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A Study on (Q, L) - Fuzzy Subsemiring of a Semiring

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Abstract

In this paper, we introduce the concept of (Q,L)- fuzzy subsemirings of a semiring and establish some results on these. We also made an attempt to study the properties of (Q,L)-fuzzy subsemirings of semiring under homomorphism and anti-homomorphism , and study the main theorem for this. We shall also give new results on this subject.

Keywords: *(Q,L)-fuzzy subset, (Q,L)-fuzzy subsemiring, (Q,L)-fuzzy relation, Product of (Q,L)-fuzzy subsets, pseudo (Q,L)-fuzzy coset, (Q,L)-anti-fuzzy subsemiring.*

Introduction

There are many concepts of universal algebras generalizing an associative ring (*R; +; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra *(R ; +, .)* is said to be a semiring if *(R;+)* and *(R; .)* are semigroups satisfying *a.(b+c)=a.b+a.c* and *(b+c).a=b.a+c.a* for all *a*, *b* and *c* in *R*. A semiring *R* is said to be additively commutative if *a+b = b+a* for all *a*, *b* in *R*. A semiring *R* may have an identity 1, defined by *1. a = a = a. 1* and a zero 0, defined by *0+a=a=a+0* and *a.0=0=0.a* for all *a* in *R*. After the introduced of fuzzy sets by L.A. Zadeh [7], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld [2] defined a fuzzy group. Asok Kumer Ray [1] defined a product of fuzzy subgroups and Fuzzy subgroups and Anti-fuzzy subgroups have introduced R. Biswas [14] A. Solairaju and R. Nagarajan [3] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of (Q, L)-fuzzy subsemiring of a semiring and established some results.

1 Preliminaries:

1.1 Definition: *Let X be a non–empty set. A fuzzy subset A of X is a function A : X → [0, 1].*

1.2 Definition: *Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function A: X×Q → L.*

1.3 Definition: *Let (R, +, ·) be a semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:*

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,

(ii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

1.4 Definition: *Let A and B be any two (Q,L)-fuzzy subsets of sets R and H, respectively. The product of A and B, denoted by A×B, is defined as A×B={<((x, y), q),A×B((x,y),q)>/ for all x in R and y in H and q in Q}, where A×B((x,y),q) =A(x, q)∧ B(y, q).*

1.5 Definition: *Let (R,+,·) and (R^{l},+,·) be any two semirings and Q be a non empty set. Let f:R→R^{l} be any function and A be a (Q, L)-fuzzy subsemiring in R, V be a (Q, L)-fuzzy subsemiring in f(R)=R^{l}, defined by* $V(y,q)= \sup_{x\in f^{-1}(y)} A(x,q)$, *for all x in R and y in R^{l} and q in Q. Then A is called a pre-image of V under f and is denoted by* $f^{-1}(V)$.

1.6 Definition: *Let A be an (Q,L)-fuzzy subsemiring of a semiring (R, +, ·) and a in R. Then the pseudo (Q, L)-fuzzy coset* $(aA)^p$ *is defined by* $((aA)^p)(x,q) = p(a)A(x,q)$*, for every x in R and for some p in P and q in Q.*

1.7 Definition: *Let A be a (Q,L)-fuzzy subset in a set S, the strongest (Q, L)-fuzzy relation on S, that is a (Q,L)-fuzzy relation V with respect to A given by* $V((x, y), q) = A(x, q) \wedge A(y, q)$*, for all x and y in S and q in Q.*

1.8 Definition: *Let (R, +, ·) be a semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-anti-fuzzy subsemiring (QLAFSSR) of R if the following conditions are satisfied:*

(i) $A(x+y, q) \leq A(x, q) \vee A(y, q)$,
(ii) $A(xy, q) \leq A(x, q) \vee A(y, q)$, for all x and y in R and q in Q.

1.9 Definition: *Let X be a non-empty set and A be a (Q,L)-fuzzy subsemiring of a semiring R. Then* A^0 *is defined as* $A^0(x,q)=A(x,q)/A(0,q)$*,for all x in R and q in Q, where 0 is the identity element of R.*

1.10 Definition: *Let A be a (Q,L)-fuzzy subset of X. For* α *in L, a Q-level subset of A is the set* $A_\alpha = \{ x \in X : A(x,q) \geq \alpha \}$.

2 Properties of (Q,L)-Fuzzy Subsemiring of a Semiring

2.1 Theorem: *If A and B are two (Q, L)-fuzzy subsemiring of a semiring R, then their intersection A∩B is a (Q, L)-fuzzy subsemiring of R.*

Proof: Let x and y belongs to R and q in Q, $A=\{<(x, q),A(x, q)>/x$ in R and q in Q} and $B=\{<(x, q),B(x,q)>/x$ in R and q in Q}. Let $C=A\cap B$ and $C=\{<(x,q),C(x,q)>/ x$ in R and q in Q}.

(i) $C(x+y,q)=A(x+y,q)\wedge B(x+y,q)\geq\{A(x,q)\wedge A(y,q)\}\wedge\{B(x,q)\wedge B(y,q)\}\geq \{A(x,q)\wedge B(x,q)\}\{A(y,q)\wedge B(y,q)\}=C(x,q)\wedge C(y,q).$

Therefore, $C(x+y,q) \geq C(x,q)\wedge C(y,q)$, for all x and y in R and q in Q.

(ii) $C(xy,q)=A(xy,q)\wedge B(xy,q)\geq\{A(x, q)\wedge A(y,q)\}\wedge\{ B(x,q)\wedge B(y,q)\}\geq\{A(x,q) \wedge B(x,q)\}\wedge\{A(y,q)\wedge B(y,q)\}=C(x,q)\wedge C(y,q).$

Therefore, $C(xy, q) \geq C(x,q)\wedge C(y,q)$, for all x and y in R and q in Q. Hence A∩B is a (Q, L)-fuzzy subsemiring of a semiring R.

2.2 Theorem: *The intersection of a family of (Q, L)-fuzzy subsemiring of a semiring R is a (Q, L)-fuzzy subsemiring of R.*

Proof: Let $\{A_i\}_{i \in I}$ be a family of (Q,L)-fuzzy subsemiring of a semiring R and $A = \bigcap_{i \in I} A_i$ Then for x and y belongs to R and q in Q, we have $A(x+y,q) = \inf_{i \in I} A(x+y,q) \geq \inf_{i \in I} \{A_i(x,q) \wedge A_i(y,q)\} = \inf_{i \in I} (A_i(x,q)) \wedge \inf_{i \in I} (A_i(y,q)) = A(x,q) \wedge A(y,q)$. Therefore, $A(x+y,q) \geq A(x,q) \wedge A(y,q)$, for all x and y in R and q in Q. $A(xy,q) = \inf_{i \in I} A_i(xy,q) \geq \inf_{i \in I} \{A_i(x,q) \wedge A_i(y,q)\} = \inf_{i \in I} (A_i(x,q)) \wedge \inf_{i \in I} (A_i(y,q)) = A(x,q) \wedge A(y,q)$.

Therefore, $A(xy,q) \geq A(x,q) \wedge A(y,q)$, for all x and y in R and q in Q. Hence the intersection of a family of (Q, L)- fuzzy subsemiring of a semiring R is a (Q, L)-fuzzy subsemiring of R.

2.3 Theorem: *If A and B are (Q, L)-fuzzy subsemiring of a semiring R and H, respectively, then A×B is a (Q, L)-fuzzy subsemiring of R×H.*

Proof: Let A and B be (Q,L)-fuzzy subsemiring of a semiring R and H respectively. Let x_1 and x_2 be in R, y_1 and y_2 be in H. Then (x_1,y_1) and (x_2,y_2) are in R×H and q in Q. Now,

$A \times B[(x_1,y_1)+(x_2,y_2),q] = A \times B((x_1+x_2,y_1+y_2),q) = A(x_1+x_2,q) \wedge B(y_1+y_2,q) \geq \{A(x_1,q) \wedge A(x_2,q)\} \wedge \{B(y_1,q) \wedge B(y_2,q)\} = \{A(x_1,q) \wedge B(y_1,q)\} \wedge \{A(x_2,q) \wedge B(y_2,q)\} = A \times B((x_1,y_1),q) \wedge A \times B((x_2,y_2),q)$.

Therefore, $A \times B[(x_1,y_1)+(x_2,y_2),q] \geq A \times B((x_1,y_1),q) \wedge A \times B((x_2,y_2),q)$.

$A \times B[(x_1,y_1)(x_2,y_2),q] = A \times B((x_1x_2,y_1y_2),q) = A(x_1x_2,q) \wedge B(y_1y_2,q) \geq \{A(x_1,q) \wedge A(x_2,q)\} \wedge \{B(y_1,q) \wedge B(y_2,q)\} = \{A(x_1,q) \wedge B(y_1,q)\} \wedge \{A(x_2,q) \wedge B(y_2,q)\} = A \times B((x_1,y_1),q) \wedge A \times B((x_2,y_2),q)$.

Therefore, $A \times B[(x_1,y_1)(x_2,y_2),q] \geq A \times B((x_1,y_1),q) \wedge A \times B((x_2,y_2),q)$.

Hence A×B is a (Q, L)-fuzzy subsemiring of R×H.

2.4 Theorem: *Let A be a (Q, L)-fuzzy subset of a semiring R and V be the strongest (Q, L)-fuzzy relation of R. Then A is an (Q, L)-fuzzy subsemiring of R if and only if V is an (Q, L)-fuzzy subsemiring of R×R.*

Proof: Suppose that A is an (Q, L)-fuzzy subsemiring of a semiring R. Then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and q in Q.

We have,

$V(x+y,q) = V[(x_1,x_2)+(y_1,y_2),q] = V((x_1+y_1,x_2+y_2),q) = A((x_1+y_1),q) \wedge A((x_2+y_2),q) \geq \{A(x_1,q) \wedge A(y_1,q)\} \wedge \{A(x_2,q) \wedge A(y_2,q)\} = \{A(x_1,q) \wedge A(x_2,q)\} \wedge \{A(y_1,q) \wedge A(y_2,q)\} = V((x_1,x_2),q) \wedge V((y_1,y_2),q) = V(x,q) \wedge V(y,q)$.

Therefore, $V(x+y,q) \geq V(x,q) \wedge V(y,q)$, for all x and y in R×R.

And,
$V(xy,q)=V[(x_1,x_2)(y_1,y_2),q]=V((x_1y_1,x_2y_2),q)=A(x_1y_1,q) \wedge A(x_2y_2,q) \geq \{A(x_1,q) \wedge A(y_1,q)\} \wedge \{A(x_2,q) \wedge A(y_2,q)\}=\{A(x_1,q) \wedge A(x_2,q)\} \wedge \{A(y_1,q) \wedge A(y_2,q)\}=V((x_1,x_2),q) \wedge V((y_1,y_2),q)=V(x,q) \wedge V(y,q).$

Therefore, $V(xy,q) \geq V(x,q) \wedge V(y,q)$, for all x and y in R×R. This proves that V is an (Q,L)-fuzzy subsemiring of R×R. Conversely assume that V is an (Q,L)-fuzzy subsemiring of R×R, then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R,

We have

$A((x_1+y_1),q) \wedge A((x_2+y_2),q)=V((x_1+y_1,x_2+y_2),q)=V[((x_1,x_2)+(y_1,y_2)),q]=V((x+y),q) \geq V(x,q) \wedge V(y,q)=V((x_1,x_2),q) \wedge V((y_1,y_2),q)=\{\{A(x_1,q) \wedge A(x_2,q)\} \wedge \{A(y_1,q) \wedge A(y_2,q)\}\}$

If $A((x_1+y_1),q) \leq A((x_2+y_2),q), A(x_1,q) \leq A(x_2,q), A(y_1,q) \leq A(y_2,q)$, we get,
$A((x_1+y_1),q) \geq A(x_1,q) \wedge A(y_1,q)$, for all x_1 and y_1 in R.

And, $A(x_1y_1,q) \wedge A(x_2y_2,q)=V((x_1y_1,x_2y_2),q)=V[((x_1,x_2)(y_1,y_2),q)]=V(xy,q) \geq V(x,q) \wedge V(y,q)=V((x_1,x_2),q) \wedge V((y_1,y_2),q)=\{\{A(x_1,q) \wedge A(x_2,q)\} \wedge \{A(y_1,q) \wedge A(y_2,q)\}\}.$

If $A(x_1y_1,q) \leq A(x_2y_2,q), A(x_1,q) \leq A(x_2,q), A(y_1,q) \leq A(y_2,q)$, we get $A(x_1y_1,q) \geq A(x_1,q) \wedge A(y_1,q)$, for all x_1, y_1 in R. Therefore A is an (Q, L)-fuzzy subsemiring of R.

2.5 Theorem: *A is an (Q, L)-fuzzy subsemiring of a semiring (R, +, ·) if and only if* $A((x+y),q) \geq A(x,q) \wedge A(y,q)$, $A(xy,q) \geq A(x,q) \wedge A(y,q)$ *, for all x and y in R.*

Proof: It is trivial.

2.6 Theorem: *If A is an (Q, L)-fuzzy subsemiring of a semiring (R, +, ·), then* $H = \{ x / x \in R: A(x,q) =1\}$ *is either empty or is a subsemiring of R.*

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A((x+y),q) \geq A(x,q) \wedge A(y,q)=1 \wedge 1=1$. Therefore, $A((x+y),q)=1$. And, $A(xy,q) \geq A(x,q) \wedge A(y,q)= 1 \wedge 1= 1$. Therefore, $A(xy,q)=1$. We get $x+y$, xy in H. Therefore, H is a subsemiring of R. Hence H is either empty or is a subsemiring of R.

2.7 Theorem: *If A be an (Q, L)-fuzzy subsemiring of a semiring (R, +, ·) , then if* $A((x+ y),q) =0$, *then either* $A(x,q) =0$ *or* $A(y,q) = 0$, *for all x and y in R and q in Q.*

Proof: Let x and y in R and q in Q. By the definition $A((x+ y),q) \geq A(x,q) \wedge A(y,q)$, which implies that $0 \geq A(x,q) \wedge A(y,q)$. Therefore, either $A(x,q) = 0$ or $A(y,q) = 0$.

2.8 Theorem: *Let A be a (Q,L)-fuzzy subsemiring of a semiring R. Then A^0 is a (Q,L)-fuzzy subsemiring of a semiring R.*

Proof: For any x in R and q in Q, we have $A^0(x+y,q)=A(x+y,q)/A(0,q)\geq [1/A(0,q)]\{A(x,q)\wedge A(y,q)\}=[A(x,q)/A(0,q)]\wedge[A(y,q)/A(0,q)]=A^0(x,q)\wedge A^0(y,q)$.

That is $A^0(x+y,q)\geq A^0(x,q)\wedge A^0(y,q)$ for all x and y in R and q in Q.

$A^0(xy,q)=A(xy,q)/A(0,q)\geq[1/A(0,q)]\{A(x,q)\wedge A(y,q)\}=$
$[A(x,q)/A(0,q)]\wedge[A(y,q)/A(0,q)] = A^0(x,q)\wedge A^0(y,q)$.

That is $A^0(xy,q)\geq A^0(x,q)\wedge A^0(y,q)$ for all x and y in R and q in Q. Hence A^0 is a (Q,L)- fuzzy subsemiring of a semiring R.

2.9 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring R. A^+ be a fuzzy set in R defined by $A^+(x,q)=A(x,q)+1-A(0,q)$, for all x in R and q in Q , where 0 is the identity element. Then A^+ is an (Q,L)- fuzzy subsemiring of a semiring R.*

Proof: Let x and y in R and q in Q. We have,
$A^+(x+y,q)=A(x+y,q)+1-A(0,q)\geq\{A(x,q)\wedge A(y,q)\}+1-A(0,q)=\{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\}=A^+(x,q)\wedge A^+(y,q)$,

which implies that $A^+(x+y,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x, y in R and q in Q. $A^+(xy,q)=A(xy,q)+1-A(0,q)\geq\{A(x,q)\wedge A(y,q)\}+1-A(0,q)=\{A(x,q)+1-A(0,q)\}\wedge\{A(y,q)+1-A(0,q)\}= A^+(x,q)\wedge A^+(y,q)$.

Therefore, $A^+(xy,q)\geq A^+(x,q)\wedge A^+(y,q)$ for all x, y in R and q in Q. Hence A^+ is an (Q,L)-fuzzy subsemiring of a semiring R.

2.10 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring R, A^+ be a fuzzy set in R defined by $A^+(x,q)=A(x,q)+1-A(0,q)$, for all x in R and q in Q , where 0 is the identity element. Then there exists 0 in R such that A(0,q)=1 if and only if $A^+(x,q)=A(x,q)$.*

Proof: It is trivial.

2.11 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring R, A^+ be a fuzzy set in R defined by $A^+(x,q)=A(x,q)+1-A(0,q)$, for all x in R and q in Q , where 0 is the identity element. Then there exists x in R such that $A^+(x,q)=1$ if and only if x=0.*

Proof: It is trivial.

2.12 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring R, A^+ be a fuzzy set in R defined by $A^+(x,q)=A(x,q)+1-A(0,q)$, for all x in R and q in Q, where 0 is the identity element . Then $(A^+)^+=A^+$.*

Proof: Let x and y in R and q in Q. We have, $(A^+)^+(x,q)=A^+(x,q)+1-A^+(0,q) = \{A(x,q)+1-A(0,q)\}+1-\{A(0,q) +1-A(0,q)\}= A(x,q)+1-A(0,q)=A^+(x,q)$.

Hence $(A^+)^+=A^+$.

2.13 Theorem: *Let A and B be (Q,L)-fuzzy subsets of the sets R and H respectively, and let α in L. Then* $(A\times B)_\alpha =A_\alpha \times B_\alpha$.

Proof: Let α in L. Let (x, y) be in $(A\times B)_\alpha$ if and only if $A\times B((x,y),q)\geq\alpha$, if and only if $\{A(x,q)\wedge B(x,q)\}\geq \alpha$, if and only if $A(x,q)\geq\alpha$ and $B(x,q) \geq \alpha$, if and only if $x\epsilon A_\alpha$ and $y\epsilon B_\alpha$, if and only if $(x,y) \epsilon A_\alpha \times B_\alpha$. Therefore, $(A\times B)_\alpha =A_\alpha \times B_\alpha$.

In the following Theorem ◦ is the composition operation of functions:

2.14 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then A◦f is an (Q, L)-fuzzy subsemiring of R.*

Proof: Let x and y in R and A be an (Q, L)-fuzzy subsemiring of a semiring H and Q be a non-empty set. Then we have,

$(A\circ f)((x+y),q)=A(f(x+y),q)=A(f(x,q)+f(y,q))\geq A(f(x,q))\wedge A(f(y,q))\geq(A\circ f)(x,q)\wedge(A\circ f)(y,q)$, which implies that $(A\circ f)((x+y,q)) \geq (A\circ f)(x,q)\wedge(A\circ f)(y,q)$.

And $(A\circ f)(xy,q)=A(f(xy),q)=A(f(x,q)f(y,q))\geq A(f(x,q))\wedge A(f(y,q))\geq(A\circ f)(x,q)\wedge (A\circ f) (y,q)$, which implies that $(A\circ f)(xy,q)\geq(A\circ f)(x,q)\wedge (A\circ f)(y,q)$.

Therefore (A◦f) is an (Q, L)-fuzzy subsemiring of a semiring R.

2.15 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then A◦f is an (Q, L)-fuzzy subsemiring of R.*

Proof: Let x and y in R and A be an (Q, L)-fuzzy subsemiring of a semiring H and Q be a non-empty set. Then we have,

$(A\circ f)((x+y),q)=A(f(x+y),q)=A(f(y,q)+f(x,q))\geq A(f(x,q))\wedge A(f(y,q))\geq (A\circ f)(x,q)\wedge (A\circ f)(y,q)$, which implies that $(A\circ f)((x+y),q)\geq(A\circ f) (x,q)\wedge(A\circ f)(y,q)$.

And $(A\circ f)(xy,q)=A(f(xy,q))=A(f(y,q)f(x,q))\geq A(f(x,q))\wedge A(f(y,q))\geq(A\circ f)(x,q)\wedge(A\circ f) (y,q)$, which implies that $(A\circ f)(xy,q)\geq (A\circ f)(x,q) \wedge(A\circ f)(y,q)$. Therefore A◦f is an (Q, L)-fuzzy subsemiring of a semiring R.

2.16 Theorem: *Let A be an (Q, L)-fuzzy subsemiring of a semiring (R, +, .), then the pseudo (Q, L)-fuzzy coset* $(aA)^p$ *is an (Q, L)-fuzzy subsemiring of a semiring R, for a in R and p in P.*

Proof: Let A be an (Q, L)-fuzzy subsemiring of a semiring R. For every x and y in R and q in Q. we have,

$((aA)^p)(x+y,q)=p(a)A(x+y,q)\geq p(a)\{(A(x,q)\wedge A(y,q)\}=$
$\{p(a)A(x,q)\wedge p(a)A(y,q)\}=\{((aA)^p)(x,q)\wedge((aA)^p)(y,q)\}$.

Therefore, $((aA)^p)((x+y),q)\geq\{((aA)^p)(x,q)\wedge((aA)^p)(y,q)\}$. Now, $((aA)^p)(xy,q) = p(a)A(xy,q)\geq p(a)\{A(x,q)\wedge A(y,q)\}=\{p(a)A(x,q)\wedge p(a)A(y,q)\}=\{((aA)^p)(x,q)\wedge((aA)^p)(y,q)\}$.

Therefore, $((aA)^p)(xy,q)\geq\{((aA)^p)(x,q)\wedge((aA)^p)(y,q)\}$.

Hence $(aA)^p$ is an (Q, L)-fuzzy subsemiring of a semiring R.

2.17 Theorem: *Let (R, +, .) and (R$^{/}$, +, .) be any two semirings Q be a non-empty set. The homomorphic image of an (Q, L)-fuzzy subsemiring of R is an (Q, L)-fuzzy subsemiring of R$^{/}$.*

Proof: Let *(R,+,.)* and *(R$^{/}$,+,.)* be any two semirings. Let $f:R\to R^{/}$ be a homomorphism. Then, $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R. Let $V=f(A)$, where A is an (Q,L)-fuzzy subsemiring of R. We have to prove that V is an (Q,L)-fuzzy subsemiring of R^{l}. Now, for f(x), f(y) in R^{l}, $V(f(x)+f(y),q)=V(f(x+y),q)\geq A((x+y),q)\geq A(x,q)\wedge A(y,q)$ which implies that $V(f(x)+f(y),q)\geq V(f(x),q)\wedge V(f(y),q)$.

Again, $V(f(x)f(y),q)=V(f(xy),q)\geq A(xy,q)\geq A(x,q)\wedge A(y,q)$ which implies that $V(f(x)f(y),q)\geq V(f(x),q)\wedge V(f(y),q)$. Hence V is an (Q, L)-fuzzy subsemiring of R^{l}.

2.18 Theorem: *Let (R, +, .) and (R$^{/}$, +, .) be any two semirings Q be a non-empty set. The homomorphic preimage of an (Q, L)-fuzzy subsemiring of R$^{/}$ is an (Q, L)-fuzzy subsemiring of R.*

Proof: Let *(R,+,.)* and *(R$^{/}$,+, .)* be any two semirings. Let $f:R\to R^{/}$ be a homomorphism. Then, $f(x+y)= f(x) +f(y)$ and $f(xy)= f(x)f(y)$, for all x and y in R. Let $V=f(A)$, where V is an (Q,L)-fuzzy subsemiring of R^{l}. We have to prove that A is an (Q,L)- fuzzy subsemiring of R. Let x and y in R and q in Q. Then, $A(x+y,q)=V(f(x+y),q) = V(f(x)+f(y),q)\geq V(f(x),q)\wedge V(f(y),q)=A(x,q)\wedge A(y,q)$ which implies that $A(x+y,q)\geq A(x,q)\wedge A(y,q)$.

Again, $A(xy,q)=V(f(xy,q))=V(f(x)f(y),q)\geq V(f(x),q)\wedge V(f(y),q)=A(x,q)\wedge A(y,q)$ which implies that $A(xy,q)\geq A(x,q)\wedge A(y,q)$.

Hence A is an (Q, L)-fuzzy subsemiring of R.

2.19 Theorem: *Let (R, +, .) and ($R^{/}$, +, .) be any two semirings Q be a non-empty set. The anti-homomorphic image of an (Q, L)-fuzzy subsemiring of R is an (Q, L)-fuzzy subsemiring of $R^{/}$.*

Proof: Let *(R,+,.)* and *($R^{/}$,+,.)* be any two semirings. Let $f:R\to R^{/}$ be an anti-homomorphism. Then, *f(x+y)= f(y)+f(x)* and *f(xy)=f(y) f(x),* for all $x, y \in R$ and q in Q. Let *V=f(A),* where A is an (Q,L)-fuzzy subsemiring of R. We have to prove that V is an (Q,L)-fuzzy subsemiring of R^{l}. Now, for f(x), f(y) in R^{l}, $V(f(x)+f(y),q)=V(f(y+x),q)\geq A(y+x),q)\geq A(y,q)\wedge A(x,q)=A(x,q)\wedge A(y,q)$ which implies that $V(f(x)+f(y),q) \geq V(f(x),q) \wedge V(f(y),q)$.

Again, $V(f(x)f(y),q)=V(f(yx),q)\geq A(yx,q)\geq A(y,q)\wedge A(x,q)=A(x,q)\wedge A(y,q)$, which implies that $V(f(x)f(y),q)\geq V(f(x),q)\wedge V(f(y),q)$. Hence *V* is an (Q,L)-fuzzy subsemiring of R^{l}.

2.20 Theorem: *Let (R, +, .) and ($R^{/}$, +, .) be any two semirings Q be a non-empty set. The anti-homomorphic preimage of an (Q, L)-fuzzy subsemiring of $R^{/}$ is an (Q, L)-fuzzy subsemiring of R.*

Proof: Let *(R,+,.)* and *($R^{/}$,+,.)* be any two semirings. Let $f :R\to R^{/}$ be an anti-homomorphism. Then, *f(x+y)= f(y)+f(x)* and *f(xy)=f(y) f(x),* for all x and y in R and q in Q. Let V=f(A), where V is an (Q,L)-fuzzy subsemiring of R^{l}. We have to prove that A is an (Q,L)-fuzzy subsemiring of R. Let x and y in R and q in Q.

Then
$A(x+y,q)=V(f(x+y),q)=V(f(y)+f(x),q)\geq V(f(y),q)\wedge V(f(x),q)=V(f(x),q)\wedge V(f(y),q)= A(x,q) \wedge A(y,q)$, which implies that

$A(x+y,q)\geq A(x,q)\wedge A(y,q)$.

Again, $A(xy,q)=V(f(xy),q)=V(f(y)f(x),q)\geq V(f(y),q)\wedge V(f(x),q) =V(f(x),q)\wedge V(f(y),q)= A(x,q)\wedge A(y,q)$ which implies that $A(xy,q)\geq A(x,q)\wedge A(y,q)$.

Hence A is an (Q,L)-fuzzy subsemiring of R.

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