Gen. Math. Notes, Vol. 24, No. 2, October 2014, pp.26-36
ISSN 2219-7184; Copyright ©ICSRS Publication, 2014
www.i-csrs.org
Available free online at http://www.geman.in

On Fuzzy Maximal θ-Continuous Functions in Fuzzy Topological Spaces

M. Pritha[1], V. Chandrasekar[2] and A. Vadivel[3]

[1]Department of Mathematics
Pachaiyappas College, Chennai-600030
E-mail: prithasrinivasan@gmail.com
[2]Department of Mathematics, Kandaswamy Kandars College
P-velur, Tamil Nadu-638182
E-mail: vcsekar5@yahoo.co.in
[3]Mathematics Section, FEAT, Annamalai University
Annamalainagar, Tamil Nadu-608002
E-mail: avmaths@gmail.com

(Received: 5-5-14 / Accepted: 1-7-14)

Abstract

The purpose of this paper is to introduce the notion of fuzzy maximal θ-continuous (fuzzy maximal θ-semi-continuous), fuzzy maximal θ-irresolute (fuzzy maximal θ-semi irresolute) functions. Some basic properties and characterization theorems are also to be investigated.

Keywords: *Fuzzy maximal θ-continuous, fuzzy maximal θ-semi-continuous, fuzzy maximal θ-irresolute and fuzzy maximal θ-semi irresolute functions.*

1 Introduction and Preliminaries

The notion of fuzzy sets due to Zadeh [8] plays important role in the study of fuzzy topological spaces which introduced by Chang [2]. In 1992, Azad [1] introduced and investigated fuzzy semi open sets and fuzzy semi closed sets. M.E. El. Shafei and A. Zakeri [7] defined fuzzy θ-open sets. Thereafter mathematicians gave in several papers in different and interesting new open sets. Other preliminary ideas on fuzzy set theory can be found in [3, 4, 5, 9]. In this paper we introduce a new class of mappings viz., fuzzy maximal θ-continuous, fuzzy maximal θ-semi-continuous, fuzzy maximal θ-irresolute and fuzzy maximal θ-semi irresolute functions and establish interrelationship among them and some of their properties, characterizations theorems and some applications in details. Some fundamental theorems and their applications are also studied. As for basic preliminaries some definitions and results are given for ready references.

Through this paper X, Y and Z mean fuzzy topological space (fts, for short) in Chang's sense. For a fuzzy set λ of a fts X, the notion I^X, $\lambda^c = 1_X - \lambda$, $Cl(\lambda)$, $Int(\lambda)$, $FM_a\theta$-$Int(\lambda)$, $FM_i\theta$-$Cl(\lambda)$ will respectively stand for the set of all fuzzy subsets of X, fuzzy complement, fuzzy closure, fuzzy interior, fuzzy maximal θ-interior, fuzzy minimal θ-closure of λ. By 1_ϕ(or 0_X or ϕ) and 1_X (or X) we will mean the fuzzy null set and fuzzy whole set with constant membership function 0 (zero function) and 1 (unit function) respectively.

A fuzzy point $x_p \in \lambda$, where λ is a fuzzy subset in X if and only if $p \leq \lambda(x)$. A fuzzy point x_p is quasi-coincident with λ, denoted by $x_p q \lambda$, if and only if $p \geq \lambda'(x)$ or $p + \lambda(x) > 1$ where λ' denotes the complement of λ defined by $\lambda' = 1 - \lambda$. A fuzzy subset λ in a fuzzy topological space X is said to be q-neighbourhood for a fuzzy point x_p if and only if there exist a fuzzy open subset η such that $x_p q \eta \leq \lambda$. A fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy subset λ if and only if for every open q-neighbourhood η of x_p, $Cl\eta$ is quasi-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and is denoted by $Cl_\theta(\lambda)$. The complement of a fuzzy θ-closed subset is a fuzzy θ-open which is equivalent to the condition: a fuzzy subset μ is called fuzzy θ-open if and only if $Int_\theta(\mu) = \mu$, where the fuzzy set $\vee\{x_p \in X :$ for some open q- neighborhood η of $x_p, Cl\eta \subseteq \mu\}$is the fuzzy θ-interior of μ and is denoted by $Int_\theta(\mu)$ and $Int_\beta(\mu)$ is the largest fuzzy β-open set contained in μ.

Definition 1.1. *[7] A fuzzy set λ in a fuzzy topological space (X, τ) is called fuzzy θ-closed set if $\lambda = [\lambda]_\theta$ and it's complement $1_X - \lambda$ is called fuzzy θ-open set in X.*

The collection of all fuzzy θ-open sets and fuzzy θ-closed sets are respectively, denoted by $F\theta$-$O(X)$ and $F\theta$-$C(X)$.

Definition 1.2. *[1] A fuzzy subset λ of fuzzy space (X, τ) is said to be*
(i) fuzzy regular open set if $Int(Cl(\lambda)) = \lambda$
(ii) fuzzy regular closed set if $Cl(Int(\lambda)) = \lambda$. Or if $1_X - \lambda$ is fuzzy regular open set in X.

The class of all fuzzy regular open and fuzzy regular closed sets are, respectively denoted by $FRO(X)$ and $FRC(X)$.

Definition 1.3. *A fuzzy set $\lambda \in I^X$ is said to be fuzzy θ-semi open set in X if $\exists$ a fuzzy θ-open set μ such that $\mu \leq \lambda \leq Cl(\mu)$ (or $\lambda \leq Cl(F_\theta Int(\lambda))$) and it's complement 1_X-λ is called fuzzy θ-semi closed set of X.*

The family of all fuzzy θ-semi open and fuzzy θ-semi closed set are respectively, denoted by $F_\theta O(X)$ and $F_\theta C(X)$.

Definition 1.4. *A nonempty proper fuzzy θ-open set λ of any fuzzy space (X, τ) is said to be fuzzy maximal θ-open set if any fuzzy θ-open set which contains λ is either λ or 1_X.*

Definition 1.5. *A nonempty proper fuzzy θ-closed set β of any fuzzy space (X, τ) is said to be fuzzy minimal θ-closed set if any fuzzy θ-closed set contained in β is either 1_ϕ or β or equivalently, if β^c is fuzzy maximal θ-open set in (X, τ).*

The family of all fuzzy maximal θ-open and fuzzy minimal θ-closed set are respectively, denoted by $FM_a\theta$-$O(X)$ and $FM_i\theta$-C(X).

Lemma 1.1. *[1] If a fuzzy topological space (fts, for short) (X, τ) is product related to anothar fts (Y, σ), then for $\lambda \in I^X$ and $\mu \in I^Y$, $Cl(\lambda \times \mu) = Cl(\lambda) \times Cl(\mu)$.*

Lemma 1.2. *[1] If $f_i : (X_i,\ \tau_i) \longrightarrow (Y_i, \sigma_i)$ fuzzy mapping and λ_i be fuzzy set of Y_i $(i = 1, 2)$. Then, $(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2) = (f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2))$.*

Definition 1.6. *[6] A non empty proper fuzzy subset $\lambda \in I^X$ of any fts (X, τ) is said to be fuzzy maximal θ-semi open set in X if $\exists$ a fuzzy maximal θ-open set δ_1 such that $\delta_1 \leq \lambda \leq Cl(\delta_1)$ or if $\lambda \leq Cl(FM_a\theta\text{-}Int(\lambda))$.*

Definition 1.7. *[6] A non empty proper fuzzy subset $\beta \in I^X$ of any fts (X, τ) is said to be fuzzy minimal θ-semi closed set in X if $\exists$ a fuzzy minimal θ-closed set β_1 such that $Int(\beta_1) \leq \beta \leq \beta_1$ or if $FM_i\theta\text{-}Cl(Int(\lambda)) \leq \lambda$.*

Or, equivalently, if the complement (i.e $1_X - \beta$) of β is fuzzy maximal θ-semi open set in X.

Or, equivalently, the complement of a fuzzy maximal θ-semi open set is called fuzzy minimal θ-semi closed set in X.

The family of all fuzzy maximal θ-semi open and fuzzy minimal θ-semi closed sets are respectably denoted by $FM_a\theta$-$SO(X)$ and $FM_i\theta$-$SC(X)$.

2 Fuzzy Maximal θ-Continuous (resp. Fuzzy Maximal θ-Semi Continuous) and Fuzzy Maximal θ-Irresolute (resp. Fuzzy Maximal θ-Semi Irresolute) Functions

In this section we introduce some new notions of fuzzy mappings viz., fuzzy maximal θ-continuous (fuzzy maximal θ-semi continuous) and fuzzy maximal θ-irresolute, fuzzy θ-semiirresolute functions. We also establish some of their characterization theorems and show some interrelationships among these new classes of functions.

Definition 2.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy maximal θ-continuous (shortly, $FM_a\theta$-continuous) iff for each $\lambda \in FO(Y)$, $f^{-1}(\lambda) \in FM_a\theta\text{-}O(X)$.*

Example 2.1 Consider the function $f : (X,\ \tau_1) \to (Y, \tau_2)$ defined by $f(x) = x$, $\forall x \in X$, where (X, τ_1) defined in Example 2.1 [6] and (Y, τ_2) is defined as $Y = \{a, b, c\}$, $\tau_2 = \{0_Y, 1_Y, C\}$, where $C(a) = 0$, $C(b) = 1$, $C(c) = 1$. Here C is the only non-empty proper fuzzy open set in Y and also it is fuzzy maximal θ-open set in X such that $f^{-1}(C(x)) = C(f(x)) = C(x) = A_1(x) \in FM_a\theta\text{-}O(X)$. Thus f is fuzzy maximal θ-continuous function on X.

Theorem 2.1. *For the mapping $f : (X,\ \tau) \to (Y, \sigma)$ the following statements are equivalent:*
(a) f is $FM_a\theta$-continuous function.
(b) for every fuzzy point x_r of X and for every fuzzy neighbourhood η of $f(x_r)$ in $(Y,\ \sigma)$, $\exists$ a fuzzy maximal θ-open neighbourhood ν of x_r in $(X,\ \tau)$ such that $f(\nu) \leq \eta$.
(c) $f^{-1}(\beta) \in FM_i\theta\text{-}C(X)$, $\forall \beta \in FC(Y)$.

Proof. We need to prove the following implications: (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c) and (c) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b). Let f be $FM_a\theta$-continuous function and let $x_r \in X$ and η be any fuzzy neighbourhood of $f(x_r)$ in Y. Then $\exists$ a $\mu \in \sigma$ such that $f(x_r) \leq \mu \leq \eta \Rightarrow x_r \in f^{-1}(\mu) \leq f^{-1}(\eta)$. As f is $FM_a\theta$-continuous and $\mu \in \sigma$. So, $f^{-1}(\mu) \in FM_a\theta\text{-}O(X)$ so that $\nu = f^{-1}(\eta)$ is a fuzzy maximal θ-neighbourhood of x_r in X such that $f(\nu) = \eta \leq \eta$.

(b) $\Rightarrow$ (c). Let (b) is true for the function f and let β be a closed set in Y and $x_r \in f^{-1}(\beta^c) \Rightarrow f(x_r) \in \beta^c$. Since β^c is open neighbourhood of $f(x_r)$, so by hypothesis, $\exists$ a fuzzy maximal θ-neighbourhood ν of x_r in X such that

$f(\nu) \leq \beta^c$ so that $\nu \leq f^{-1}(\beta^c)$. Since ν is fuzzy maximal θ-neighbourhood of x_r, $\exists$ a fuzzy maximal θ-open set G such that $x_r \in G \leq f^{-1}(\beta^c) \Rightarrow \bigcup\{x_r\} \leq \bigcup\{G\} \leq \bigcup\{f^{-1}(\beta^c)\} \Rightarrow f^{-1}(\beta^c)\} \leq \bigcup\{G\} \leq \bigcup f^{-1}(\beta^c) \Rightarrow (f^{-1}(\beta))^c = \bigcup\{G\} = G$ or $1_X \in FM_a\theta\text{-}O(X) \Rightarrow f^{-1}(\beta) \in FM_i\theta\text{-}C(X)$.

(c) $\Rightarrow$ (a). Let (c) is true and let $\lambda \in FO(Y)$. Then $f^{-1}(\lambda) = f^{-1}((\lambda^c)^c) = (f^{-1}(\lambda^c))^c$. Since, $\lambda^c \in FC(Y)$, by hypothesis, we have, $f^{-1}(\lambda^c) \in FM_i\theta\text{-}C(X)$ and hence $(f^{-1}(\lambda^c))^c = f^{-1}(\lambda) \in FM_a\theta\text{-}O(X)$ showing that f is $FM_a\theta$-continuous function on X.

Theorem 2.2. *For $FM_a\theta$-continuous mapping f from a fts (X, τ) into another fts (Y, σ) following statements hold:*
(i) $f(FM_a\theta\text{-}Int(\mu)) \geq Intf(\mu)$, for every fuzzy set μ in X.
(ii) $FM_a\theta\text{-}Int(f^{-1}(\lambda)) \geq f^{-1}(Int(\lambda))$, for every fuzzy set λ in Y and for onto map f.
(iii) $f(FM_i\theta\text{-}Cl(\mu)) \leq Clf(\mu)$, for every fuzzy set μ in X.
(iv) $FM_i\theta\text{-}Cl(f^{-1}(\lambda)) \leq f^{-1}(Cl(\lambda))$, for every fuzzy set λ in Y and for onto map f.

Proof. (i) Since, $Int(f(\mu))$ is fuzzy open set in Y and f is $FM_a\theta$-continuous, $f^{-1}(Intf(\mu)) \in FM_a\theta\text{-}O(X)$. As we know that $f(\mu) \geq Intf(\mu) \Rightarrow \mu \geq f^{-1}(Intf(\mu)) \Rightarrow FM_a\theta\text{-}Int(\mu) \geq f^{-1}(Intf(\mu))$ so that $f(FM_a\theta\text{-}Int(\mu)) \geq Intf(\mu)$.

(ii) Since, $f^{-1}(\lambda)$ is a fuzzy set in X, so far $\mu = f^{-1}(\lambda)$ (i) must holds i.e., $f(FM_a\theta\text{-}Int(f^{-1}(\lambda))) \geq Int(f(f^{-1}(\lambda))) = Int(\lambda$ [As f is onto mapping]. Hence, $FM_a\theta\text{-}Int(f^{-1}(\lambda)) \geq f^{-1}(Int(\lambda))$.

(iii) Since, $Cl(f(\mu))$ is fuzzy closed set in Y and f is $FM_a\theta$-continuous, $f^{-1}(Cl(f(\mu)) \in FM_a\theta\text{-}C(X)$. Now $f(\mu) \leq Cl(f(\mu)) \Rightarrow \mu \leq f^{-1}(Cl(f(\mu))) \Rightarrow FM_i\theta\text{-}Cl(\mu) \leq f^{-1}(Cl(f(\mu)))$. Thus $f(FM_i\theta\text{-}Cl(\mu)) \leq Cl(f(\mu))$.

(iv) Since, $f^{-1}(\lambda) \in I^X$, $\forall\lambda \in I^Y$, so for $\mu = f^{-1}(\lambda)$ we have from (iii) $f(FM_i\theta\text{-}Cl(f^{-1}(\lambda))) \geq Clf(f^{-1}(\lambda)) = Cl(\lambda)$ [Being f an onto map]. Hence, $FM_a\theta\text{-}Int(f^{-1}(\lambda)) \geq f^{-1}(Cl(\lambda))$.

Definition 2.2. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be fuzzy maximal θ-irresolute (shortly, $FM_a\theta$-irresolute) iff for each $\lambda \in FM_a\theta\text{-}O(Y)$, $f^{-1}(\lambda) \in FM_a\theta\text{-}O(X)$.*

Example 2.2 Let $f : (X, \tau) \rightarrow (X, \tau)$ be a function defined by $f(x) = x$, $\forall x \in X$, where, (X, τ) is defined in Example 2.1 [6]. Since, for $A_1 \in FM_a\theta\text{-}O(X)$, $f^{-1}(A_1(x)) = A_1(f(x)) = A_1(x) \in FM_a\theta\text{-}O(X)$, f is fuzzy maximal θ-irresolute function on X.

Definition 2.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy maximal θ-semi irresolute iff for each $\lambda \in FM_a\theta$-$SO(Y)$, $f^{-1}(\lambda) \in FM_a\theta$-$SO(X)$.*

Example 2.3 Let $f : (X, \tau) \to (X, \tau)$ be a function defined by $f(x) = x$, $\forall x \in X$, where, (X, τ) is defined in Example 2.2 [6]. Since, for $A_4 \in FM_a\theta$-$SO(X)$, $f^{-1}(A_4(x)) = A_4(f(x)) = A_4(x) \in FM_a\theta$-$SO(X)$, f is fuzzy maximal θ-semi irresolute function on X.

Theorem 2.3. *If $f : (X, \tau_1) \to (Y, \tau_2)$ be $FM_a\theta$-continuous function and $g : (Y, \tau_2) \to (Z, \tau_3)$ be fuzzy continuous function. Then $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ is also $FM_a\theta$-continuous function.*

Proof. $\lambda \in FO(Z)$. Now, $(g \circ f)^{-1}(\lambda) = (f^{-1} \circ g^{-1})(\lambda) = (f^{-1}(g^{-1}(\lambda))$. Since g is fuzzy continuous, $g^{-1}(\lambda)$ is fuzzy open and then $(g \circ f)^{-1}(\lambda) = (f^{-1}($ fuzzy open in $Y))$. But f being $FM_a\theta$-continuous $(g \circ f)^{-1}(\lambda) \in FM_a\theta$-$O(X)$. This shows that $g \circ f$ is $FM_a\theta$-continuous function.

Theorem 2.4. *If $f : (X, \tau_1) \to (Y, \tau_2)$ be $FM_a\theta$-irresolute function and $g : (Y, \tau_2) \to (Z, \tau_3)$ be fuzzy $FM_a\theta$-continuous function. Then $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ is also $FM_a\theta$-continuous function.*

Proof. $\lambda \in FO(Z)$. Now, $(g \circ f)^{-1}(\lambda) = (f^{-1} \circ g^{-1})(\lambda) = (f^{-1}(g^{-1}(\lambda))$. Since g is fuzzy $FM_a\theta$-continuous, $g^{-1}(\lambda)$ is fuzzy $FM_a\theta$-open and then $(g \circ f)^{-1}(\lambda) = (f^{-1}(FM_a\theta$ fuzzy open set in $Y))$. But f being $FM_a\theta$-irresolute $(g \circ f)^{-1}(\lambda) \in FM_a\theta$-$O(X)$. This shows that $g \circ f$ is $FM_a\theta$-continuous function.

Theorem 2.5. *Composition of two $FM_a\theta$-irresolute function is again a $FM_a\theta$-irresolute function.*

Proof. Straight forward.

Definition 2.4. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy maximal θ-semi continuous (shortly, $FM_a\theta$-S-continuous) iff for each $\lambda \in FO(Y)$, $f^{-1}(\lambda) \in FM_a\theta$-$SO(X)$.*

Example 2.4 Consider the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = x$, $\forall x \in X$, where, (X, τ_1) defined in Example 2.2 [6] and (Y, τ_2) is defined as $Y = \{a, b\}$, $\tau_2 = \{0_Y, C, 1_Y\}$, where, $C(a) = \frac{9}{10}$, $C(b) = \frac{7}{8}$ Here C is the only non-empty proper fuzzy open set in Y and in Example 2.2 [6] we have shown that $C \in FM_a\theta$-$SO(X)$. Since, $f^{-1}(C(x)) = C(f(x)) = C(x) = A_4(x) \in FM_a\theta$-$SO(X)$. Thus f is fuzzy maximal θ-semi continuous function on X.

Theorem 2.6. *Let X_i, Y_i $(i = 1,\ 2)$ be fts. s. t. X_1 is product related to X_2 and $f_i : (X_i, \tau_i) \to (Y_i, \sigma_i)$ $(i = 1, 2)$ fuzzy maximal θ-semi continuous function. Then, $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is also fuzzy maximal θ-semi continuous function on $X_1 \times X_2$.*

Proof. Let $\lambda \in FO(Y_1)$, $\mu \in FO(Y_2)$. Then $\lambda \times \mu \in FO(Y_1 \times Y_2)$. Using Lemma 1.8 [1] we have, $(f_1 \times f_2)^{-1}(\lambda \times \mu) = f_1^{-1}(\lambda) \times f_2^{-1}(\mu)$. Since, f_i is fuzzy maximal θ-semi continuous function on X_i. So $f_i^{-1}(\lambda)$ is fuzzy maximal θ-semi open set of X_1. Again, since, X_1 is product related to X_2. So, by Theorem 2.10 [6], $(f_1 \times f_2)^{-1}(\lambda \times \mu) = f_1^{-1}(\lambda) \times f_2^{-1}(\mu) \in FM_a\theta\text{-}SO(X_1 \times X_2)$ and hence, $f_1 \times f_2$ is fuzzy maximal θ-semi continuous function on $X_1 \times X_2$.

Theorem 2.7. *Let X_i, Y_i $(i = 1, 2, 2, ..., n)$ be fts. s. t. X_i is product related to X_j $(i \neq j)$ and $f_i : (X_i, \tau_i) \to (Y_i, \sigma_i)$ $(i = 1, 2, 3, ..., n)$ fuzzy maximal θ-semi continuous function. Then, $\prod_{i=1}^{n} f_i : \prod_{i=1}^{n} X_i \to \prod_{i=1}^{n} Y_i$ is also fuzzy maximal θ-semi continuous function on $\prod_{i=1}^{n}$.*

Proof. Obvious.

Theorem 2.8. *Let $f : X \to Y$ be a function, defined by $f(x) = y$, $\forall x \in X$ and $g : X \to X \times Y$ a graph of the map f defined by $g(x) = (x, f(x))$, $\forall x \in X$. If g is fuzzy maximal θ-semi continuous, then so is f.*

Proof. Let $\mu \in FO(Y)$. Then for $1_X \in FO(X)$, $1_X \times \mu$ is a fuzzy open set in $X \times Y$. Since, g is a graph of the map f, so, $g(x) = (x, y) = (x, f(x))$, $\forall x \in X$. Now $\forall x \in X$ we have, $g^{-1}(1_X \times \mu)(x) = (1_X \times \mu)(g(x)) = (1_X \times \mu)(x, f(x)) = min\{1_X(x), \mu f(x)\} = 1_X(x) \wedge f^{-1}(\mu)(x) = (1_X \wedge f^{-1}(\mu))(x) = f^{-1}(\mu)(x)$. Since g is fuzzy maximal θ-semi continuous, so, $g^{-1}(1_X \times \mu) = f^{-1}(\mu) \in FM_a\theta\text{-}SO(X)$, $\forall \mu \in FO(Y)$. Hence, f is fuzzy maximal θ-semi continuous function on X.

Theorem 2.9. *Every fuzzy maximal θ-continuous function is fuzzy maximal θ-semi continuous function.*

Proof. Proof follows from Corollary 2.1 (a) [6], i.e., from the fact that Every fuzzy maximal θ-open set is fuzzy maximal θ-semi-open set in a fts (X, τ).

The converse of the above Theorem need not be true as seen from the following Example.

Example 2.5 Consider the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = x$, $\forall x \in X$, where, (X, τ_1) defined in Example 2.2 [6] and (Y, τ_2) is defined as $Y = \{a, b\}$, $\tau_2 = \{0_Y, C, 1_Y\}$, where, $C(a) = \frac{9}{10}$, $C(b) = \frac{7}{8}$. Here C is the only non-empty proper fuzzy open set in Y and in Example 2.2 [6] we have shown that $C \in FM_a\theta\text{-}SO(X)$. Since, $f^{-1}(C(x)) = C(f(x)) = C(x) = A_4(x)$ which is a $FM_a\theta$-semi open but not $FM_a\theta$-open set in X. Thus f is fuzzy maximal θ-semi continuous function but not fuzzy maximal θ-continuous function on X.

Theorem 2.10. *Every fuzzy maximal θ-irresolute function is fuzzy maximal θ-semi irresolute function.*

Proof. Obvious.

Definition 2.5. *(a) A collection M is said to be fuzzy maximal θ-open cover (shortly, $FM_a\theta$-open cover) of a fuzzy set $\mu \in I^X$ iff M covers μ and each member of M is fuzzy maximal θ-open set in X i.e., $\mu \leq Sup\{\mu_\alpha \in FM_a\theta\text{-}O(X) : \mu_\alpha \in M,\ \forall \alpha \in \wedge\}$.*
(b) A collection M is said to be fuzzy maximal θ-semi open cover (shortly, $FM_a\theta$-semi open cover) of a fuzzy set $\mu \in I^X$ iff M covers μ and each member of M is fuzzy maximal θ-semi open set in X i.e., $\mu \leq Sup\{\mu_\alpha \in FM_a\theta\text{-}SO(X) : \mu_\alpha \in M,\ \forall \alpha \in \wedge\}$.

Definition 2.6. *(a) A fuzzy set $\lambda \in I^X$ of a fts (X, τ) is said to be fuzzy maximal θ-compact (shortly, $FM_a\theta$-compact) iff for each $FM_a\theta$-open cover M of λ has a finite subcover M_0 which also covers λ.*
(b) A fuzzy set $\lambda \in I^X$ of a fts (X, τ) is said to be fuzzy maximal θ-semi compact (shortly, $FM_a\theta$-semi compact) iff for each $FM_a\theta$-semi open cover M of λ has a finite subcover M_0 which also covers λ.

Theorem 2.11. *(a) Fuzzy maximal θ-continuous image of a $FM_a\theta$-compact set is fuzzy compact.*
(b) Fuzzy maximal θ-semi continuous image of a $FM_a\theta$-S-compact set is fuzzy compact.

Proof. (a) Let $f : X \to Y$ be fuzzy maximal θ-continuous and $B \in I^X$, a $FM_a\theta$-compact set of a fts X and $P = \{\mu_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $f(B)$ such that $f(B) \leq SupP \Rightarrow B \leq f^{-1}(f(B)) \leq f^{-1}(Sup\{\mu_\alpha : \alpha \in \Lambda\}) = Sup\{f^{-1}(\mu_\alpha) : \alpha \in \Lambda\}$. Then, $Q = \{f^{-1}(\mu_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of B. Since f is fuzzy maximal θ-continuous function, $f^{-1}(\mu_\alpha) \in FM_a\theta\text{-}O(X)$, $\forall \alpha \in \Lambda$, an arbitrary index set and then Q is $FM_a\theta$-open cover of B. Since, B is $FM_a\theta$-compact, $\exists$ a finite sub cover $Q = \{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\}$ of Q such that $B \leq Sup\{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\}$. Since each $f^{-1}(\mu_\alpha)$ is distinct $FM_a\theta$-open set in X. So, $Sup\{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\} = 1_X$ so that $A \leq 1_X \Rightarrow f(A) \leq f(1_X) = 1_Y$. This shows that $P_\epsilon = \{1_Y\}$ is the existing finite subcover of P. Hence, $f(B)$ is compact set in Y.

(b) Same as the proof of (a).

Theorem 2.12. *(a) If $f : X \to Y$ is fuzzy maximal θ-irresolute function and $A \in I^X$, a $FM_a\theta$-compact set of X, then $f(A)$ is $FM_a\theta$-compact set in Y.*
(b) If $f : X \to Y$ is fuzzy maximal θ-semi irresolute function and $A \in I^X$, a $FM_a\theta$-semi compact set of X, then $f(A)$ is $FM_a\theta$-semi compact set in Y.

Proof. (a) Let A be a $FM_a\theta$-compact set of X and $Q = \{\mu_\alpha : \alpha \in \Lambda\}$ be a fuzzy $FM_a\theta$-open cover of $f(A)$ such that $f(A) \leq SupQ$. Then, $P = \{f^{-1}(\mu_\alpha) : \alpha \in \Lambda\}$ is a cover of A. Since f is fuzzy maximal θ-irresolute function, each $f^{-1}(\mu_\alpha) \in FM_a\theta\text{-}O(X)$, $\forall \alpha \in \Lambda =$ arbitrary index set and then P is $FM_a\theta$-open cover of A. Since, A is $FM_a\theta$-compact, $\exists$ a finite sub cover $P_\epsilon = \{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\}$ of P such that $A \leq Sup\{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\}$. Since, each $f^{-1}(\mu_\alpha)$ is distinct $FM_a\theta$-open set in X. So, $Sup\{f^{-1}(\mu_\alpha) : \alpha = 1, 2, 3, ..., n\} = 1_X$ so that $A \leq 1_X \Rightarrow f(A) \leq f(1_X) = 1_Y$. This shows that $Q_0 = \{1_Y\}$ is existing finite $FM_a\theta$-open subcover of Q. Hence, $f(B)$ is $FM_a\theta$-compact set in Y.

(b) Same as the proof of (a).

Definition 2.7. *(a) Two non-empty fuzzy sets λ and μ of a fuzzy space (X, τ) are said to be fuzzy maximal θ-separated (in short, $FM_a\theta$-separated) if $FM_a\theta$-$Cl(\lambda)q\mu$ and $FM_a\theta$-$Cl(\mu)q\lambda$.*
(b) Two non-empty fuzzy sets λ and μ of a fuzzy space (X, τ) are said to be fuzzy maximal θ-semi separated (in short, $FM_a\theta$-S-separated) if $FM_a\theta$-S-$Cl(\lambda)q\mu$ and $FM_a\theta$-S-$Cl(\mu)q\lambda$.
(c) A fuzzy set β is said to be fuzzy maximal θ-connected (shortly, $FM_a\theta$-connected) iff β can't be expressed as the union of two $FM_a\theta$-separated sets λ and μ of X.
(d) A fts X is said to be fuzzy maximal θ-connected (shortly, $FM_a\theta$-connected) iff X can't be expressed as the union of two non empty disjoint $FM_a\theta$-open sets λ and μ i.e., $X \neq \lambda \vee \mu$, where $\lambda, \mu \in FM_a\theta\text{-}O(X)$.
(e) A fuzzy set β is said to be fuzzy maximal θ-semi connected (shortly, $FM_a\theta$-S-connected) iff β can't be expressed as the union of two $FM_a\theta$-semi separated sets λ and μ of X.
(f) A fts X is said to be fuzzy maximal θ-semi connected (shortly, $FM_a\theta$-semi connected) iff X can't be expressed as the union of two non empty disjoint $FM_a\theta$-semi open sets λ and μ i.e., $X \neq \lambda \vee \mu$, where $\lambda, \mu \in FM_a\theta\text{-}S\text{-}O(X)$.

Theorem 2.13. *A fuzzy subset $\lambda \in I^X$ of a fts (X, τ) is $FM_a\theta$-connected (resp. $FM_a\theta$-semi connected) iff X can't be expressed as the union of two non empty disjoint $FM_a\theta$-closed sets ($FM_a\theta$-semi closed sets).*

Proof. Follows from Definition.

Theorem 2.14. *(a) If $f : X \to Y$ is $FM_a\theta$-continuous surjection map and X is $FM_a\theta$-connected, then Y is fuzzy connected.*
(b) If $f : X \to Y$ is $FM_a\theta$-semi continuous surjection map and X is $FM_a\theta$-semi connected, then Y is fuzzy connected.

Proof. (a) Suppose that $f(X) = Y$ is not fuzzy connected space. Then, $\exists$ non empty fuzzy open sets λ and μ such that $f(X) = \lambda \vee \mu \Rightarrow$ Both λ and

μ are fuzzy clopen sets in Y. Then $X = f^{-1}(\lambda) \vee f^{-1}(\mu)$. Since f is $FM_a\theta$-continuous and λ and μ are non empty disjoint fuzzy closed sets, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are also non empty disjoint and $\in FM_a\theta\text{-}C(X)$. This shows that X is not $FM_a\theta$-connected which is a contradiction to the given hypothesis. Hence, Y is fuzzy connected.

(b) Similar to the proof of (a).

Theorem 2.15. *(a) If $f : X \to Y$ is $FM_a\theta$-irresolute surjection map and X is $FM_a\theta$-connected, then Y is fuzzy $FM_a\theta$-connected.*
(b) If $f : X \to Y$ is $FM_a\theta$-semi irresolute surjection map and X is $FM_a\theta$-semi connected, then Y is fuzzy semi-connected.

Proof. (a) Suppose that $f(X) = Y$ is not $FM_a\theta$-connected space. Then, $\exists$ non empty fuzzy open sets λ and μ such that $f(X) = \lambda \vee \mu \Rightarrow$ Both λ and μ are fuzzy $FM_a\theta$-open as well as $FM_a\theta$-closed sets in Y. Then $X = f^{-1}(\lambda) \vee f^{-1}(\mu)$. Since λ and μ are non empty disjoint $FM_i\theta$-closed sets and f is f is $FM_a\theta$-irresolute surjection, $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are also non empty disjoint and $\in$ $FM_a\theta\text{-}C(X)$ such that $X = f^{-1}(\lambda) \vee f^{-1}(\mu)$. This shows from Theorem 2.13. that X is not $FM_a\theta$-connected which is a contradiction to the given hypothesis that X is $FM_a\theta$-connected. Hence, Y is fuzzy $FM_a\theta$-connected.

(b) Similar to the proof of (a).

3 Conclusion

In this paper, we introduce fuzzy maximal θ-semi continuous to create some applications which is fuzzy maximal θ-semi generalized continuity, fuzzy maximal θ-semi generalized irresolute and fuzzy maximal θ-semi generalized closed maps. We also investigate the relationship of some maximal closed sets which is related to fuzzy maximal θ-semi generalized closed sets. This will give some new relationships which have be found to be useful in study of generalized closed sets and generalized continuities in fuzzy topological spaces.

References

[1] K.K. Azad, On fuzzy continuity, Fuzzy almost continuity and Fuzzy semi continuituy, *J. Math. Anal. Appl.*, 82(1981), 14-32.

[2] C.L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24(1968), 182-190.

[3] F. Nakoka and N. Oda, Some properties of maximal open sets, *Int. J. Math. and Math. Sci.*, 21(2003), 1331-1340.

[4] F. Nakoka and N. Oda, Some applications of minimal open sets, *Int. J. Math. and Math. Sci.*, 27(2001), 471-476.

[5] P. Pao-Ming and L. Ying-Ming, Fuzzy topology I: Neighbourhood structure of a fuzzy pointand Moore-smith convergence, *J. Math. Anal. and Appl.*, 76(2) (1980), 571-599.

[6] M. Pritha, V. Chandrasekar and A. Vadivel, On fuzzy maximal θ-semi open and fuzzy minimal θ-semi closed sets, *(to appear)*.

[7] M.E. El-Shafei and A. Zakari, θ-generalized closed sets in fuzzy topological spaces, *Arab. J. Sci. Eng. Sect. A. Sci.*, 31(2) (2006), 197-206.

[8] L.A. Zadeh, Fuzzy sets, *Information and Control*, 8(1965), 338-353.

[9] C.K. Wong, Fuzzy topology: Product and quotient theorems, *J. Math. Anal. Appl.*, 45(1974), 512-552.