Gen. Math. Notes, Vol. 30, No. 2, October 2015, pp.54-73
ISSN 2219-7184; Copyright ©ICSRS Publication, 2015
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On Soft Generalized $\alpha b-$Closed Sets in Soft Topological Spaces

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(Received: 14-8-15 / Accepted: 21-9-15)

Abstract

In this paper, a new class of soft generalized closed sets in soft topological spaces called soft $g\alpha b-$closed sets, is introduced and its soft topological properties is studied and investigated. Moreover, we discussed the relationship among, $gb-$closed, $\alpha g-$closed, $sw-$closed, $sg-$closed, s^*g-closed, $wg-$closed, $gp-$closed, $g-$closed, $rwg-$closed, $gpr-$closed, $rg-$closed, $gs-$closed and nowhere dense sets. Finally, we defined and discussed the properties of soft $g\alpha b-$open sets and $g\alpha b-$neighbourhood.

Keywords: *Soft $g\alpha b-$closed, soft $\alpha-$open sets, soft $b-$open sets, soft $b-$interior, $b-$closure.*

1 Introduction

The concept of soft set theory has been introduced in 1999 by Molodtsov [1] this set designed to solve the sophisticated problems in economic, engineering environment, etc. It has been applied to several branches of mathematics such as operation research, game theory and among others.

The soft set theory and it's applications increase after time to several researchers, especially in the recent years. This is because of the general nature of parameterizations expressed by a soft set. Therefore due to these facts, several special sets have been introduced in the soft set theory and their properties have been studied, within the soft topological space. The notion of topological spaces for soft sets was formulated by Shabir and Naz [2], which is defined over an initial universe with a fixed set of parameters. Levine [3] introduced generalized closed sets in general topology. Kannan [4] introduced soft generalized closed and open sets in soft topological spaces which are defend over an initial universe with a fixed set of parameters. He studied their some properties. After then Saziye et al. [5] studied behavior relative to soft subspaces of soft generalized closed sets and continued investigating the properties of soft generalized closed and open sets. Nazmul and Samanta [6] introduced neighborhood properties of soft topological spaces. Hussain and Ahmad [7] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Soft $\alpha-$open sets and it's properties were introduced and studied by Ilango et al. [8]. Akdag and Ozkan [9] introduced soft $b-$open sets and soft $b-$continues functions. Characterization of b-open soft sets in soft topological spaces were introduced and studied by El-Sheikh and El-latif [10]. Let (F, E) be a soft set over X, the soft closure of (F, E) and soft interior of (F, E) will be denoted by $cl(F, E)$ and $int(F, E)$ respectively, union of all soft $b-$open sets over X contained in (F, E) is called soft b-interior of (F, E) and it is denoted by $bint(F, E)$, the intersection of all soft $b-$closed sets over X containing (F, E) is called soft $b-$closure of (F, E) and it is denoted by $bcl(F, E)$. In this paper we introduce a new type of generalized closed sets in soft topological spaces. Further we investigate some soft topological properties of this set.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 *[1] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \Phi$ i.e $F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.*

Definition 2.2 *[11] Let F_A, $G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \widetilde{\subseteq} G_B$, if*

1. *$A \subseteq B$, and*
2. *$F(e) \subseteq G(e)$, $\forall e \in A$.*

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $F_A \widetilde{\subseteq} G_B$.

Definition 2.3 *[11] Two soft sets F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4 *[12] The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall e \in A$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.*

Definition 2.5 *[13] The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E)-(G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.*

Definition 2.6 *[13] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.7 *[13] Let $x \in X$. A soft set (x, E) over X, where $x_E(e) = \{x\}$, $\forall e \in E$, is called the singleton soft point and denoted by x_E.*

Definition 2.8 *[11] A soft set (F, A) over X is said to be a NULL soft set denoted by Φ or Φ_A if for all $e \in A$, $F(e) = \Phi$ (null set).*

Definition 2.9 *[11] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X_A^c = \Phi_A$ and $\Phi_A^c = X_A$.*

Definition 2.10 *[11] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

Definition 2.11 *[11] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C, H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.*

Definition 2.12 *[10] Let I be an arbitrary indexed set and $L = \{(F_i, E); i \in I\}$ be a subfamily of $SS(X)_E$.*

1. *The union of L is the soft set (H, E), where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cup}_{i \in I}(F_i, E) = (H, E)$.*

2. *The intersection of L is the soft set (M, E), where $M(e) = \cap_{i \in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cap}_{i \in I}(F_i, E) = (M, E)$.*

Definition 2.13 *[13] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

1. *$X, \Phi \in \tau$, where $\Phi(e) = \Phi$ and $X(e) = X$, $\forall e \in E$,*

2. *The union of any number of soft sets in τ belongs to τ,*

3. *The intersection of any two soft sets in τ belongs to τ.*

(X, τ, E) is called a soft topological space over X.

Definition 2.14 *[6] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Define $\tau_{(F,E)} = \{(G, E)\widetilde{\cap}(F, E) : (G, E) \in \tau\}$ which is a soft topology on (F, E). The soft topology is called soft relative topology of τ on (F, E), and $((F, E), \tau_{(F,E)})$is called soft subspace of (X, τ, E).*

Definition 2.15 *[13] A soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$, $F(e) = \{x\}$ and $F(e^c) = \Phi$ for each $e^c \in E - \{e\}$, and the soft point (F, A) is denoted by x_e, the soft point x_e is said to be belonging to the soft set (G, E), $x_e \widetilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.*

Definition 2.16 *[6] A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of a soft point $F(e)$ if there exists a soft open set (H, E) such that $F(e) \in (H, E)\widetilde{\subseteq}(G, E)$.*

A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of a soft set (F, E) if there exists a soft open set (H, E) such that $(F, E)\widetilde{\subseteq}(H, E)\widetilde{\subseteq}(G, E)$. The neighborhood system of a soft point $F(e)$ denoted by $N_\tau(F(e))$, is the family of all it's neighborhood.

Definition 2.17 *[14] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$, (F, E) is said to be $b-$open soft set if $(F, E)\widetilde{\subseteq}int(cl(F, E))\widetilde{\cup}cl(int(F, E))$ and it's complement is said to be b-closed soft. The set of all b-open soft sets is denoted by $BOS(X, \tau, E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X, \tau, E)$,or $BCS(X)$.*

Definition 2.18 *[15] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$, (F, E) is said to be,*

1. *Soft preopen set if $(F, E)\widetilde{\subseteq}int(cl(F, E))$.*
2. *Soft semi open set if $(F, E)\widetilde{\subseteq}cl(int(F, E))$.*
3. *Soft $\alpha-$open set if $(F, E)\widetilde{\subseteq}int(cl(int(F, E)))$.*

Definition 2.19 *[15] A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft pre generalized closed (in short soft $pg-$closed) set, if soft $pcl(F, E)\widetilde{\subseteq}(G, E)$ whenever $(F, E)\widetilde{\subseteq}(G, E)$ and (G, E) is a soft open set in X.*

Definition 2.20 *[15] A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft generalized pre closed set (in short soft $gp-$closed) sets, if soft $cl(F, E)\widetilde{\subseteq}(G, E)$ whenever $(F, E)\widetilde{\subseteq}(G, E)$ and (G, E) is soft preopen set in X.*

Definition 2.21 *[16] A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft generalized α-closed set (in short soft $g\alpha-$closed) sets, if soft $cl(F, E)\widetilde{\subseteq}(G, E)$ whenever $(F, E)\widetilde{\subseteq}(G, E)$ and (G, E) is soft $\alpha-$open set.*

Definition 2.22 *[17] A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft $s{*}g-$closed sets, if soft $cl(F, E)\widetilde{\subseteq}(G, E)$ whenever $(F, E)\widetilde{\subseteq}(G, E)$ and (G, E) is soft semi-open set.*

Definition 2.23 *[18, 4] A soft set (A, E) in a soft topological space (X, τ, E) is called*

1. *a soft generalized closed set (Soft $g-$closed) in a soft topological space (X, τ, E) if $cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft open in (X, τ, E).*
2. *a soft $sg-$closed set if $sscl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft semi-open.*
3. *a soft $gs-$closed set if $sscl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft open.*

4. *a soft $rwg-$closed set if $cl(Int(A,E))\widetilde{\subseteq}(U,E)$ whenever $(A,E)\widetilde{\subseteq}(U,E)$ and (U,E) is soft regular open.*

5. *a soft $wg-$closed set if $cl(Int(A,E))\widetilde{\subseteq}(U,E)$ whenever $(A,E)\widetilde{\subseteq}(U,E)$ and (U,E) is soft open.*

Definition 2.24 *[19] A soft set (F,E) in a soft topological space (X,τ,E) is called soft generalized preregular closed (in short soft $gpr-$closed) set, if soft $pcl(F,E)\widetilde{\subseteq}(G,E)$ whenever $(F,E)\widetilde{\subseteq}(G,E)$ and (G,E) is soft regular open set.*

Definition 2.25 *[20] A soft set (F,E) in a soft topological space (X,τ,E) is called a soft regular generalized closed (in short soft $rg-$closed) set, if soft $cl(F,E)\widetilde{\subseteq}(G,E)$ whenever $(F,E)\widetilde{\subseteq}(G,E)$ and (G,E) is soft regular open set.*

Definition 2.26 *[21] A soft set (F,E) in a soft topological space (X,τ,E) is called soft Weakly closed (in short soft $SW-$closed) set, if soft $cl(F,E)\widetilde{\subseteq}(G,E)$ whenever $(F,E)\widetilde{\subseteq}(G,E)$ and (G,E) is soft semi open.*

3 Soft Generalized $\alpha b-$Closed Sets

The present section gives the definition of soft generalized $\alpha b-$closed set and investigates some of it's properties.

Definition 3.1 *A soft set (F,E) in a soft topological space (X,τ,E) is said to be soft generalized $\alpha b-$closed (in short soft $g\alpha b-$closed) set, if soft $bcl(F,E)\widetilde{\subseteq}(G,E)$ whenever $(F,E)\widetilde{\subseteq}(G,E)$ and (G,E) is soft $\alpha-$open. The collection of all soft $g\alpha b-$closed sets in (X,τ,E) is denoted by $sg\alpha b-C(X)$.*

Definition 3.2 *A soft set (F,E) in a soft topological space (X,τ,E) is said to be soft generalized $b-$closed (in short soft $gb-$closed) set, if soft $bcl(F,E)\widetilde{\subseteq}(G,E)$ whenever $(F,E)\widetilde{\subseteq}(G,E)$ and (G,E) is a soft open set. The collection of all soft $gb-$closed sets in (X,τ,E) is denoted by $sgb-C(X)$.*

Theorem 3.3 *Every $b-$closed soft set in soft topological space (X,τ,E) is soft $g\alpha b-$closed.*

Proof. Let (F,E) be a soft $b-$closed set and (U,E) be any soft $\alpha-$open such that $(F,E)\widetilde{\subseteq}(U,E)$, then $bcl(F,E)=(F,E)\widetilde{\subseteq}(U,E)$. Therefore (F,E) is soft $g\alpha b-$closed.

Theorem 3.4 *A soft set (F,E) is a soft $g\alpha b-$closed, iff soft $bcl(F,E)-(F,E)$, does not contain any non-empty soft $\alpha-$closed sets.*

Proof. Suppose that (F, E) is a soft $g\alpha b$-closed and (F_1, E) is a non-empty $\alpha-$closed soft set such that $(F_1, E)\widetilde{\subseteq} bcl(F, E)-(F, E)$. $\Rightarrow (F_1, E)\widetilde{\subseteq} bcl(F, E)\tilde{\cap}(F, E)^c$. $\Rightarrow (F_1, E)\widetilde{\subseteq} bcl(F, E)$ and $(F_1, E)\widetilde{\subseteq}(F, E)^c$, $(F, E)\widetilde{\subseteq}(F_1, E)^c$ since $(F_1, E)^c$ is soft $\alpha-$open and (F, E) is $g\alpha b-$closed $bcl(F, E)\widetilde{\subseteq}(F_1, E)^c \Rightarrow (F_1, E)\widetilde{\subseteq}[bcl(F, E)]^c$. Thus $(F_1, E)\widetilde{\subseteq} bcl(F, E)\tilde{\cap}[bcl(F, E)]^c = \Phi$. That is $(F_1, E) = \Phi$. $\Rightarrow bcl(F, E) - (F, E) = \Phi$ contains no non empty $\alpha-$closed set.

Conversely: Suppose that soft $bcl(F, E)-(F, E)$, does not contain any non-empty soft $\alpha-$closed sets, $(F, E)\widetilde{\subseteq}(G, E)$ and (G, E) is a soft $\alpha-$open, If it is possible that $bcl(F, E)\widetilde{\nsubseteq}(G, E)$, then $bcl(F, E)\tilde{\cap}(G, E)^c$ is not empty $\alpha-$closed set of $bcl(F, E)-(F, E)$ which is a contradiction. Therefore, $bcl(F, E)\widetilde{\subseteq}(G, E)$. Hence (F, E) is soft $g\alpha b-$closed.

Theorem 3.5 *If (F, E) is a soft $g\alpha b-$closed set, then (F, E) is $gb-$closed iff $bcl(F, E) - (F, E) = \Phi$ is closed.*

Proof. Assume that (F, E) is soft g$\alpha b-$closed. Since $bcl(F, E) = (F, E)$, $bcl(F, E) - (F, E) = \Phi$ is $gb-$closed and hence closed. Now assume that $bcl(F, E) - (F, E)$ is closed. By (Theorem 3.4) $bcl(F, E) - (F, E)$ does not contain any non-empty soft $\alpha-$closed set. That is $bcl(F, E)-(F, E) = \Phi$, thus $bcl(F, E) = (F, E)$. Hence, (F, E) is $gb-$closed.

Theorem 3.6 *If a set (F, E) is soft $g\alpha b-$closed in X then $bcl(F, E)-(F, E)$ contains only null soft closed set.*

Proof. Let (F, E) be a soft $g\alpha b-$closed in X and (H, E) be a soft closed set such that $(H, E)\widetilde{\subseteq} bcl(F, E) - (F, E)$. Since (H, E) is soft closed its relative complement is soft open, $(H, E)\widetilde{\subseteq}(F, E)^c$. Thus $(F, E)\widetilde{\subseteq}(H, E)^c$. Consequently $bcl(F, E)\widetilde{\subseteq}(H, E)^c$. Therefore, $(H, E)\widetilde{\subseteq}(bcl(F, E))^c$. Hence $(H, E) = \Phi$ and thus $bcl(F, E) - (F, E)$ contains only null soft closed set.

Lemma 3.7 *In a soft topological space we have the following:*

1. *Every soft regular open set is soft $g\alpha b-$closed.*

2. *Every soft regular closed set is soft $g\alpha b-$closed.*

3. *Every soft semi-closed set is soft $g\alpha b-$closed.*

4. *Every soft pre-closed set is soft $g\alpha b-$closed.*

5. *Every soft $\alpha-$closed set is soft $g\alpha b-$closed.*

6. *If (F, E) is soft $\alpha-$open and soft $g\alpha b-$closed then (F, E) is soft $b-$closed.*

Proof. Obvious.

Theorem 3.8 *If (F,E) is a soft $g\alpha b-$closed and (G,E) is a soft $\alpha-$closed, then $(F,E)\tilde{\cap}(G,E)$ is $g\alpha b-$closed.*

Proof. To show that $(F,E)\tilde{\cap}(G,E)$ is $g\alpha b-$closed, it is enough to show that $bcl(F\cap G,E)\widetilde{\subseteq}(U,E)$, where (U,E) is a soft $\alpha-$open and $(F\cap G,E)\widetilde{\subseteq}(U,E)$. Let $(H,E)=(G,E)^c$ then $(F,E)\widetilde{\subseteq}(U,E)\widetilde{\cup}(H,E)$ since (H,E) is $\alpha-$open set and (F,E) is $g\alpha b-$ closed then, $bcl(F,A)\widetilde{\subseteq}(U,E)\widetilde{\cup}(H,E)$.

Now $bcl(F,A)\widetilde{\cap}(G,E)\widetilde{\subseteq}[(U,E)\widetilde{\cup}(H,E)]\widetilde{\cap}(G,E)\widetilde{\subseteq}(U,E)$, since every $\alpha-$closed is $b-$closed then $bcl(F,E)\widetilde{\cap}bcl(G,E)\widetilde{\subseteq}(U,E)$, by Theorm 3.3 [10], $bcl[(F,E)\widetilde{\cap}(G,E)]$ $\widetilde{\subseteq}(U,E)$, this implies that $bcl(F\widetilde{\cap}G,E)\widetilde{\subseteq}(U,E)$. Hence $(F\widetilde{\cap}G,E)$ is $g\alpha b-$closed set.

Theorem 3.9 *If (F,E), (B,E) are two soft sets in a soft topological space (X,τ,E), (B,E) $\widetilde{\subseteq}$ (F,E), (B,E) is soft $g\alpha b-$closed set relative to (F,E) and (F,E) is a soft $g\alpha b-$closed set in (X,τ,E). Then (B,E) is a soft $g\alpha b-$closed set relative to (X,τ,E).*

Proof. Let $(B,E)\widetilde{\subseteq}(U,E)$ and (U,E) be a soft $\alpha-$open set in (X,τ,E). Given $(B,E)\widetilde{\subseteq}(F,E)\widetilde{\subseteq}(X,\tau,E)$, then $(B,E)\widetilde{\subseteq}(F,E)$ and $(B,E)\widetilde{\subseteq}(U,E)$ this implies $(B,E)\widetilde{\subseteq}(F\cap U,E)$ given (B,E) is a soft $g\alpha b-$closed set relative to (F,E), $bcl(B,E)\widetilde{\subseteq}(F\cap U,E)\widetilde{\subseteq}(U,E)$ hence $(F,E)\widetilde{\cup}bcl(B,E)\widetilde{\subseteq}(U,E)$.
Now $[(F,E)\widetilde{\cup}(bcl(B,E))]\widetilde{\cap}(bcl(B,E))^c\widetilde{\subseteq}(U,E)\widetilde{\cap}(bcl(B,E))^c$
$\Rightarrow$ $[(F,E)\widetilde{\cap}(bcl(B,E))^c]\widetilde{\cup}[(bcl(B,E))\widetilde{\cap}(bcl(B,E))^c]\widetilde{\subseteq}(U,E)\widetilde{\cap}(bcl(B,E))^c$
$\Rightarrow (F,E)\cap(bcl(B,E))^c\widetilde{\subseteq}(U,E)\widetilde{\cap}(bcl(B,E))^c$ since (F,E) is a soft $g\alpha b-$closed set and $(B,E)\widetilde{\subseteq}(F,E)$ we get $bcl(B,E)\widetilde{\subseteq}bcl(F,E)\widetilde{\subseteq}(U,E)\widetilde{\cap}(bcl(B,E))^c$
$\Rightarrow bcl(B,E)\widetilde{\subseteq}(U,E)\widetilde{\cap}(bcl(B,E))^c$
$\Rightarrow bcl(B,E)\widetilde{\subseteq}(U,E)$ but $bcl(B,E)\widetilde{\nsubseteq}(bcl(B,E))^c$
$\Rightarrow (B,E)$ is a soft $g\alpha b-$closed set relative to (X,τ,E).

Theorem 3.10 *If (F,E) is a soft $g\alpha b-$closed set in a soft topological space (X,τ,E) and $(F,A)\widetilde{\subseteq}$ $(B,E)\widetilde{\subseteq}bcl(F,E)$, then (B,E) is a soft $g\alpha b-$closed set.*

Proof. Let $(B,E)\widetilde{\subseteq}(U,E)$, (U,E) be a soft $\alpha-$open, then $(F,E)\widetilde{\subseteq}(B,E)$ and $(F,E)\widetilde{\subseteq}(U,E)$. Since (F,E) is a soft $g\alpha b-$closed, $bcl(F,E)\widetilde{\subseteq}(U,E)$ since (B,E) $\widetilde{\subseteq}bcl(F,E)$, this implies $bcl(B,E)\widetilde{\subseteq}bcl(F,E)$. Thus $bcl(B,E)\widetilde{\subseteq}bcl(F,E)\widetilde{\subseteq}(U,E)$. Therefore, $bcl(B,E)\widetilde{\subseteq}(U,E)$, and (U,E) is given soft $\alpha-$open. Hence, (B,E) is soft $g\alpha b-$closed.

Theorem 3.11 *If (F,E) is a soft set in soft topological space (X,τ,E), (F,E) $\widetilde{\subseteq}(Y,A)\widetilde{\subseteq}(X,\tau,E)$ and (F,E) is soft $g\alpha b-$closed in (X,τ,E), then (F,E) is soft $g\alpha b-$closed relative to (Y,τ_Y,E).*

Proof. Let $(F,E)\widetilde{\subseteq}(Y,E)\widetilde{\subseteq}(X,\tau,E)$ and (F,E) is soft $g\alpha b-$closed set in soft topological space (X,τ,E). To show that (F,E) is a soft $g\alpha b-$closed set relative to subspace (Y,τ_Y,E), suppose that $(F,E)\widetilde{\subseteq}(Y,E)\tilde{\cap}(U,E)$ where (U,E) is soft $\alpha-$open set in (X,τ,E), then $bcl(F,E)\widetilde{\subseteq}(U,E)$ and $bcl(F,E)\tilde{\cap}(Y,E)\widetilde{\subseteq}(U,E)\tilde{\cap}(Y,E)$. Thus, $bcl(F,E)\widetilde{\cap}(Y,E)$ is $b-$closure of (F,E) in (Y,τ_Y,E). Hence, (F,E) is soft $g\alpha b-$closed set relative to (Y,τ_Y,E).

Theorem 3.12 *The intersection of any two soft $g\alpha b-$closed sets in a soft topological space (X,τ,E) is soft $g\alpha b-$closed.*

Proof. Let (F,E) and (B,E) be two soft in $g\alpha b-$closed sets in (X,τ,E), $(F,E)\widetilde{\subseteq}(U,E)$ then $bcl(F,E)\widetilde{\subseteq}(U,E)$, $(B,E)\widetilde{\subseteq}(H,E)$ and $bcl(B,E)\widetilde{\subseteq}(H,E)$ where (U,E) and (H,E) are any soft $\alpha-$open sets in (X,τ,E), we obtained that $(F,E)\tilde{\cap}(B,E)\widetilde{\subseteq}(U,E)\widetilde{\cap}(H,E)=(V,E)$ where (V,E) is any soft $\alpha-$open set, since $(F,E)\tilde{\cap}(B,E)\widetilde{\subseteq}(F,E)$ and $bcl[(F,E)\tilde{\cap}(B,E)]\widetilde{\subseteq}bcl(F,E)\widetilde{\subseteq}(U,E)$, $(F,E)\tilde{\cap}(B,E)\widetilde{\subseteq}(B,E)$ and $bcl[(F,E)\widetilde{\cap}(B,E)]\widetilde{\subseteq}bcl(B,E)\widetilde{\subseteq}(H,E)$, therefore $bcl[(F,E)\widetilde{\cap}(B,E)]\widetilde{\subseteq}bcl(F,E)\tilde{\cap}bcl(B,E)\widetilde{\subseteq}(V,E)$ where (V,E) is any soft $\alpha-$open set, $(F,E)\widetilde{\subseteq}(U,E)$, $(B,E)\widetilde{\subseteq}(H,E)$, $bcl(F,E)\widetilde{\subseteq}(U,E)$ and $bcl(B,E)\widetilde{\subseteq}(H,E)$ whenever $(F,E)\ \tilde{\cap}(B,E)\widetilde{\subseteq}(U,E)\tilde{\cap}(H,E)=(V,E)$. Hence, $(F,E)\tilde{\cap}(B,E)$ is a soft $g\alpha b-$closed sets in (X,τ,E).

The union of two soft $g\alpha b-$closed sets generally need not be soft $g\alpha b-$closed set as shown by the following example:

Example 3.13 *Let $X=\{h_1,h_2,h_3\}$ and $E=\{e_1,e_2\}$ be a set of parameters and $F_1,F_2,F_3,F_4,\ F_5,\ F_6$ are mappings from E to $P(X)$ defined by*
$(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$,
$(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\})\},(F_3,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$,
$(F_4,E)=\{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\},(F_5,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$,
$(F_6,E)=\{(e_1,\{h_3\}),(e_2,\{h_3\})\}$, *then* $\tau=\{\Phi,\ X,\ (F_1,E),\ (F_2,E),\ (F_3,E)\}$ *is a soft topology over X. The family of all soft open sets is* $\{\Phi,\ X,\ (F_1,E),(F_2,E),(F_3,E)\}$, *and the family of all soft closed sets is* $\{\Phi,\ X,\ (F_4,E)=(F_1,E)^c,\ (F_5,E)=(F_2,E)^c,\ (F_6,E)=(F_3,E)^c\}$, *where $(F_1,E),(F_2,E)$ are soft open sets and also are soft $\alpha-$open set, $bcl(F_1,E)=(F_1,E)$ and $bcl(F_2,E)=(F_2,E)$, then (F_1,E) and (F_2,E) are soft $g\alpha b-$closed sets. $(F_1,E)\widetilde{\cup}(F_2,E)=(F_3,E)$, which is soft α-open sets and $(F_3,E)\widetilde{\subseteq}(F_3,E)$ but $bcl(F_3,E)=X\widetilde{\not\subseteq}(F_3,E)$. Hence, Their union is not soft $g\alpha b-$closed set.*

Theorem 3.14 *If a soft set (F,E) of a soft topological space (X,τ,E) is soft nowhere dense, then it is soft $g\alpha b-$closed.*

Proof. Suppose that (F,E) be a soft set nowhere dense and (U,E) be a soft $\alpha-$open set in (X,τ,E) such that $(F,E)\widetilde{\subseteq}(U,E)$, $bcl(F,E)=sscl(F,E)\tilde{\cap}spcl(F,E)$ $=[(F,E)\widetilde{\cup}intcl(F,E)]\tilde{\cap}[(F,E)\widetilde{\cup}clint(F,E)]$ since $int(cl(F,E))=\Phi$. Then

$bcl(F,E) = (F,E)$. Therefore, $bcl(F,E)\widetilde{\subseteq}(U,E)$, which implies that (F,E) is $g\alpha b-$closed in (X,τ,E).

The converse of the above theorem generally need not be true as shown by the following example:

Example 3.15 *Let $X = \{h_1,h_2,h_3,h_4\}$ and $E = \{e_1,e_2\}$ be the set of parameters and $\tau = \{\Phi,\ X,\ (F_1,E),\ (F_2,E)\}$ where $(F_1,E) = \{(e_1,\{h_1,h_3\}),\ (e_2,\{h_1,h_3\})\}$, $(F_2,E) = \{(e_1,\{h_2,h_4\})(e_2,\{h_{2,}h_4\})\}$, so (F_1,E), (F_2,E) are soft open sets and also are soft $\alpha-$open set, $bcl(F_1,E) = cl(F_1,E) = (F_1,E)$ and $bcl(F_2,E) = cl(F_2,E) = (F_2,E)$, then (F_1,E) and (F_2,E) are soft $g\alpha b-$closed sets in (X,τ,E), but not soft nowhere dense.*

Theorem 3.16 *If a soft set (F,E) of a soft topological space (X,τ,E) is $g\alpha b-$closed, then it is $gb-$closed.*

Proof. Let (F,E) be a soft $g\alpha b-$closed set in (X,τ,E) and (U,E) be any soft open set such that $(F,E)\widetilde{\subseteq}(U,E)$. Therefore, $bcl(A))\widetilde{\subseteq}(U,E)$ and (U,E) is a soft open. Thus, (F,E) is a soft $gb-$closed.

(i) Opposite direction of the above theorem can be achieved if every soft $\alpha-$open set is soft open set.
(ii) The converse of the above theorem generally need not be true as shown by the following example:

Example 3.17 *Let $X = \{h_1,h_2,h_3\}$ and $E = \{e_1,e_2\}$ be the set of parameters with a soft topology $\tau = \{\Phi,X,(F_1,E),(F_2,E),\}$ where $(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E) = \{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$. $(F_3,E) = \{(e_1\ ,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$ is a soft $gb-$closed set but not soft $g\alpha b-$closed.*

Theorem 3.18 *let (F,E) be a soft set of (X,τ,E). If $clint(F,E)\widetilde{\subseteq}(U,E)$, whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) be a soft $\alpha-$open set, then (F,E) is soft $g\alpha b-$closed.*

Proof. Assume that $clint(F,E)\widetilde{\subseteq}(U,E)$, whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) is a soft $\alpha-$open subset of (X,τ,E), $bcl(F,E) = [(F,E)\widetilde{\cup}(clint(F,E)\widetilde{\cap}intcl(F,E))]$ $= [(F,E)\widetilde{\cup}clint(F,E)]\widetilde{\cap}[(F,E)\widetilde{\cup}intcl(F,E)]\widetilde{\subseteq}[(F,E)\widetilde{\cup}clint(F,E)]\widetilde{\subseteq}[(F,E)\widetilde{\cup}(U,E)]$, since $(F,E)\widetilde{\subseteq}(U,E)$, then $bcl(F,E)\widetilde{\subseteq}(U,E)$. Hence, (F,E) is a soft $g\alpha b-$closed set.

Theorem 3.19 *let (F,E) be a soft set of (X,τ,E), if $intcl(F,E)\widetilde{\subseteq}(U,E)$, whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) be a soft $\alpha-$open set ,then (F,E) is soft $g\alpha b-$closed.*

Proof. Assume that $intcl(F,E)\widetilde{\subseteq}(U,E)$, whenever $(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) be a soft $\alpha-$open set of (X,τ,E), $bcl(F,E) = [(F,E)\widetilde{\cup}(clint(F,E)\widetilde{\cap}intcl(F,E))] = [(F,E)\widetilde{\cup}(clint(F,E))]\widetilde{\cap}[(F,E)\widetilde{\cup}intcl(F,E)]\widetilde{\subseteq}[(F,E)\widetilde{\cup}intcl(F,E)]\widetilde{\subseteq}[(F,E)\widetilde{\cup}(U,E)]$, since $(F,E)\widetilde{\subseteq}(U,E)$ then $bcl(F,E)\widetilde{\subseteq}(U,E)$. Hence, (F,E) is a soft $g\alpha b-$closed set.

Theorem 3.20 *If a soft set (F,E) of a soft topological space (X,τ,E) is soft $gp-$closed , then it is soft $g\alpha b-$closed.*

Proof. Let (F,E) be a soft $gp-$closed and $(F,E)\widetilde{\subseteq}(U,E)$, such that (U,E) is a soft $\alpha-$open set and also soft preopen set, since every soft closed set is soft $b-$closed set, $bcl(F,E)\widetilde{\subseteq}cl(F,E)\widetilde{\subseteq}(U,E)$. Then, $bcl(F,E)\widetilde{\subseteq}(U,E)$ and (U,E) is a soft $\alpha-$open set. Therefore, (F,E) is soft $g\alpha b-$closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.21 *Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ be a set of parameters and $\tau = \{\Phi, X, (F_1,E)\}$ where $(F_1,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$. A soft set $(F_2,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, is soft $g\alpha b-$closed, but not soft $gp-$closed, because $(F_2,E)\widetilde{\subseteq}(F_1,E)$, $bcl(F_2,E) = (F_2,E)$ but $cl(F_2,E) = X\widetilde{\nsubseteq}(F_1,E)$.*

The following examples show that the concepts of soft $g\alpha b-$closeness and soft $pg-$closeness are independent.

Example 3.22 *Let $X = \{h_1,h_2,h_3,h_4\}$ and $E = \{e_1,e_2\}$ be a set of parameters with a soft topology $\tau = \{\Phi, X, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E)\}$ where $(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\}),\}$, $(F_2,E) = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$, $(F_3,E) = \{(e_1,\{h_3\}),(e_2,\{h_3\})\}$, $(F_4,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$, $(F_5,E) = \{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$, $(F_6,E) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$, $(F_7,E) = \{(e_1,\{h_1,h_2,h_3\}),(e_2,\{h_1,h_2,h_3\})\}$. A soft set $(F_6,E) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$ is soft open, also is soft $\alpha-$open and $(F_6,E)\widetilde{\subseteq}(F_6,E)$, $bcl(F_6,E) = (F_6,E)$, $pcl(F_6,E) = \{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}\widetilde{\nsubseteq}(F_6,E)$. Then it is $g\alpha b-$closed but not soft $pg-$closed.*

Example 3.23 *Let $X = \{h_1,h_2,h_3\}$ and $E = \{e_1,e_2,e_3\}$ be a set of parameters with a soft topology $\tau = \{\Phi, X, (F_1,E)\}$ where $(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\}),(e_3,\{h_1\})\}$. A soft set $(F_2,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\}),(e_3,\{h_1,h_2\})\}$ is soft $pg-$closed but not soft $g\alpha b-$closed.*

Theorem 3.24 *If a soft set (F,E) in a soft topological space (X,τ,E) is soft $g\alpha-$closed, then it is soft $g\alpha b-$closed.*

Proof. Let (F,E) be a soft $g\alpha-$closed and (U,E) is a soft $\alpha-$open set such that $(F,E)\widetilde{\subseteq}(U,E)$. Now $bcl(F,E)\widetilde{\subseteq}cl(F,E)\widetilde{\subseteq}(U,E)$, implies that $bcl(F,E)\widetilde{\subseteq}(U,E)$, and (U,E) is a soft $\alpha-$open set. Thus, (F,E) is $g\alpha b-$closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.25 *Let $X=\{h_1,h_2,h_3\}$ and $E=\{e_1,e_2\}$ be a set of parameters with a soft topology $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E)\}$ where$(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$, $(F_3,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$, the family of all soft closed sets is $\{X,\Phi,(F_1,E)^c,(F_2,E)^c,(F_3,E)^c\}$, where $(F_1,E)^c=\{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$, $(F_2,E)^c=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$, $(F_3,E)^c=\{(e_1,\{h_3\}),(e_2,\{h_3\})\}$, (F_1,E), is a soft open sets and also soft $\alpha-$open set, $bcl(F_1,E)=(F_1,E)\widetilde{\subseteq}(F_1,E)$ but $cl(F_1,E)=(F_2,E)^c=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}\widetilde{\nsubseteq}(F_1,E)$. Thus (F_1,E) is soft $g\alpha b-$closed but not soft $g\alpha-$closed.*

Theorem 3.26 *If a soft set (F,E) in a soft topological space (X,τ,E) is soft s^*g-closed, then it is soft $g\alpha b-$closed.*

Proof. Let (F,E) be a soft s^*g-closed set and (U,E) be a soft $\alpha-$open set such that $(F,E)\widetilde{\subseteq}(U,E)$. Since every soft $\alpha-$open set is soft semi-open, then $cl(F,E)\widetilde{\subseteq}(U,E)$, $bcl(F,E)\widetilde{\subseteq}cl(F,E)$, this implies to $bcl(F,E)\widetilde{\subseteq}(U,E)$, and (U,E) is a soft α-open set. Thus, (F,E) is $g\alpha b-$closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.27 *Let $X=\{h_1,h_2,h_3,h_4\}$ and $E=\{e_1,e_2\}$ be a set of parameters with a soft topology $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E)\}$, where $(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$, $(F_3,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$, the family of all soft closed sets is $\{\Phi,X,(F_1,E)^c,(F_2,E)^c,(F_3,E)^c\}\}$, where $(F_1,E)^c=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$, $(F_2,E)^c=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_1,h_3,h_4\})\}$ and $(F_3,E)^c=\{(e_1,\{h_3,h_4\}),(e_2,\{h_3,h_4\})\}$. (F_1,E), is a soft open set, soft $\alpha-$open and also soft semi open, $bcl(F_1,E)\widetilde{\subseteq}(F_1,E)$ where $(F_1,E)\widetilde{\subseteq}(F_1,E)$ but $cl(F_1,E)=(F_2,E)^c\widetilde{\nsubseteq}(F_1,E)$. Thus a soft set (F_1,E) is soft $g\alpha b-$closed, but not soft s^*g-closed.*

The following examples show that the concepts of soft $g\alpha b-$closeness and soft $gs-$closeness are independent.

Example 3.28 *Consider $X=\{h_1,h_2,h_3,F_4\}$ and $E=\{e_1,e_2\}$ be the set of parameters with a soft topology $\tau=\{\Phi,X,(F_1,E)\}$, where $(F_1,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$ $(F_2,E)=\{(e_1,\{h_3,F_4\}),(e_2,\{h_3,F_4\})\}$, and*

$(F_3, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\}), \}$. *The family of all soft closed sets is* $\{\Phi, X, (F_2, E)\}$. (F_3, E) *is a soft set, whenever* $(F_3, E)\widetilde{\subseteq}(F_1, E)$, (F_1, E) *is soft open, also is soft* $\alpha-$*open and* $scl(F_3, E) = X\widetilde{\not\subseteq}(F_1, E)$, $bcl(F_3, E) = (F_3, E)\widetilde{\subseteq}(F_1, E)$, *then it is* $g\alpha b-$*closed but not soft* $gs-$*closed.*

Example 3.29 *Let* $X = \{h_1, h_2, h_3, h_4\}$ *and* $E = \{e_1, e_2, e_3, e_4\}$ *be the set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E)\}$ *where* $(F_1, E) = \{(e_1, \{h_4\}), (e_2, \{h_4\}), (e_3, \{h_4\}), (e_4, \{h_4\})\}$. *The family of all soft closed sets is* $\{\Phi, X, (F_1, E)^c\}$ *where* $(F_1, E)^c = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2, h_3\}), (e_3, \{h_1, h_2, h_3\}), (e_4, \{h_1, h_2, h_3\})\}$. $(F_2, E) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_3, h_4\}), (e_3, \{h_1, h_4\}), (e_4, \{h_4\})\}$ *is soft* $\alpha-$*open and also soft semi open and* X *is the only soft open set which containing* (F_2, E), $bcl(F_1, E) = scl(F_1, E) = X\widetilde{\subseteq}X$ *but* $(F_1, E)\widetilde{\subseteq}(F_2, E)$ *and* $bcl(F_1, E) = X\widetilde{\not\subseteq}(F_2, E)$. *Hence,* (F_1, E) *is soft* $gs-$*closed but not soft* $g\alpha b-$*closed.*

Theorem 3.30 *If a soft set* (F, E) *of a soft topological space* (X, τ, E) *is soft* $sg-$*closed,then it is soft* $g\alpha b-$*closed.*

Proof. Let (F, E) be a soft sg-closed and $(F, E)\widetilde{\subseteq}(U, E)$, where (U, E) is a soft $\alpha-$open set and also soft semi open set, since every soft semi-closed set is soft $b-$closed set, then $bcl(F, E)\widetilde{\subseteq}scl(F, E)\widetilde{\subseteq}(U, E)$. Therefore (F, E) is soft $g\alpha b-$closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.31 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}\}$ *be the set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E)\}$, $(F_1, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$. $(F_2, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ *is soft* $g\alpha b-$*closed, but not soft* $sg-$*closed, since* $(F_2, E)\widetilde{\subseteq}(F_1, E)$, $bcl(F_2, E) = (F_2, E)$ *but* $scl(F_2, E) = X\widetilde{\not\subseteq}(F_1, E)$.

The following examples show that the concepts of soft $g\alpha b-$closeness and soft $gpr-$closeness are independent.

Example 3.32 *Let* $X = \{h_1, h_2, h_3, F_4\}$ *and* $E = \{e_1, e_2, e_3\}$ *be the set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$, *where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\}), (e_3, \{h_1\})\}$, $(F_2, E) = \{(e_1, \{h_2\}), (e_2, \{h_2\}), (e_3, \{h_2\})\}$, $(F_3, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\}), (e_3, \{h_1, h_2\})\}$. (F_1, E) *is soft regular open,* $(F_1, E)\widetilde{\subseteq}(F_1, E)$, $pcl(F_1, E) = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_3, h_4\}), (e_3, \{h_1, h_3, h_4\})\}\widetilde{\not\subseteq}(F_1, E)$, $bcl(F_1, E) = (F_1, E)\widetilde{\subseteq}(F_1, E)$. *Hence,* (F_1, E) *is a* $g\alpha b-$*closed, but not soft* $gpr-$*closed.*

Example 3.33 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be the set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ *where*

$(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_3\}),(e_2,\{h_3\})\}$, $(F_3,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$ *and* $(F_4,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$.
(F_1,E) *is soft* $gpr-$*closed, but not soft* $g\alpha b-$*closed, since* (F_4,E) *is the only soft regular open set which containing* (F_1,E) *and* $pcl(F_1,E)=bcl(F_1,E)=(F_4,E)\widetilde{\subseteq}(X,E)$, $(F_1,E)\widetilde{\subseteq}(F_1,E)$ *and* (F_1,E) *is soft open and also* $\alpha-$*open,* $bcl(F_1,E)=(F_4,E)\widetilde{\nsubseteq}(F_1,E)$.

The following examples show that the concepts of soft $g\alpha b-$closeness and soft $rg-$closeness are independent.

Example 3.34 *Let* $X=\{h_1,h_2,h_3,F_4\}$ *and* $E=\{e_1,e_2,e_3\}$ *be the set of parameters with a soft topology* $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E)\}$, *where* $(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\}),(e_3,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\}),(e_3,\{h_2\})\}$, $(F_3,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\}),(e_3,\{h_1,h_2\})\}$ *and* $(F_4,E)=\{(e_1,\{h_1\}),(e_2,\Phi),(e_3,\{h_1\})\}$. (F_1,E) *is soft regular open,* $(F_4,E)\widetilde{\subseteq}(F_1,E)$, $cl(F_4,E)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_1,h_3,h_4\}),\ (e_3,\{h_1,h_3,h_4\})\}\widetilde{\nsubseteq}(F_1,E)$, bcl $(F_4,E)=(F_4,E)\widetilde{\subseteq}(F_1,E)$. *Hence,* (F_4,E) *is* $g\alpha b-$ *closed, but not soft* $rg-$*closed.*

Example 3.35 *Let* $X=\{h_1,h_2,h_3\}$ *and* $E=\{e_1,e_2\}$ *be the set of parameters with a soft topology* $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$, *where* $(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_3\}),(e_2,\{h_3\})\}$, $(F_3,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$ *and* $(F_4,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$.
(F_1,E) *is soft* $rg-$*closed, but not soft* $g\alpha b-$*closed,since* X *itself is the only soft regular open set which containing* (F_1,E) *and* $cl(F_1,E)=bcl(F_1,E)=(F_4,E)\widetilde{\subseteq}(X,E)$, $(F_1,E)\widetilde{\subseteq}(F_1,E)$ *and* (F_1,E) *is soft open and also soft* $\alpha-$*open,* $bcl(F_1,E)=(F_4,E)\widetilde{\nsubseteq}(F_1,E)$. *Hence,* (F_1,E) *is not soft* $g\alpha b-$*closed set.*

Theorem 3.36 *If a soft set* (F,E) *of a soft topological space* (X,τ,E) *is soft* $SW-$*closed, then it is soft* $g\alpha b-$*closed.*

Proof. Let (F,E) be a soft $SW-$closed set and $(F,E)\widetilde{\subseteq}(U,E)$, where (U,E) is a soft $\alpha-$open set and also soft semi open, $cl(F,E)\widetilde{\subseteq}(U,E)$, $bcl(F,E)\widetilde{\subseteq}cl(F,E)$ $\widetilde{\subseteq}(U,E)$, $bcl(F,E)\widetilde{\subseteq}(U,E)$. Therefore, (F,E) is soft $g\alpha b-$closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.37 *Let* $X=\{h_1,h_2,h_3\}$ *and* $E=\{e_1,e_2\}$ *be the set of parameters with a soft topology* $\tau=\{\Phi,X,(F_1,E)\}$, *where* $(F_1,E)=\{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$. $(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ *is soft* $g\alpha b-$*closed, but not soft* $SW-$*closed, since* $(F_2,E)\widetilde{\subseteq}(F_1,E)$ *and* $bcl(F_2,E)=(F_2,E)$, $cl(F_2,E)=X$.

The following examples show that the concepts of soft $g\alpha b-$closeness and soft $g-$closeness are independent.

Example 3.38 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E)\}$ *where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$*.* $(F_2, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ *is soft g-closed, but not soft* $g\alpha b-$*closed, since the only soft open set containing* (F_2, E) *is* X *and* $bcl(F_2, E) = cl(F_2, E) = X$*.*

Example 3.39 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$*, where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$*,* $(F_2, E) = \{(e_1, \{h_3\}), (e_2, \{h_3\})\}$ *and* $(F_3, E) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_3\})\}$*.* (F_1, E) *is soft* $g\alpha b-$*closed, but not soft* $g-$*closed.*

The following examples show that the concepts of soft $g\alpha b-$ closeness and soft $wg-$closeness are independent.

Example 3.40 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E)\}$ *where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$*,* $(F_2, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ *is soft* $wg-$*closed, but not soft* $g\alpha b-$*closed, since* X *is the only soft open set containing* (F_2, E) *and* $bcl(F_2, E) = clint(F_2, E) = X$*.*

Example 3.41 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$*, where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$*,* $(F_2, E) = \{(e_1, \{h_3\}), (e_2, \{h_3\})\}$ *and* $(F_3, E) = \{(e_1, \{h_1, h_3\})\}, (e_2, \{h_1, h_3\})\}$*.* (F_1, E) *is soft* $g\alpha b-$*closed but not soft* $wg-$*closed.*

The following examples show that the concepts of soft $g\alpha b-$ closeness and soft $rwg-$closeness are independent.

Example 3.42 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E)\}$ *where* $(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$*,* $(F_2, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ *is soft* $rwg-$*closed, but not soft* $g\alpha b-$*closed, since* X *is the only soft regular open set which containing* (F_2, E) *and* $bcl(F_2, E) = clint(F_2, E) = X$*.*

Example 3.43 *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$ *be a set of parameters with a soft topology* $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$*, where* $(F_1, E) = \{(e_1, \{h_1\}), , (e_2, \{h_1\})\}$*,* $(F_2, E) = \{(e_1, \{h_3\}), (e_2, \{h_3\})\}$*,* $(F_3, E) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_3\})\}$*.* (F_1, E) *is soft* $g\alpha b-$*closed, but not soft* $rwg-$*closed, since the soft sets* $\Phi, X, (F_1, E)$ *and* (F_2, E) *are soft regular open sets of* X*, and* $bcl(F_2, E) = (F_2, E)\widetilde{\subseteq}(F_2, E)$*,* $clint(F_2, E)\widetilde{\nsubseteq}(F_2, E)$*.*

Figure 1 gives the implication relationship of soft $g\alpha b-$closed sets based on the above results, where

1. $A \rightarrow B$ represent A implies B,
2. $A \not\leftrightarrow B$ represent A and B are independent,
3. $A \not\rightarrow B$ represent A does not implies B.

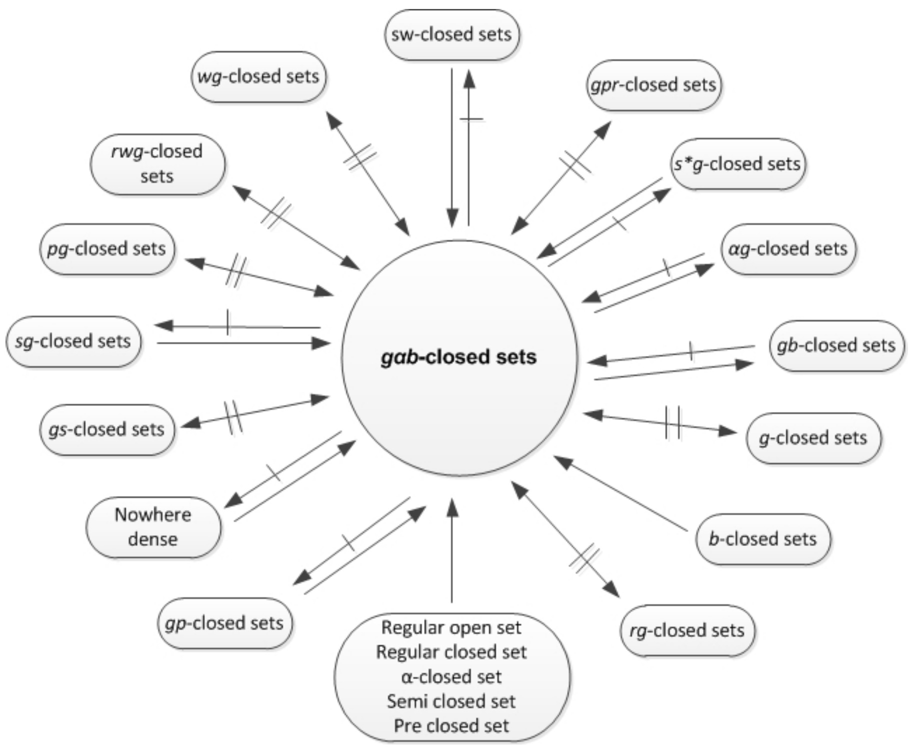

Figure 1: Implication of soft gαb-closed sets

4 Soft Generalized $\alpha b-$Open Sets and Soft Generalized $\alpha b-$Neighbourhoods

This section introduces the concept of soft generalized $\alpha b-$open sets in soft topological space and studies some of their properties.

Definition 4.1 *A soft set (F, E) of a soft topological space (X, τ, E) is called a soft generalized αb-open (briefly soft g$\alpha b-$open) set, if its complement $(F, E)^c$ is soft g$\alpha b-$closed. The collection of all soft g$\alpha b-$open sets in (X, τ, E) is denoted by sg$\alpha b-$O(X).*

Theorem 4.2 *A soft set (F, E) of a topological space (X, τ, E) is soft g$\alpha b-$open if and only if $(U, E)\widetilde{\subseteq}bint(F, E)$, whenever (U, E) is a soft $\alpha-$closed and (U, E) $\widetilde{\subseteq}(F, E)$.*

Proof. Let (F, E) be soft $g\alpha b-$open and (U, E) be a soft $\alpha b-$closed contained in (F, E). Then $(F, E)^c$ is soft $g\alpha b-$closed and $(U, E)^c$ is soft $\alpha-$open containing $(F, E)^c$, $bcl(F, E)^c \widetilde{\subseteq} (U, E)^c$. Therefore, $(U, E) \widetilde{\subseteq}\ bint(F, E)$.

Conversely: Let $(U, E) \widetilde{\subseteq} bint(F, E)$ whenever $(U, E) \widetilde{\subseteq} (F, E)$ and (U, E) is a soft $\alpha-$closed. Let (G, E) be a soft $\alpha-$open set containing $(F, E)^c$, then $(G, E)^c \widetilde{\subseteq} bint(F, E)$, $bcl(F, E)^c \widetilde{\subseteq} (G, E)$. Hence, $(F, E)^c$ is soft $g\alpha b-$closed. Therefore, (F, E) is soft $g\alpha b-$open.

Theorem 4.3 *If $bint(A, E) \widetilde{\subseteq} (B, E) \widetilde{\subseteq} (A, E)$ and (A, E) is soft $g\alpha b-$open then (B, E) is soft $g\alpha b-$open.*

Proof. $bint(A, E) \widetilde{\subseteq} (B, E) \widetilde{\subseteq} (A, E)$ implies $(A, E)^c \widetilde{\subseteq} (B, E)^c \widetilde{\subseteq} bcl((A, E)^c)$ and $(A, E)^c$ is soft $g\alpha b-$-closed. By the Theorem 3.10, $(B, E)^c$ is soft $g\alpha b-$closed. Hence (B, E) is soft $g\alpha b-$open.

Theorem 4.4 *If (A, E) and (B, E) are soft $g\alpha b-$open in X then $(A, E) \widetilde{\cup} (B, E)$ is also soft $g\alpha b-$open.*

Proof. Since (A, E) and (B, E) are soft $g\alpha b-$open and by the Definition 4.1 their relative complements are soft $g\alpha b-$closed sets and by Theorem 3.12, $(A, E) \widetilde{\cap} (B, E)^c$ is soft $g\alpha b-$closed. Hence $(A, E) \widetilde{\cup} (B, E)$ is soft $g\alpha b-$open.

Definition 4.5 *A soft set (N, E) in a soft topological space (X, τ, E) is called a soft $g\alpha b-$neighborhood of the soft point $F(e) \widetilde{\in} (X, \tau, E)$ if there exists a soft $g\alpha b-$open set (H, E) such that $F(e) \widetilde{\in} (H, E) \widetilde{\subseteq} (N, E)$. A soft set (N, E) in a soft topological space (X, τ, E) is called a soft $g\alpha b-$neighborhood of the soft set (F, E) if there exists a soft $g\alpha b-$open set (H, E) such that $(F, E) \widetilde{\subseteq} (H, E) \widetilde{\subseteq} (N, E)$.*

A soft $g\alpha b-$neighborhood generally need not be soft $g\alpha b-$open as shown by the following example.

Example 4.6 *Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E) = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, $(F_2, E) = \{(e_1, \{a, b\}), (e_2, \{a\})\}, (F_3, E) = \{(e_1, \{b\}), (e_2, \Phi)\}$, $(F_4, E) = \{(e_1, \{a\}), (e_2, \{a\})\}$ the family of all soft closed sets is $\{X, \Phi, (F_1, E)^c, (F_2, E)^c, (F_3, E)^c, (F_4, E)^c\}$, where $(F_1, E)^c = \{(e_1, \{b\}), (e_2, \Phi)\}, (F_2, E)^c = \{(e_1, \Phi), (e_2, \{b\})\}, (F_3, E)^c = \{(e_1, \{a\}), (e_2, \{a, b\})\}, (F_4, E)^c = \{(e_1, \{b\}), (e_2, \{b\})\}$ and soft $\alpha-O(X) = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$. A soft set (N, E) such that $(N, E) = \{(e_1\{b\}), (e_2, \{b\})\}$, is not soft $g\alpha b-$open, but it is soft $g\alpha b-$neighborhood of a point $G(e_1) = \{b\}$, and also it is soft $g\alpha b-$neighborhood of a soft $g\alpha b-$open set (G, E), where $(G, E) = \{(e_1, \{b\}), (e_2, \Phi)\}$. Hence, $(G, E)\ \widetilde{\subseteq} (N, E)$.*

Theorem 4.7 *Every soft neighborhood is a $g\alpha b-$neighborhood.*

Proof. Let (N, E) be a soft neighborhood of a soft point $F(e)\widetilde{\in}(X, \tau, E)$, then there exists a soft open set (G, E) such that $F(e)\widetilde{\in}(G, E)\widetilde{\subseteq}(N, E)$. As every soft open set (G, E) is a soft $g\alpha b-$open set, such that $F(e)\widetilde{\in}(G, E)\widetilde{\subseteq}(N, E)$. Hence, (N, E) is soft $g\alpha b-$neighborhood of $F(e)$.

A soft $g\alpha b-$neighborhood generally need not be a soft neighborhood as shown by the following example.

Example 4.8 *Let $X = \{a, b\}$ and $E = \{e_1, e_2\}$ ba a set of parameters with a soft topology $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$ $(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{a\})\}$, $(F_2, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$, $(F_3, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$ the family of all soft closed sets is $\{X, \Phi, (F_1, E)^c, (F_2, E)^c, (F_3, E)^c\}$ where $(F_1, E)^c = \{(e_1, \Phi), (e_2, \{b\})\}, (F_2, E)^c = \{(e_1, \{a\}), (e_2, \Phi)\}, (F_3, E)^c = \{(e_1, \{a\}), (e_2, \{b\})\}$ and soft $\alpha - O(X) = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$. A soft set (N, E) such that $(N, E) = \{(e_1, \Phi), (e_2, \{a, b\})\}$ is soft $g\alpha b-$open set and also soft $g\alpha b-$neighborhood of a point $G(e_2) = \{b\}$ since $G(e_2) = \{b\}\widetilde{\in}(N, E)\widetilde{\subseteq}(N, E)$. However, the soft set (N, E) is not a soft neighborhood of the point $G(e_2) = \{b\}$, since (F_2, E) is the only soft open set containing $G(e_2) = \{b\}$ without X and $G(e_2) = \{b\}\widetilde{\in}(F_1, E)\widetilde{\nsubseteq}(N, E)$.*

Theorem 4.9 *A soft $g\alpha b-$closed set is a soft $g\alpha b-$closed neighborhood of each of it's soft points.*

Proof. Let (N, E) be a soft $g\alpha b-$closed subset of a soft topological space (X, τ, E) and $Fe\widetilde{\in}(N, E)$, it can be claimed that (N, E) is a soft $g\alpha b-$closed neighborhood of Fe. (N, E) is a soft $g\alpha b-$closed set such that $Fe\widetilde{\in}(N, E)\widetilde{\subseteq}(N, E)$, since $F(e)$ is an arbitrary point of (N, E), then (N, E) is a $g\alpha b-$closed neighborhood of each of it's points.

The converse of the above theorem generally need not be true as shown by the following example.

Example 4.10 *Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\Phi, X, (F_1, E)\}$ is a soft topology over X, where $(F_1, E) = \{(e_1, \{a, b\}, (e_2, \{a, b\})\}$, the family of all soft closed sets is $\{X, \Phi, (F_2, E)\}$, where $(F_2, E) = \{(e_1, \{c, d\}, (e_2, \{c, d\})\}$, the soft set $(F_3, E) = \{(e_1, \{a, b, d\}, (e_2, \{a, b, d\})\}$ is a soft $\alpha-$open set , $(F_4, E) = \{(e_1, \{a, d\}, (e_2, \{a, d\})\}$ and $(F_5, E) = \{(e_1, \{b, d\}, (e_2, \{b, d\})\}$ are soft $g\alpha b-$closed sets, $(F_4, E)\widetilde{\subseteq}(F_3, E)$ and $(F_5, E)\widetilde{\subseteq}(F_3, E)$, then the soft set $(F_3, E) = \{(e_1, \{a, b, d\}, (e_2, \{a, b, d\})\}$ is a soft $g\alpha b-$closed neighborhood of of each of its points but $bcl(F_3, E) = X\widetilde{\nsubseteq}(F_3, E)$. However, (F_3, E) is not soft $g\alpha b-$closed in X.*

5 Conclusion

In the present, we have introduced soft generalized $\alpha b-$closed and soft generalized $\alpha b-$open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have presented it's fundamental properties with the help of some counterexamples. In future these results may be extended to new types of soft generalized closed and open sets in soft topological spaces.

References

[1] D. Molodtsov, Soft set theory-first results, *Computers & Mathematics with Applications*, 37(4) (1999), 19-31.

[2] M. Shabir and M. Naz, On soft topological spaces, *Computers & Mathematics with Applications*, 61(7) (2011), 1786-1799.

[3] N. Levine, On generalized closed sets in topology, *Rendiconti del Circolo Matematico di Palermo*, 19(2) (1970), 89-96.

[4] K. Kannan, Soft generalized closed sets in soft topological spaces, *Journal of Theoretical and Applied Information Technology*, 37(2012), 17-21.

[5] Y. Saziye, N. Tozlu and Z.E. Guzel, On soft generalized closed sets in soft topological spaces, *Computers & Mathematics with Applications*, 55(2013), 273-279.

[6] S. Nazmul and S. Samanta, Neighbourhood properties of soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 6(2012), 1-15.

[7] S. Hussain and B. Ahmad, Some properties of soft topological spaces, *Computers & Mathematics with Applications*, 62(2011), 4058-4067.

[8] G. Ilango, B. Arun and K.S. Kumar, Some properties of soft $\alpha-$open sets in soft topological space, *IOSR Journal of Mathematics (IOSR-JM)*, 9(2014), 20-24.

[9] M. Akdag and A. Ozkan, Soft $b-$open sets and soft $b-$continuous functions, *Mathematical Sciences*, 8(2) (2014), 1-9.

[10] S. El-Sheikh and A.A. El-latif, Characterization of $b-$open soft sets in soft topological spaces, *Journal of New Theory*, 2(2015), 8-18.

[11] P. Maji, R. Biswas and A.R. Roy, Soft set theory, *Computers & Mathematics with Applications*, 45(2) (2003), 555-562.

[12] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, On some new operations in soft set theory, *Computers & Mathematics with Applications*, 57(9) (2009), 1547-1553.

[13] I. Zorlutuna, M. Akdag and W.K. Min, Remarks on soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 3(2) (2012), 171-185.

[14] A. Kandil, O. Tantawy, S. El-Sheikh and A.A. El-Latif, $\gamma-$operation and decompositions of some forms of soft continuity in soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 7(2) (2014), 181-196.

[15] I. Arockiarani and T.A. Albinaa, Soft generalized preclosed sets and space, *Proceedings of ICMSCA*, (2014), 183-187.

[16] T. Nandhini and A. Kalaichelvi, Soft $\hat{g}$-closed sets in soft topological spaces, *International Journal of Innovative Research in Science, Engineering and Technology*, 3(7) (2014), 14595-14600.

[17] K. Kannan and D. Rajalakshmi, Soft semi star generalized closed sets, *Malaysian Journal of Mathematical Sciences*, 3(2015), 77-88.

[18] C. Janaki and V. Jeyanthi, Soft rˆg-closed sets in soft topological spaces, *International Journal of Engineering Research and Technology*, 3(4) (2014), 2084-2091.

[19] Z.E. Guzel, S. Yksel and N. Tozlu, On soft generalized preregular closed and open sets in soft topological spaces, *Applied Mathematical Sciences*, 8(2014), 7875-7884.

[20] S. Yuksel, N. Tozlu and Z.G. Ergul, Soft regular generalized closed sets in soft topological spaces, *International Journal of Mathematical Analysis*, 8(2014), 355-367.

[21] T. Nandhini and A. Kalaichelv, SRW-closed sets in soft topological spaces, *International Journal of Innovative Research in Science, Engineering and Technology*, 3(2014), 13343-13348.