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Eccentric Connectivity Index, Hyper and Reverse-Wiener Indices of the Subdivision Graph

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Abstract

If G *is a connected graph with vertex set* V*, then the* eccentric connectivity index *of* G, $\xi^{(c)}(G)$ *is defined as* $\sum deg(v).ec(v)$ *where* $deg(v)$ *is the degree of a vertex* v *and* $ec(v)$ *is its eccentricity. The* Wiener index $W(G) = \frac{1}{2}[\sum d(u,v)]$, *the* hyper-Wiener index $WW(G) = \frac{1}{2}[\sum d(u,v) + \sum d^2(u,v)]$ *and the* reverse-Wiener index $\wedge(G) = \frac{n(n-1)D}{2} - W(G)$, *where* $d(u,v)$ *is the distance of two vertices* u, v *in* G, $d^2(u,v) = d(u,v)^2$, $n = |V(G)|$ *and* D *is the diameter of* G. *In this paper, we determine the* eccentric connectivity index *of the* subdivision graph *of the* complete graphs, tadpole graphs and the wheel graphs. *Also, derive an expressions for the* hyper and reverse-Wiener indices *of the same class of graphs.*

Keywords: *Eccentric connectivity index, Wiener index, Hyper-Wiener index, Reverse-Wiener index, Subdivision Graph.*

1 Introduction

Critical step in pharmaceutical drug design continues to be the identification and optimization of compounds in a rapid and cost effective way. An important tool in this work is the prediction of physico-chemical, pharmacological and toxicological properties of a compound directly from its molecular structure. This analysis is known as the study of the *Quantitative Structure Activity relationship (QSAR)*. In chemistry, a *molecular graph* represents the topology of a molecule, by considering how the atoms are connected. This can be modelled by a graph, where the points represent the atoms, and the edges symbolize the covalent bonds. Relevant properties of these graph models are studied, giving rise to numerical graph invariants. The parameters derived from this graph-theoretic model of a chemical structure are being used not only in $QSAR$ studies pertaining to molecular design and pharmaceutical drug design, but also in the environmental hazard assessment of chemicals.

Many such graph invariant *topological indices* have been studied. The first, and most well-known parameter, the *Wiener index*, was introduced in the late 1940's in an attempt to analyze the chemical properties of paraffins (alkanes) [20]. This is a distance-based index, whose mathematical properties and chemical applications have been widely researched. Numerous other indices have been defined, and more recently, indices such as the eccentric distance sum, and the adjacency- cum-distance-based *eccentric connectivity index* have been considered [6, 7, 8, 10, 13]. These topological models have been shown to give a high degree of predictability of pharmaceutical properties, and may provide leads for the development of safe and potent anti-HIV compounds.

The *hyper-Wiener index* of acyclic graphs was introduced by Randic in 1993. Then Klein et al. generalized Randic's definition for all connected graphs, as a generalization of the *Wiener index* [12]. For the mathematical properties of *hyper- Wiener index*[9] and its applications in chemistry we refer to [3, 4, 9]. The *reverse-Wiener index*[2] was proposed by Balaban et al. in 2000 [1]; it is important for a reverse problem and applications in modelling of structure property relations. Some mathematical properties of the reverse-Wiener index may be found in [2].

Consider a simple connected graph G, and let $V(G)$ and $E(G)$ denote its vertex and edge sets, respectively. The distance between u and v in $V(G)$, $d(u,v)$ is the length of the shortest $u-v$ path in G. The *eccentricity*, $ec(u)$ of a vertex $u \in V(G)$ is the maximum distance between u and any other vertex in G. The diameter D of G, is defined as the maximum value of the eccentricities of the vertices of G. Finally, the degree of a vertex $v \in V(G)$, $deg(v)$ is the number of edges incident to v. We recall definitions which are essential to our work.

- The eccentric connectivity index $\xi^c(G) = \sum deg(v).ec(v)$.
- The Wiener index $W(G) = \frac{1}{2}[\sum d(u,v)]$.
- The hyper Wiener index $WW(G) = \frac{1}{2}[\sum d(u,v) + \sum d^2(u,v)]$.
- The reverse Wiener index $\wedge(G) = \frac{n(n-1)D}{2} - W(G)$.

The $T_{n,k}$ Tadpole graph [5, 18] is the graph obtained by joining a cycle graph C_n to a path of length k. The wheel graph W_{n+1} [14] is defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph. The *subdivision graph* [15, 19] $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently, by inserting an additional vertex into each edge of G [16, 17]. For all terminologies and notations which are not defined in this paper, we refer to F. H. Harary [11].

2 Eccentric Connectivity Index, Hyper and Reverse-Wiener Indices of the Subdivision Graph

In this section, we derived an expression for the *eccentric connectivity index, Wiener index, hyper and reverse-Wiener indices* of the *subdivision graphs* of the *complete graphs, tadpole graphs and the wheel graphs.*

Theorem 2.1. *The eccentric connectivity index of the subdivision graph of the complete graph S_n is*

$$\xi^{(c)}(S(S_n)) = \begin{cases} 7n(n-1), & \text{for } n \geq 4; \\ 18, & \text{for } n=3. \end{cases}$$

Proof. The cardinality of the vertex set of the $S(S_n) = \frac{n(n+1)}{2}$, among which the n vertices are of valency $n-1$ and the remaining $\frac{n(n-1)}{2}$ vertices are the subdivision vertices. For the case $n = 3$ $S(S_n)$ is the cycle C_6, in which each vertex of same eccentricity 3. Hence $\xi^{(c)}(S(S_n)) = 18$, for the case $n = 3$. For $n \geq 4$, the vertex with degree $n-1$ is of *eccentricity* 3 and the *subdivision* vertices are of *eccentricity* 4. Hence $\xi^{(c)}(S(S_n)) = 7n(n-1)$.

Theorem 2.2. *The eccentric connectivity index of the subdivision graph of the wheel graph W_{n+1} is*

$$\xi^{(c)}(S(W_{n+1})) = \begin{cases} 38n, & \text{when } n \geq 6. \\ 180, & \text{when } n=5; \\ 124, & \text{when } n=4; \\ 84, & \text{when } n=3. \end{cases}$$

Proof. The $S(W_{n+1})$ contains the subgraph $S(C_n)$ and the *hub* of the wheel is of degree n and the n vertices are of degree 3, and the remaining $2n$ vertices are the *subdivision* vertices. For all values of n, $n \geq 3$, the *hub* of the wheel is of eccentricity 3 and the *subdivision* vertices on the spokes are of eccentricity 4. The vertex of degree 3 is of eccentricity 3 for the case $n = 3$ and is of eccentricity 4 for the case $n = 4$ and is of eccentricity 5 for the case $n \geq 5$. The *subdivision* vertices of C_n in $S(W_{n+1})$ are of eccentricity 4 for the

case $n = 3$ and $n = 4$ and are of eccentricity 5 for the case $n = 5$ and are of eccentricity 6 for the case $n \geq 6$.
Hence,
$\xi^{(c)}(S(W_{n+1})) = 3.n + 2.4.n + 3.3.n + 2.4.n = 28n = 84,$ *for the case* $n = 3$.
$\xi^{(c)}(S(W_{n+1})) = 3.n + 2.4.n + 3.4.n + 2.4.n = 31n = 124,$ *for the case* $n = 4$.
$\xi^{(c)}(S(W_{n+1})) = 3.n + 2.4.n + 3.5.n + 2.5.n = 36n = 180,$ *for the case* $n = 5$.
$\xi^{(c)}(S(W_{n+1})) = 3.n + 2.4.n + 3.5.n + 2.6.n = 38n,$ *for the case* $n \geq 6$.

Theorem 2.3. *The eccentric connectivity index of the subdivision graph of the tadpole graph $T_{n,k}$ is*

$$\xi^{(c)}(S(T_{n,k})) = \begin{cases} 12k(k+n), & \textit{for the case } n{=}2k; \\ 12nk + 12k^2 + 4n + 2n(2n-4k-2), & \textit{for the case } n > 2k; \\ \frac{5}{2}n^2 + 10nk + 6k^2, & \textit{when } n < 2k \textit{ and } n \textit{ even;} \\ \frac{5}{2}n^2 + 10nk + 6k^2 + \frac{1}{2}, & \textit{when } n < 2k \textit{ and } n \textit{ odd.} \end{cases}$$

Proof. The $S(T_{n,k})$ contains the the subgraph $S(C_n)$ and the path $S(P_k)$. We calculate $\sum d(u)e(u)$ along $S(C_n)$ and along $S(P_k)$. Let v_1 be the vertex in $S(C_n)$ with maximum eccentricity $n+2k$. Then the two neighbors of v_1 say v_2 and $v_{2'}$ are of eccentricity $n + 2k - 1$, and the neighbors of v_2 and $v_{2'}$ are of eccentricity $n+2k-2$, and so on. Let v_l be the vertex of degree 3 in $S(T_{n,k})$.

For the case $n > 2k$, the $4k + 1$ vertices in $S(C_n)$ are of eccentricity $n + 2k, n + 2k - 1, n + 2k - 1, n + 2k - 2, n + 2k - 2, ..., n, n$. Then from the remaining $2n - 4k - 1$ vertices in $S(C_n)$, the $2n - 4k - 2$ vertices of degree 2 and v_l of degree 3 are of eccentricity n.
Hence $\sum d(u)e(u)$ in $S(C_n)$ for the case $n > 2k$ is

$$8nk + 8k^2 + 5n + 2n(2n - 4k - 2) \tag{1}$$

For the case $n = 2k$, the vertex v_l of degree 3 is of eccentricity n. Hence $\sum d(u)e(u)$ in $S(C_n)$ for the case $n = 2k$ is

$$8nk + 8k^2 + n \tag{2}$$

For the case $n < 2k$, for any value of n, the vertices in $S(C_n)$ are of eccentricity $n+2k, n+2k-1, n+2k-1, ..., 2k+1, 2k+1$ and the vertex v_l is of eccentricity $2k$. Hence $\sum d(u)e(u)$ in $S(C_n)$ for the case $n < 2k$ is

$$2n^2 + 8nk + 2k \tag{3}$$

Now, we calculate $\sum d(u)e(u)$ along the vertices in the path. Let $v_{l_1}e_1v_{l_2}e_2...v_{l_{2k-1}}v_l$ be the path $S(P_k)$ in $S(T_{n,k})$. The pendent vertex v_{l_1} is of maximum eccentricity $n + 2k$.
For the case $n \geq 2k$, for all values of n, the vertices of $S(P_k)$ starting from v_{l_1}

is of eccentricity $n+2k, n+2k-1, ..., n+1$. Hence $\sum d(u)e(u)$ along the path for the case $n \geq 2k$ is

$$4nk + 4k^2 - n \tag{4}$$

For the case $n < 2k$, $e(v)$ in $S(P_k)$ when n odd, starting from v_{l_1}, the first $\lceil \frac{n+2k}{2} \rceil$ vertices are of eccentricity $n+2k, n+2k-1, n+2k-2, ..., \lceil \frac{n+2k}{2} \rceil$ and the remaining $2k - \lceil \frac{n+2k}{2} \rceil$ vertices are of eccentricity $\lceil \frac{n+2k}{2} \rceil, \lceil \frac{n+2k}{2} \rceil + 1, ..., 2k-1$. Hence $\sum d(u)e(u)$ along the path for the case $n < 2k$ and n odd is

$$\frac{1}{2}n^2 + 2nk + 6k^2 - 2k + \frac{1}{2} \tag{5}$$

For the case $n < 2k$, $e(v)$ in $S(P_k)$ when n even , starting from v_{l_1} , the vertices in $S(P_k)$ are of eccentricity $n+2k, n+2k-1, n+2k-2, ..., \frac{n+2k}{2}, \frac{n+2k}{2}+1, ..., 2k-1$. Hence $\sum d(u)e(u)$ along the path $S(P_k)$ for the case $n < 2k$ and n even is

$$\frac{1}{2}n^2 + 2nk + 6k^2 - 2k \tag{6}$$

Adding equations 1 and 4, the *eccentric connectivity index of* $S(T_{n,k})$ for the case $n > 2k$ and n even or odd is
$\xi^{(c)}(S(T_{n,k})) = 12nk + 12k^2 + 4n + 2n(2n - 4k - 2)$.
Adding equations 2 and 4, the *eccentric connectivity index of* $S(T_{n,k})$ for the case $n = 2k$ and n even or odd is
$\xi^{(c)}(S(T_{n,k})) = 12k(k + n)$.
Adding equations 3 and 5, the *eccentric connectivity index of* $S(T_{n,k})$ for the case $n < 2k$ and n odd is
$\xi^{(c)}(S(T_{n,k})) = \frac{5}{2}n^2 + 10nk + 6k^2 + \frac{1}{2}$.
Adding equations 3 and 6, *eccentric connectivity index of* $S(T_{n,k})$ for the case $n < 2k$ and n even is
$\xi^{(c)}(S(T_{n,k})) = \frac{5}{2}n^2 + 10nk + 6k^2$.

Theorem 2.4. *The Wiener index, the hyper Wiener index and the reverse Wiener index of the subdivision graph of the complete graph* S_n *is* $W(S(S_n)) = \frac{n^3(n-1)}{2}$, $WW(S(S_n)) = \frac{n(n-1)(5n^2-7n+4)}{2}$ *and* $\wedge(S(S_n)) = \frac{n(3n+2)(n-1)}{2}$.

Proof. The cardinality of the vertex set of $S(S_n)$ is $\frac{n(n+1)}{2}$ among which the $\frac{n(n-1)}{2}$ vertices are the subdivision vertices and the n vertices are of degree $n-1$. All the vertices in S_n are at a distance 2 in $S(S_n)$. The $\sum d(u,v)$ and $\sum d(u,v)^2$ among the vertices of S_n in $S(S_n)$ is

$$n(n-1) \tag{7}$$

$$2n(n-1) \tag{8}$$

Each vertex of degree $n-1$ is at a distance 1 with its adjacent $n-1$ subdivision vertices and at a distance 3 with the remaining $\frac{(n-1)(n-2)}{2}$ subdivision vertices. Hence $\sum d(u,v)$ and $\sum d(u,v)^2$ among the vertices of degree $n-1$ and the subdivision vertices is

$$\frac{n(n-1)(3n-4)}{2} \tag{9}$$

$$\frac{n(n-1)(9n-16)}{2} \tag{10}$$

Each subdivision vertex is at a distance 2 with the $2n-4$ subdivision vertices and at a distance 4 with the remaining $\frac{(n-2)(n-3)}{2}$ subdivision vertices. Hence, the $\sum d(u,v)$ and $\sum d(u,v)^2$ among the subdivision vertices of S_n is

$$\frac{n(n-1)^2(n-2)}{2} \tag{11}$$

$$\frac{2n(n-1)(n-2)^2}{2} \tag{12}$$

Adding equations 7,a1.9 and 11, The *Wiener index of* $S(S_n)$ is

$$W(S(S_n)) = \frac{n^3(n-1)}{2} \tag{13}$$

Adding equations 8,10 and 12, $\sum d(u,v)^2$ among all the vertices of $S(S_n)$ is

$$\frac{n(n-1)(4n^2-7n+4)}{2} \tag{14}$$

From equation 13 and 14 the *hyper wiener index of* $S(S_n)$ is
$WW(S(S_n)) = \frac{n(n-1)(5n^2-7n+4)}{2}$.
The *diameter* of $S(S_n)$ is 4 with which the reverse wiener index is
$\wedge(S(S_n)) = \frac{n(3n+2)(n-1)}{2}$.

Theorem 2.5. *The Wiener index, the hyper Wiener index and the reverse Wiener index of the subdivision graph of the wheel graph* W_{n+1} *is*

$$W(S(W_{n+1})) = \begin{cases} 2n(9n-13), & \text{for the case } n \geq 5; \\ 188, & \text{for the case } n=4; \\ 96, & \text{for the case } n=3. \end{cases}$$

$$WW(S(W_{n+1})) = \begin{cases} 24n(4n-9), & \text{for the case } n \geq 5; \\ 716, & \text{for the case } n=4; \\ 336, & \text{for the case } n=3. \end{cases}$$

and

$$\wedge(S(W_{n+1})) = \begin{cases} n(9n+35), & \textit{for the case } n \geq 6; \\ 280, & \textit{for the case } n=5; \\ 124, & \textit{for the case } n=4; \\ 84, & \textit{for the case } n=3. \end{cases}$$

Proof. In $S(W_{n+1})$, a vertex of degree 3 is at a distance 1, 2, 3 with the neighboring subdivision vertices on a spoke, to the hub and to the remaining $n-1$ subdivision vertices on the remaining spokes. Hence the $\sum d(u,v)$ and $\sum d(u,v)^2$ from the vertices of degree 3 to the subdivision vertices on the spokes and to the hub is respectively as

$$3n^2 \tag{15}$$

$$n(9n-4) \tag{16}$$

Also the vertex of degree 3 is at a distance 2 with the two neighboring vertices of degree 3 and at a distance 4 with the remaining $n-3$ vertices of degree 3. So $\sum d(u,v)$ and $\sum d(u,v)^2$ among the vertices of degree 3 is

$$2n(n-2) \tag{17}$$

$$4n(2n-5) \tag{18}$$

Also $\sum d(u,v)$ and $\sum d(u,v)^2$ from the vertex of degree 3 to the subdivision vertices of C_n for the case $n=3$ is

$$5n \tag{19}$$

$$11n \tag{20}$$

For the case $n \geq 4$, the vertex of degree 3 is at a distance 1 with the two neighboring subdivision vertices vertices of C_n and at a distance 3 with two more neighboring subdivision vertices of C_n and at a distance 5 with the remaining $n-4$ subdivision vertices of C_n. So $\sum d(u,v)$ and $\sum d(u,v)^2$ from the vertex of degree 3 to the subdivision vertices of C_n for the case $n \geq 4$ is

$$n(5n-12) \tag{21}$$

$$n(5n-16) \tag{22}$$

The subdivision vertices of C_n are at a distance 2 with the 2 neighboring subdivision vertices on the spoke and at a distance 3 with the hub and a distance 4 with the remaining $n-2$ subdivision vertices on the spoke. Hence $\sum d(u,v)$ and $\sum d(u,v)^2$ from the subdivision vertices of C_n to the vertices on the spoke and to the hub for all values of $n \geq 3$ is

$$n(4n-1) \tag{23}$$

$$n(16n - 15) \tag{24}$$

For the case $n = 3$, the subdivision vertices of C_n are at a distance 2 with each other. Hence $\sum d(u, v)$ and $\sum d(u, v)^2$ among the subdivision vertices of C_n for the case $n = 3$ is

$$2n \tag{25}$$

$$4n \tag{26}$$

For the case $n = 4$, the the subdivision vertices of C_n are at a distance 2, 2, 4 with each other. Hence the $\sum d(u, v)$ and $\sum d(u, v)^2$ among the subdivision vertices of C_n for the case $n = 4$ is

$$4n \tag{27}$$

$$12n \tag{28}$$

For the case $n \geq 5$, a subdivision vertex of C_n are at a distance 2, 2, 4, 4 and 6 respectively with the four adjacent subdivision vertices and remaining $n - 5$ subdivision vertices of C_n. Hence $\sum d(u, v)$ and $\sum d(u, v)^2$ among the subdivision vertices of C_n for the case $n \geq 5$ is

$$3n(n - 3) \tag{29}$$

$$2n(9n - 35) \tag{30}$$

Now to calculate the $\sum d(u, v)$ and $\sum d(u, v)^2$ among the subdivision vertices on the spoke and to the hub, the distance between a pair of subdivision vertices on the spoke is 2 and with the hub is 1. Hence the $\sum d(u, v)$ and $\sum d(u, v)^2$ among the subdivision vertices on the spoke and to the hub is

$$n^2 \tag{31}$$

$$n(2n - 1) \tag{32}$$

Hence adding equations 16, 18, 20, 24, 26 and 32, the Wiener index of the subdivision graph of the wheel graph is

$$W(S(W_{n+1})) = 2n(5n + 1) = 96, \textit{ for the case } n = 3 \tag{33}$$

Adding the equations 16, 18, 22, 24, 28 and 32, the *Wiener index of the subdivision graph of the wheel graph* is

$$W(S(W_{n+1})) = n(15n - 13) = 188, \textit{ for the case } n = 4 \tag{34}$$

Adding the equations 16, 18, 22, 24, 30 and 32, the *Wiener index of the subdivision graph of the wheel graph* is

$$W(S(W_{n+1})) = 2n(9n - 13), \textit{ for the case } n \geq 5 \tag{35}$$

Adding the equations 17, 19, 21, 25, 27 and 32, $\sum d(u,v)^2$ when $n = 3$ is

$$5n(7n-5), \text{ for the case } n = 3 \tag{36}$$

Adding the equations 17, 19, 23, (25, 29 and 33, $\sum d(u,v)^2$ when $n = 4$ is

$$12n(5n-9), \text{ for the case } n = 4 \tag{37}$$

Adding equations 17, 19, 23, (25, 31 and 33, $\sum d(u,v)^2$ when $n = 4$ is

$$2n(39n-95) \text{ for the case } n \geq 5 \tag{38}$$

From equations 34 and 38, the *hyper wiener index of* $S(W_{n+1})$ for the case $n = 3$ is
$WW(S(W_{n+1})) = n(45n-23) = 336$.
From equations 35 and 38, the *hyper wiener index of* $S(W_{n+1})$ for the case $n = 4$ is
$WW(S(W_{n+1})) = n(75n-121) = 716$.
From equations 36 and 38, the *hyper wiener index of* $S(W_{n+1}))$ for the case $n \geq 5$ is
$WW(S(W_{n+1})) = 24n(4n-9)$.
When $n = 3, 4, 5$ the diameter of $S(W_{n+1})$ is 4, 4 and 5 respectively and for all values of $n \geq 6$ the diameter is 6. The cardinality of the vertex set of $S(W_{n+1})$ is $3n+1$ for all $n \geq 3$. Hence from equation (2.33),
$\wedge(S(W_{n+1})) = 4n(2n+1) = 84$, *for the case* $n = 3$.
From equation 35,
$\wedge(S(W_{n+1})) = n(3n+19) = 124$, *for the case* $n = 4$.
From equation 36,
$\wedge(S(W_{n+1})) = \frac{n(9n+67)}{2} = 280$, *for the case* $n = 5$.
From equation 36, with the additional concept the diameter of $S(W_{n+1})$ is 6 for $n \geq 6$
$\wedge(S(W_{n+1})) = n(9n+35)$, *for the case* $n \geq 6$.

Theorem 2.6. *The Wiener index, the hyper Wiener index and the reverse Wiener index of the subdivision graph of the tadpole graph* $T_{n,k}$ *is*
$W(S(T_{n,k})) = \frac{(4k^2-1)k}{3} + 2kn + \frac{n(n+1)}{2} + 2kn(2k+n) + n^3$,
$WW(S(T_{n,k}) = \frac{(4k^3-k+10nk+2n+4n^3+4k^4-k^2+16k^3n+4kn^3+2n^4)}{3} + (n^2+8k^2n+4n^2k+4n^2k^2)$ *and* $\wedge(S(T_{n,k})) = n^2(k-1-\frac{n}{2}) + 2k^2(n-1+\frac{4k}{3}) + (-4nk+\frac{k}{3}-\frac{n}{2})$.

Proof. The $S(T_{n,k})$ contains a path $S(P_k)$ of length $2k$ and a cycle $S(C_n)$. Let v_{l_1} be the unique vertex of degree 3 in $S(T_{n,k})$ and let $v_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ be the path $S(P_k)$ of length $2k$. Starting from v_{l_1}, the vertices in $S(P_k)$ are at a distance $1, 2, 3, ..., 2k$. From v_{l_2}, the remaining vertices in $S(P_k)$ are at a distance $1, 2, 3, ..., 2k-1$. Proceeding like this the vertex $v_{l_{2k}}$ is at a distance 1 with $v_{l_{2k+1}}$. Hence $\sum d(u,v)$ and $\sum d(u,v)^2$ along the path $S(P_k)$ is

$$\frac{2k(k+1)(2k+1)}{3} \tag{39}$$

$$\frac{k(k+1)(2k+1)^2}{3} \tag{40}$$

Let $P_1 = v_1e_1v_2e_2...v_{n-1}e_nv_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ be the largest path in $S(T_{n,k})$ of length $n+2k$. Then $P_2 = v_2e_2...v_{n-1}e_nv_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ and
$P_{2'} = v_2e_{2'}...v_{n-1'}e_{n'}v_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ be the two paths in $S(T_{n,k})$ of length $n+2k-1$. $P_3 = v_3e_3...v_{n-1}e_nv_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ and
$P_{3'} = v_3e_{3'}...v_{n-1'}e_{n'}v_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ be the paths in $S(T_{n,k})$ of length $n+2k-2$.
Continuing like this $P_n = v_{n-1}e_nv_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ and
$P_{n'} = v_{n-1}e_{n'}v_{l_1}e_{l_1}v_{l_2}e_{l_2}...e_{l_{2k}}v_{l_{2k+1}}$ be the paths in $S(T_{n,k})$ of length $2k+1$. Hence $\sum d(u,v)$ and $\sum d(u,v)^2$ along P_1 is

$$\frac{(2k+n)(2k+n+1)}{2} \tag{41}$$

$$\frac{(2k+n+1)^3}{3} - \frac{(2k+n+1)^2}{2} + \frac{k}{3} + \frac{n}{6} + \frac{1}{6} \tag{42}$$

$\sum d(u,v)$ and $\sum d(u,v)^2$ along P_2 and $P_{2'}$

$$\frac{(2k+n)(2k+n-1)}{2} \tag{43}$$

$$\frac{(2k+n)^3}{3} - \frac{(2k+n)^2}{2} + \frac{k}{3} + \frac{n}{6} \tag{44}$$

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....
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Proceeding like this $\sum d(u,v)$ and $\sum d(u,v)^2$ along P_n and $P_{n'}$

$$\frac{(2k+1)(2k+2)}{2} \tag{45}$$

$$\frac{(2k+2)^3}{3} - \frac{(2k+2)^2}{2} + \frac{k}{3} + \frac{1}{3} \tag{46}$$

Adding equations 41, a1.43,..., a1.45, $\sum d(u,v)$ along $P_1, P_2, P_{2'}...P_n, P_{n'}$ is

$$-2k^2 + 2kn - k + \frac{n^2}{2} + \frac{n}{6} + 4k^2n + 2kn^2 + \frac{n^3}{3} \tag{47}$$

Adding equations 42, 2,..., 46, $\sum d(u,v)^2$ along $P_1, P_2, P_{2'}...P_n, P_{n'}$ is

$$\frac{(4nk-2k+n+n^2)(8k^2+6k+1+4nk+n+n^2)}{6} \tag{48}$$

For an even cycle with cardinality n of the vertex set V the *Wiener index* $W(C_n) = \frac{n^3}{8}$ and $\sum d(u,v)^2 = \frac{n^2(n^2+2)}{24}$. Hence for $S(C_n)$ in $S(T_{n,k})$ *Wiener index* and $\sum d(u,v)^2$ is

$$n^3 \tag{49}$$

$$\frac{n^2(4n^2+2)}{6} \tag{50}$$

$\sum d(u,v)$ and $\sum d(u,v)^2$ from v_2 to v_{l_1}, v_3 to v_{l_1},...,v_{n-1} to v_{l_1} is

$$\frac{n(n^2-1)}{3} \tag{51}$$

$$\frac{n^4}{6} - \frac{n^2}{6} \tag{52}$$

Adding equations 39,a1.47, a1.49 and subtracting equation 51, the *Wiener index*

$$W(S(T_{n,k})) = \frac{(4k^2-1)k}{3} + 2kn + \frac{n(n+1)}{2} + 2kn(2k+n) + n^3 \tag{53}$$

Adding equations 40, 48, 50 and subtracting equation 52, $\sum d(u,v)^2$ in $S(T_{n,k})$

$$\sum d(u,v)^2 = \frac{4k^4}{3} - \frac{k^2}{3} + \frac{16k^3n}{3} + 4k^2n + \frac{4kn}{3} + 4n^2k^2 + 2kn^2 + \frac{4kn^3}{3} + \frac{n}{6} + \frac{5n^2}{6} + \frac{n^3}{3} + \frac{2n^4}{3} \tag{54}$$

Adding equations 53 and **??**, the *hyper wiener index*
$WW(S(T_{n,k}) = \frac{(4k^3-k+10nk+2n+4n^3+4k^4-k^2+16k^3n+4kn^3+2n^4)}{3} + (n^2+8k^2n+4n^2k+4n^2k^2)$.
The cardinality of the vertex set of $S(T_{n,k})$ is $2(n+k)$ and the diameter is $n+2k$. Hence from equation 53, the reverse wiener index of $S(T_{n,k})$ is

$\wedge(S(T_{n,k})) = n^2(k-1-\frac{n}{2}) + 2k^2(n-1+\frac{4k}{3}) + (-4nk+\frac{k}{3}-\frac{n}{2})$.

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