Then $F(x) \cap G(x) = \varnothing$. Since F and G are point closed multifunctions and Y is a strongly c-normal space, it follows that there exist disjoint open sets U and V having connected complement containing $F(x)$ and $G(x)$, respectively we have $(U) \cap (V) = \emptyset$. Since F and G are upper weakly c-e-continuous functions, upper weakly continuous, respectively, then there exist e-open set U_1 containing x and open set U_2 containing x such that $F(U_1) \subset (V)$ and $F(U_2) \subset (V)$. Now set $H = U_1 \cap U_2$, then H is an e-open set containing x and $H \cap K = \varnothing$; hence K is e-closed in X.

Theorem 3.24 *Let $F : X \to Y$ be a multifunction and U be an δ-open subset in X. If F is a lower (upper) weakly c-e-continuous multifunction, then $F_{|_U}$: $U \to Y$ is a lower (upper) weakly c-e-continuous multifunction.*

Proof: Let V be any δ-open set of Y having connected complement. Let $x \in U$ and $x \in F^{-}_{|_U}(V)$. Since F is lower weakly c-e-continuous multifunction, then there exists an e-open set G containing x such that $x \in G \subset F^{-}((V))$. Then $x \in G \cap U \in EO(U)$ and $G \cap U \subset F^{-}_{|_U}((V))$. This shows that $F_{|_U}$ is a lower weakly c-e-continuous.
The proof of the upper weakly c-e-continuity of $F_{|_U}$ can be done by the same token.

Theorem 3.25 *Let $\{A_i\}_{i \in I}$ be an δ-open cover of a topological space X. Then a multifunction $F : X \to Y$ is upper (lower) weakly c-e-continuous if and only if $F_{|_{A_i}}$: $A_i \to Y$ is a upper (lower) weakly c-e-continuous for each $i \in I$.*

Proof: Necessity: Let $i \in I$ and V be any δ-open set of Y having connected complement. Since F is lower weakly c-e-continuous, $F^{+}(V) \subset e(F^{+}((V)))$. We obtain $(F|_{A_i})^{+}(V) = F^{+}(V) \cap A_i \subset e(F^{+}($

$(V))) \cap A_i = e(F^+((V)) \cap A_i) \subset e_{A_i}(F^+((V)))$. Hence $F_{|A_i}\colon A_i \to Y$ is a upper weakly c-e-continuous for each $i \in I$.
Suffiency: Let V be any open set of Y having connected complement. Since F_i is lower weakly c-e-continuous for each $i \in I$, from Theorem 3.3, $F_i^+(V) \subset e_{A_i}(F_i^+((V)))$ and since A_i is open, we have $F^+(V) \cap A_i \subset e_{A_i}(F^+((V)) \cap A_i)$ and $F^+(V) \cap A_i \subset e(F^+($
$(V))) \cap A_i$. Since $\{A_i\}_{i \in I}$ is an open cover of X, it follows that $F^+(V) \subset e(F^+((V)))$. Hence, from Theorem 3.3, we obtain that F is upper weakly c-e-continuous.
The proof of the lower weakly c-e-continuity of $F_{|A_i}$ can be done by the same token.

Definition 3.26 *For a multifunction, $F : X \to Y$, the graph multifunction $G_F : X \to X \times Y$ is defined as follows: $G_F(x) = \{x\} \times F(x)$ for every $x \in X$ and the subset $\{\{x\} \times F(x) : x \in X\} \subset X \times Y$ is called the graph multifunction of F and is denoted by $G(x)$.*

Lemma 3.27 *For a multifunction $F : X \to Y$, the following holds:*

(i) $G_F^+(A \times B) = A \cap F^+(B)$;

(ii) $G_F^-(A \times B) = A \cap F^-(B)$

for any subset A of X and B of Y.

Theorem 3.28 *Let $F : X \to Y$ be a multifunction and X be a connected space. If the graph multifunction of F is upper (lower) weakly c-e-continuous, then F is upper (lower) weakly c-e-continuous.*

Proof: Suppose that $G_F : X \to X \times Y$ is upper weakly c-e-continuous. Let $x \in X$ and V be any open subset of Y having connected complement and containing $F(x)$. Since $X \times V$ is an open set having connected complement relative to $X \times Y$ and $G_F(x) \subset X \times V$, there exists an e-open set U containing x such that $G_F(U) \subset (X \times V) = X \times (V)$. By Lemma 3.27, we have $U \subset G_F^+(X \times (V)) = F^+((V))$ and $F(U) \subset (V)$. Thus, F is upper weakly c-e-continuous.
The proof of the lower weakly c-e-continuity of F can be done by the same token.

Definition 3.29 *[6, 7] A topological space (X, τ) is said to e-T_2 if for each pair of distinct points x and y in X, there exist disjoint e-open sets U and V in X such that $x \in U$ and $y \in V$.*

Theorem 3.30 *If $F : X \to Y$ is an upper weakly c-e-continuous injective multifunction and point closed from a topological space X to a strongly c-normal space Y, then X is a e-T_2 space.*

Proof: Let x and y be any two distinct points in X. Then we have $F(x) \cap F(y) = \varnothing$ since F is injective. Since Y is strongly c-normal, it follows that there exist disjoint open sets U and V having connected complement containing $F(x)$ and $F(y)$, respectively such that $(U) \cap (V) = \emptyset$. Thus, there exist disjoint e-open sets U and V containing x and y, respectively such that $G \subset F^{+}((U))$ and $W \subset F^{+}((V))$. Therefore, we obtain $G \cap W = \emptyset$ and hence X is e-T_2.

Theorem 3.31 *Suppose that (X, τ) and (X_α, τ_α) are topological spaces where X_α is connected space for each $\alpha \in J$. Let $F : X \to \prod_{\alpha \in J} X_\alpha$ be a multifunction from X to the product space $\prod_{\alpha \in J} X_\alpha$ and let $P_\alpha : \prod_{\alpha \in J} X_\alpha \to X_\alpha$ be the projection for each $\alpha \in J$ which is defined by $P_\alpha((x_\alpha)) = \{x_\alpha\}$. If F is upper (lower) weakly c-e-continuous multifunction, then $P_\alpha \circ F$ is upper (lower) weakly c-e-continuous multifunction for each $\alpha \in J$.*

Proof: Take any $\alpha_0 \in J$. Let V_{α_0} be an open set having connected complement in $(X_{\alpha_0}, \tau_{\alpha_0})$. Then $(P_{\alpha_0} \circ F)^{+}(V_{\alpha_0}) = F^{+}(P^{+}_{\alpha_0}(V_{\alpha_0})) = F^{+}(V_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$. We take $x \in (P_{\alpha_0} \circ F)^{+}(V_{i_0})$. Since F is upper weakly c-e-continuous and $V_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha$ is an open set having connected complement to $\prod_{\alpha \in J} X_\alpha$, there exists an e-open set U containing x such that $U \subset F^{+}((V_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha))$. Since $F^{+}((V_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)) = F^{+}((V_{\alpha_0}) \times \prod_{\alpha \neq \alpha_0} X_\alpha) = (P_{\alpha_0} \circ F)^{+}((V_{\alpha_0}))$, $P_{\alpha_0} \circ F$ is upper weakly c-e-continuous.
The proof of the lower weakly c-e-continuity of F can be done by the same token.

Theorem 3.32 *Suppose that for each $\alpha \in J$, (X_α, τ_α), $(Y_\alpha, \sigma_\alpha)$ are topological spaces. Let $F_\alpha : X_\alpha \to Y_\alpha$ be a multifunction for each $\alpha \in J$ and let $F : \prod_{\alpha \in J} X_\alpha \to \prod_{\alpha \in J} Y_\alpha$ be defined by $F((x_\alpha)) = \prod_{\alpha \in J} F_\alpha(x_\alpha)$ from the product space $\prod_{\alpha \in J} X_\alpha$ to the product space $\prod_{\alpha \in J} Y_\alpha$. If F is upper (lower) weakly c-e-continuous multifunction and X_α is connected for each $\alpha \in J$, then each F_α is upper (lower) weakly c-e-continuous multifunction for each $\alpha \in J$.*

Proof: Similar to the proof of Theorem 3.31.

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