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Generalized Alpha Star Star Closed Sets in Bitopological Spaces

Qays Hatem Imran

Al-Muthanna University, College of Education
Department of Mathematics, Al-Muthanna, Iraq
E-mail: alrubaye84@yahoo.com

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Abstract

The aim of this paper is to introduce the concepts of generalized alpha star star closed sets, generalized alpha star star open sets and studies their basic properties in bitopological spaces.

Keywords: *Bitopological space,* $\tau_1\tau_2$ *-generalized alpha star closed sets,* $\tau_1\tau_2$ *-generalized alpha star star closed sets,* $\tau_1\tau_2$ *-generalized alpha star star open sets.*

1 Introduction

Levine, [6] initiated the study of generalized closed sets in topological spaces in 1970. In 1963, J.C. Kelly, [2] defined: a set equipped with two topologies is called a bitopological space, denoted by (X,τ_1,τ_2) where (X,τ_1) and (X,τ_2) are two topological spaces. In 1986, T. Fukutake, [7] generalized this notion to bitopological spaces and he defined a set *A* of a bitopological space *X* to be an *ij*-generalized closed set (briefly *ij*-*g*-closed) if $j\text{-}cl(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i-open in *X*, $i,j=1,2$ and $i\neq j$. Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by F. H. Khedr and H. S. Al-saadi, [1]. K. Chandrasekhara Rao and K. Kannan, [4, 5] introduced the concepts of semi star generalized closed sets in bitopological spaces.

The aim of this communication is to introduce the concepts of $\tau_1\tau_2$-generalized alpha star star closed sets, $\tau_1\tau_2$-generalized alpha star star open sets and study their basic properties in bitopological spaces.

2 Preliminaries

Throughout this paper, spaces always mean a bitopological spaces, for a subset A of X τ_i-$cl(A)$ (resp. τ_i-$\text{int}(A)$, τ_i-$\alpha cl(A)$) denote the closure (resp. interior, α-closure) of A with respect to the topology τ_i, for $i = 1,2$.

Definition 2.1: *A subset A of a bitopological space (X,τ_1,τ_2) is called*

(i) $\tau_1\tau_2$-α-*open [3] if* $A \subseteq \tau_1\text{-int}(\tau_2\text{-}cl(\tau_1\text{-int}(A)))$.

(ii) $\tau_1\tau_2$-α-*closed [3] if* $X - A$ *is* $\tau_1\tau_2$-α-*open. Equivalently, a subset A of a bitopological space* (X,τ_1,τ_2) *is called* $\tau_1\tau_2$-α-*closed if* $\tau_2\text{-}cl(\tau_1\text{-int}(\tau_2\text{-}cl(A))) \subseteq A$.

(iii) $\tau_1\tau_2$-*generalized closed (briefly* $\tau_1\tau_2$-g-*closed) [7] if* $\tau_2\text{-}cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_1-*open in X.*

(iv) $\tau_1\tau_2$-*generalized open (briefly* $\tau_1\tau_2$-g-*open) [7] if* $X - A$ *is* $\tau_1\tau_2$-g-*closed.*

(v) $\tau_1\tau_2$-*alpha generalized closed (briefly* $\tau_1\tau_2$-αg-*closed) [3] if* $\tau_2\text{-}\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_1-*open in* X.

(vi) $\tau_1\tau_2$-*alpha generalized open (briefly* $\tau_1\tau_2$-αg-*open) [3] if* $X - A$ *is* $\tau_1\tau_2$-αg-*closed.*

(vii) $\tau_1\tau_2$-*generalized alpha closed (briefly* $\tau_1\tau_2$-$g\alpha$-*closed) [3] if* $\tau_2\text{-}\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_1-α-*open in X.*

(viii) $\tau_1\tau_2$-*generalized alpha open (briefly* $\tau_1\tau_2$-$g\alpha$-*open) [3] if* $X - A$ *is* $\tau_1\tau_2$-$g\alpha$-*closed.*

Definition 2.2: *A subset A of a bitopological space* (X,τ_1,τ_2) *is called* $\tau_1\tau_2$-*generalized alpha star closed (briefly* $\tau_1\tau_2$-$g\alpha^*$-*closed) if* $\tau_2\text{-}cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* τ_1-α-*open in X. The family of all* $\tau_1\tau_2$-$g\alpha^*$-*closed sets of X is denoted by* $\tau_1\tau_2\text{-}g\alpha^*C(X)$.

Example 2.3: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{b\},\{a,b\}\}$. Then $\{a,b\}$ is $\tau_1\tau_2$ - $g\alpha^*$ - closed and $\{a\}$ is not $\tau_1\tau_2$ - $g\alpha^*$ - closed.

Definition 2.4: *A subset A of a bitopological space* (X,τ_1,τ_2) *is called* $\tau_1\tau_2$ *- generalized alpha star open (briefly* $\tau_1\tau_2$ - $g\alpha^*$ *- open) if and only if* $X-A$ *is* $\tau_1\tau_2$ - $g\alpha^*$ *- closed. The family of all* $\tau_1\tau_2$ - $g\alpha^*$ *- open sets of X is denoted by* $\tau_1\tau_2$ - $g\alpha^* O(X)$.

3 Generalized Alpha Star Star Closed Sets

In this section we define and study the concept of $\tau_1\tau_2$ - generalized alpha star star closed sets in bitopological spaces.

Definition 3.1: *A subset A of a bitopological space* (X,τ_1,τ_2) *is called* $\tau_1\tau_2$ *- generalized alpha star star closed (briefly* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed) if* τ_2 - $cl(A)\subseteq U$ *whenever* $A\subseteq U$ *and U is* τ_1 - $g\alpha^*$ *- open in X. The family of all* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed sets of X is denoted by* $\tau_1\tau_2$ - $g\alpha^{**}C(X)$.

Example 3.2: Let $X=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a,b\}\}$, $\tau_2=\{\phi,X,\{b\},\{c\},\{b,c\}\}$. Then ϕ, $X,\{c\},\{d\}$, $\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},\{b,c,d\}$ are $\tau_1\tau_2$ - $g\alpha^{**}$ - closed sets.

Now, the characterization of $\tau_1\tau_2$ - $g\alpha^{**}$ - closed sets by using different types of generalization of closed sets and τ_1 - $g\alpha^*$ - open sets are established in the following theorem.

Theorem 3.3: *Let* (X,τ_1,τ_2) *be a bitopological space and* $A\subseteq X$ *. Then the following are true.*

(i) *If A is* τ_2 *- closed, then A is* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed.*
(ii) *If A is* τ_1 - $g\alpha^*$ *- open and* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed, then A is* τ_2 *- closed.*
(iii) *If A is* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed, then A is* $\tau_1\tau_2$ - g *- closed.*
(iv) *If A is* $\tau_1\tau_2$ - $g\alpha^{**}$ *- closed, then A is* $\tau_1\tau_2$ - αg *- closed.*

Proof:

(i) It is obvious that every τ_2 - closed set is $\tau_1\tau_2$ - $g\alpha^{**}$ - closed.

(ii) Suppose that A is τ_1 - $g\alpha^*$ - open and $\tau_1\tau_2$ - $g\alpha^{**}$ - closed. Then $A\subseteq A$ implies that τ_2 - $cl(A)\subseteq A$. Obviously, $A\subseteq\tau_2$ - $cl(A)$. Therefore, A is τ_2 - closed.

(iii) Suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. Let $A \subseteq U$ and U is τ_1 - open in X. Since every τ_1 - open set is τ_1 - $g\alpha^*$ - open in X, we have U is τ_1 - $g\alpha^*$ - open in X. Then, τ_2 - $cl(A) \subseteq U$ since A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. Consequently, A is $\tau_1\tau_2$ - g -closed.

(iv) Suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. Let $A \subseteq U$ and U is τ_1 - open in X. Since every τ_1 - open set is τ_1 - $g\alpha^*$ - open in X, we have U is τ_1 - $g\alpha^*$ - open in X. Then, τ_2 - $cl(A) \subseteq U$ since A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. Since τ_2 - $\alpha cl(A) \subseteq \tau_2$ - $cl(A)$, we have τ_2 - $\alpha cl(A) \subseteq U$. Consequently, A is $\tau_1\tau_2$ - αg - closed.

In the following examples it is proved that the converses of the assertions of the above theorem are not true in general.

Example 3.4: In example (3.2), $\{c\}$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed but not τ_2 -closed. Also $\{a, d\}$ is τ_2 -closed, $\tau_1\tau_2$ - $g\alpha^{**}$ -closed but not τ_1 - $g\alpha^*$ - open.

Example 3.5: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, X, \{b,c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$. Then $\{b\}$ is $\tau_1\tau_2$ - g -closed but not $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X.

Example 3.6: In example (3.2), $\{a\}$ is $\tau_1\tau_2$ - αg - closed but not $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X.

Remark 3.7: $\tau_1\tau_2$ - $g\alpha$ - closed sets and $\tau_1\tau_2$ - $g\alpha^{**}$ -closed sets are independent in general. The following example supports our claim. In Example (3.2), $\{a\}$ is $\tau_1\tau_2$ - $g\alpha$ - closed but not $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X. Also $\{a,b,c\}$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed but not $\tau_1\tau_2$ - $g\alpha$ - closed in X.

Theorem 3.8: *If A is* $\tau_1\tau_2$ - $g\alpha^{**}$ *-closed,* τ_1 - $g\alpha^*$ *- open in X and F is* τ_2 *-closed in X then* $A \cap F$ *is* τ_2 *-closed in X.*

Proof: Since A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed, τ_1 - $g\alpha^*$ - open in X, we have A is τ_2 -closed in X [by theorem (3.3) (ii)]. Since F is τ_2 -closed in X, $A \cap F$ is τ_2 -closed in X.

Remark 3.9:

(i) $\tau_1\tau_2$ - $g\alpha^*$ - closed and $\tau_1\tau_2$ - α - closed sets are independent in general.
(ii) $\tau_1\tau_2$ - $g\alpha^{**}$ -closed and $\tau_1\tau_2$ - $g\alpha^*$ - closed sets are independent in general.

Example 3.10: In example (3.2), $\{c\}$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed but not $\tau_1\tau_2$ - $g\alpha^*$ - closed in X.

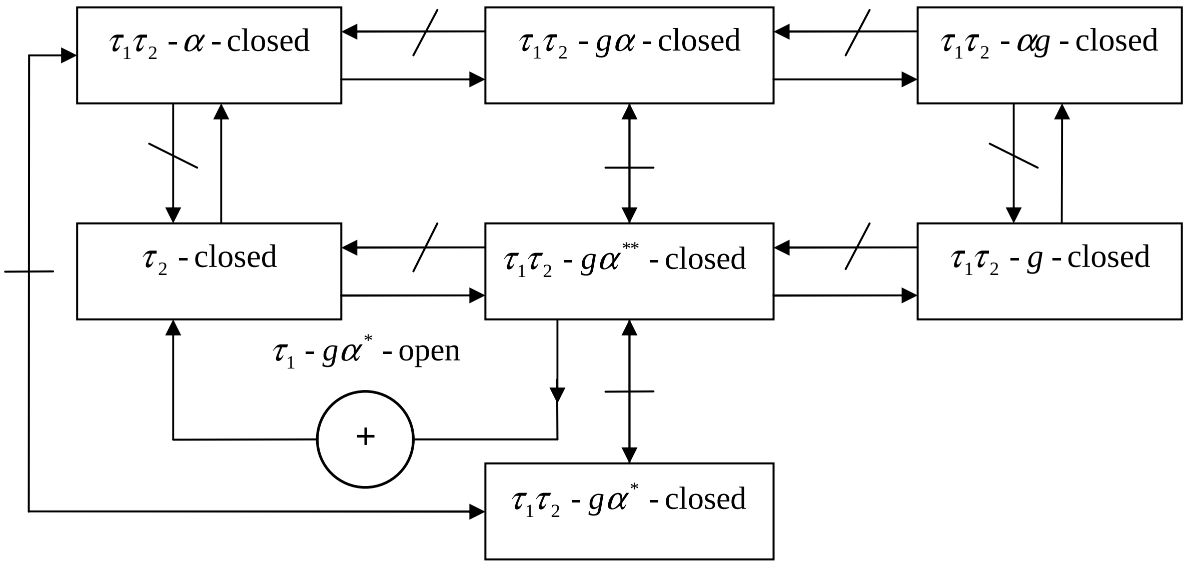
Theorem 3.11: *If A is $\tau_1\tau_2$-$g\alpha^{**}$-closed in X and $A \subseteq B \subseteq \tau_2$-$cl(A)$, then B is $\tau_1\tau_2$-$g\alpha^{**}$-closed.*

Proof: Suppose that A is $\tau_1\tau_2$-$g\alpha^{**}$-closed in X and $A \subseteq B \subseteq \tau_2$-$cl(A)$. Let $B \subseteq U$ and U is τ_1-$g\alpha^*$-open in X. Since $A \subseteq B$ and $B \subseteq U$, we have $A \subseteq U$. Hence τ_2-$cl(A) \subseteq U$ (Since A is $\tau_1\tau_2$-$g\alpha^{**}$-closed). Since $B \subseteq \tau_2$-$cl(A)$, we have τ_2-$cl(B) \subseteq \tau_2$-$cl(A) \subseteq U$. Therefore, B is $\tau_1\tau_2$-$g\alpha^{**}$-closed.

Theorem 3.12: *If A and B are $\tau_1\tau_2$-$g\alpha^{**}$-closed sets then so is $A \cup B$.*

Proof: Suppose that A and B are $\tau_1\tau_2$-$g\alpha^{**}$-closed sets. Let U be τ_1-$g\alpha^*$-open in X and $A \cup B \subseteq U$. Since $A \cup B \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Since U is τ_1-$g\alpha^*$-open in X and A and B are $\tau_1\tau_2$-$g\alpha^{**}$-closed sets, we have τ_2-$cl(A) \subseteq U$ and τ_2-$cl(B) \subseteq U$. Therefore, τ_2-$cl(A \cup B) \subseteq \tau_2$-$cl(A) \cup \tau_2$-$cl(B) \subseteq U$. Hence $A \cup B$ is $\tau_1\tau_2$-$g\alpha^{**}$-closed.

Remark 3.13: The following diagram shows the relations among the different types of weakly closed sets that were studied in this section:



Theorem 3.14: *The arbitrary union of $\tau_1\tau_2$-$g\alpha^{**}$-closed sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-$g\alpha^{**}$-closed if the family $\{A_i, i \in I\}$ is locally finite in (X, τ_1).*

Proof: Let $\{A_i, i \in I\}$ be locally finite in X and A_i is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is τ_1 - $g\alpha^*$ - open in X. Then $A_i \subseteq U$ and U is τ_1 - $g\alpha^*$ - open in X for each i. Since A_i is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X for each $i \in I$, we have τ_2 - $cl(A_i) \subseteq U$. Consequently, $\bigcup[\tau_2$ - $cl(A_i)] \subseteq U$. Since the family $\{A_i, i \in I\}$ is locally finite in X, τ_2 - $cl[\bigcup(A_i)] = \bigcup[\tau_2$ - $cl(A_i)] \subseteq U$. Therefore, $\bigcup A_i$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X.

Remark 3.15: The intersection of any two $\tau_1\tau_2$ - $g\alpha^{**}$ -closed sets is not necessary $\tau_1\tau_2$ - $g\alpha^{**}$ -closed set as in the following example.

Example 3.16: In example (3.2), $A = \{a, c\}$, $B = \{a, d\}$ are $\tau_1\tau_2$ - $g\alpha^{**}$ -closed but $A \cap B = \{a\}$ is not $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X.

Theorem 3.17: *If a set A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X , then τ_2 - $cl(A) - A$ contains no nonempty τ_1 - $g\alpha^*$ - closed set.*

Proof: Suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X. Let F be τ_1 - $g\alpha^*$ - closed and $F \subseteq \tau_2$ - $cl(A) - A$. Since F is τ_1 - $g\alpha^*$ - closed, we have F^c is τ_1 - $g\alpha^*$ - open. Since $F \subseteq \tau_2$ - $cl(A) - A$, we have $F \subseteq \tau_2$ - $cl(A)$ and $F \subseteq A^c$. Hence $A \subseteq F^c$.
Consequently τ_2 - $cl(A) \subseteq F^c$ {Since A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in X}. Therefore, $F \subseteq [\tau_2$ - $cl(A)]^c$. Hence $F \subseteq [\tau_2$ - $cl(A)]^c \cap \tau_2$ - $cl(A) = \phi$. Hence τ_2 - $cl(A) - A$ contains no nonempty τ_1 - $g\alpha^*$ - closed set.

Corollary 3.18: *Let A be $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. Then A is τ_2 -closed if and only if τ_2 - $cl(A) - A$ is τ_1 - $g\alpha^*$ - closed.*

Proof: Suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed and τ_2 -closed. Since A is τ_2 -closed, we have τ_2 - $cl(A) = A$. Therefore, τ_2 - $cl(A) - A = \phi$ which is τ_1 - $g\alpha^*$ - closed.
Conversely, suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed and τ_2 - $cl(A) - A$ is τ_1 - $g\alpha^*$ - closed. Since A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed, we have τ_2 - $cl(A) - A$ contains no nonempty τ_1 - $g\alpha^*$ -closed set {by Theorem 3.17}. Since τ_2 - $cl(A) - A$ is itself τ_1 - $g\alpha^*$ - closed, we have τ_2 - $cl(A) - A = \phi$. Therefore, τ_2 - $cl(A) = A$ implies that A is τ_2 -closed.

Theorem 3.19: *If A is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed and $A \subseteq B \subseteq \tau_2$ - $cl(A)$ then τ_2 - $cl(B) - B$ contains no nonempty τ_1 - $g\alpha^*$ - closed set.*

Proof: Let A be $\tau_1\tau_2$ - $g\alpha^{**}$ -closed and $A \subseteq B \subseteq \tau_2$ - $cl(A)$. Then B is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed {by theorem (3.11)}. Therefore, τ_2 - $cl(B) - B$ contains no nonempty τ_1 - $g\alpha^*$ - closed set {by theorem (3.17)}.

Theorem 3.20: *For each $x \in X$, the singleton {x} is either τ_1 - $g\alpha^*$ - closed or its complement $\{x\}^c$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in (X, τ_1, τ_2).*

Proof: Let $x \in X$. Suppose that $\{x\}$ is not τ_1 - $g\alpha^*$ - closed. Then $\{x\}^c$ is not τ_1 - $g\alpha^*$ - open. Consequently, X itself is the only τ_1 - $g\alpha^*$ - open set containing $X - \{x\}$. Therefore, τ_2 - $cl(X - \{x\}) \subseteq X$ which implies that $X - \{x\}$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed in (X, τ_1, τ_2).

4 Generalized Alpha Star Star Open Sets

We begin this section with a relatively new definition.

Definition 4.1: *A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - generalized alpha star star open (briefly $\tau_1\tau_2$ - $g\alpha^{**}$ - open) if and only if $X - A$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -closed. The family of all $\tau_1\tau_2$ - $g\alpha^{**}$ - open sets of X is denoted by $\tau_1\tau_2$ - $g\alpha^{**}O(X)$.*

Example 4.2: In example (3.2), ϕ, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{a, b, d\}$ are $\tau_1\tau_2$ - $g\alpha^{**}$ - open sets in X.

The following theorem will give an equivalent definition of $\tau_1\tau_2$ - $g\alpha^{**}$ - open sets.

Theorem 4.3: *A set A is $\tau_1\tau_2$ - $g\alpha^{**}$ - open if and only if $F \subseteq \tau_2$ - $\text{int}(A)$ whenever F is τ_1 - $g\alpha^*$ - closed and $F \subseteq A$.*

Proof: Suppose that A is $\tau_1\tau_2$ - $g\alpha^{**}$ - open. Then A^c is $\tau_1\tau_2$ - $g\alpha^{**}$ - closed. Suppose that F is τ_1 - $g\alpha^*$ - closed and $F \subseteq A$. Then F^c is τ_1 - $g\alpha^*$ - open and $A^c \subseteq F^c$.

Therefore, τ_2 - $cl(A^c) \subseteq F^c$ (since A^c is $\tau_1\tau_2$ - $g\alpha^{**}$ - closed). Since τ_2 - $cl(A^c) = [\tau_2$ - $\text{int}(A)]^c$, we have $[\tau_2$ - $\text{int}(A)]^c \subseteq F^c$. Hence $F \subseteq \tau_2$ - $\text{int}(A)$.

Conversely, suppose that $F \subseteq \tau_2$ - $\text{int}(A)$ whenever F is τ_1 - $g\alpha^*$ - closed and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is τ_1 - $g\alpha^*$ - open. Take $U = F^c$.
Since $F \subseteq \tau_2$ - $\text{int}(A)$, we have $[\tau_2$ - $\text{int}(A)]^c \subseteq F^c = U$. Since τ_2 - $cl(A^c) = [\tau_2$ - $\text{int}(A)]^c$, we have τ_2 - $cl(A^c) \subseteq U$. Therefore, A^c is $\tau_1\tau_2$ - $g\alpha^{**}$ - closed.

Thus, A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open.

Remark 4.4: Every τ_1-open set is $\tau_1\tau_2\text{-}g\alpha^{**}$-open but the converse is not true in general as can be seen from the following example.

Example 4.5: In example (3.2), $\{a, c\}$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X but not τ_1-open in X.

Remark 4.6: $\tau_1\tau_2\text{-}g\alpha^{**}$-open and $\tau_1\tau_2\text{-}g\alpha^{*}$-open sets are in general, independent as can be seen from the following two examples.

Example 4.7: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{a\},\{b,c\}\}$, $\tau_2=\{\phi,X,\{b\},\{a,c\}\}$. Then $\{c\}$ is $\tau_1\tau_2\text{-}g\alpha^{*}$-open in X but not $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X.

Example 4.8: In example (3.2), $\{d\}$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X but not $\tau_1\tau_2\text{-}g\alpha^{*}$-open in X.

Remark 4.9: The union of any two $\tau_1\tau_2\text{-}g\alpha^{**}$-open sets is not necessary $\tau_1\tau_2\text{-}g\alpha^{**}$-open set as in the following example.

Example 4.10: In example (3.2), $A=\{b,c\}$, $B=\{b,d\}$ are $\tau_1\tau_2\text{-}g\alpha^{**}$-open sets but $A\bigcup B=\{b,c,d\}$ is not $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X.

Theorem 4.11: *If A and B are $\tau_1\tau_2\text{-}g\alpha^{**}$-open sets then so is $A\bigcap B$.*

Proof: Suppose that A and B are $\tau_1\tau_2\text{-}g\alpha^{**}$-open sets. Let F be $\tau_1\text{-}g\alpha^{*}$-closed in X and $F\subseteq A\bigcap B$. Since $F\subseteq A\bigcap B$, we have $F\subseteq A$ and $F\subseteq B$. Since A and B are $\tau_1\tau_2\text{-}g\alpha^{**}$- open sets. Then $F\subseteq\tau_2\text{-int}(A)$ and $F\subseteq\tau_2\text{-int}(B)$. Therefore, $F\subseteq\tau_2\text{-int}(A)\bigcap\tau_2\text{-int}(B)\subseteq\tau_2\text{-int}(A\bigcap B)$. Hence $A\bigcap B$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-open.

Theorem 4.12: *The arbitrary intersection of $\tau_1\tau_2\text{-}g\alpha^{**}$-open sets $A_i, i\in I$ in a bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2\text{-}g\alpha^{**}$-open if the family $\{A_i^c, i\in I\}$ is locally finite in (X,τ_1).*

Proof: Let $\{A_i^c, i\in I\}$ be locally finite in (X,τ_1) and A_i is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X for each $i\in I$. Then A_i^c is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed in X for each $i\in I$. Then by theorem (3.14), we have $\bigcup(A_i^c)$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed in X. Consequently, let $(\bigcap A_i)^c$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed in X. Therefore, $\bigcap A_i$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X.

Theorem 4.13: *If A is* $\tau_1\tau_2$ *-* $g\alpha^{**}$ *-open in X and* $\tau_2\text{-}\mathrm{int}(A) \subseteq B \subseteq A$*, then B is* $\tau_1\tau_2$ *-* $g\alpha^{**}$ *-open.*

Proof: Suppose that A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X and $\tau_2\text{-}\mathrm{int}(A) \subseteq B \subseteq A$. Let F is $\tau_1\text{-}g\alpha^{*}$-closed and $F \subseteq B$. Since $F \subseteq B$ and $B \subseteq A$, we have $F \subseteq A$. Since A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open, we have $F \subseteq \tau_2\text{-}\mathrm{int}(A)$. Since $\tau_2\text{-}\mathrm{int}(A) \subseteq B$, we have $F \subseteq \tau_2\text{-}\mathrm{int}(A) \subseteq \tau_2\text{-}\mathrm{int}(B)$. Hence B is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in X.

Theorem 4.14: *If a set A is* $\tau_1\tau_2\text{-}g\alpha^{**}$*-closed in X, then* $\tau_2\text{-}cl(A) - A$ *is* $\tau_1\tau_2\text{-}g\alpha^{**}$*-open set.*

Proof: Suppose that A is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed in X. Let F be $\tau_1\text{-}g\alpha^{*}$-closed and $F \subseteq \tau_2\text{-}cl(A) - A$. Since A is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed in X, we have $\tau_2\text{-}cl(A) - A$ contains no nonempty $\tau_1\text{-}g\alpha^{*}$-closed set. Since $F \subseteq \tau_2\text{-}cl(A) - A$, we have $F = \phi \subseteq \tau_2\text{-}\mathrm{int}[\tau_2\text{-}cl(A) - A]$. Therefore, $\tau_2\text{-}cl(A) - A$ is $\tau_1\tau_2\text{-}g\alpha^{**}$-open.

Theorem 4.15: *If a set A is* $\tau_1\tau_2\text{-}g\alpha^{**}$*-open in a bitopological space* (X, τ_1, τ_2)*, then G = X whenever G is* $\tau_1\text{-}g\alpha^{*}$*-open and* $\tau_2\text{-}\mathrm{int}(A) \cup A^c \subseteq G$.

Proof: Suppose that A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open in a bitopological space (X, τ_1, τ_2) and G is $\tau_1\text{-}g\alpha^{*}$-open and $\tau_2\text{-}\mathrm{int}(A) \cup A^c \subseteq G$. Then $G^c \subseteq [\tau_2\text{-}\mathrm{int}(A) \cup A^c]^c =$ $\tau_2\text{-}cl(A^c) - A^c$. Since G is $\tau_1\text{-}g\alpha^{*}$-open, we have G^c is $\tau_1\text{-}g\alpha^{*}$-closed. Since A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open, we have A^c is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed. Therefore, $\tau_2\text{-}cl(A^c) - A^c$ contains no nonempty $\tau_1\text{-}g\alpha^{*}$-closed set in X {by theorem (3.17)}. Consequently, $G^c = \phi$. Hence $G = X$.

Remark 4.16: The converse of the above theorem is not true in general as can seen from the following example.

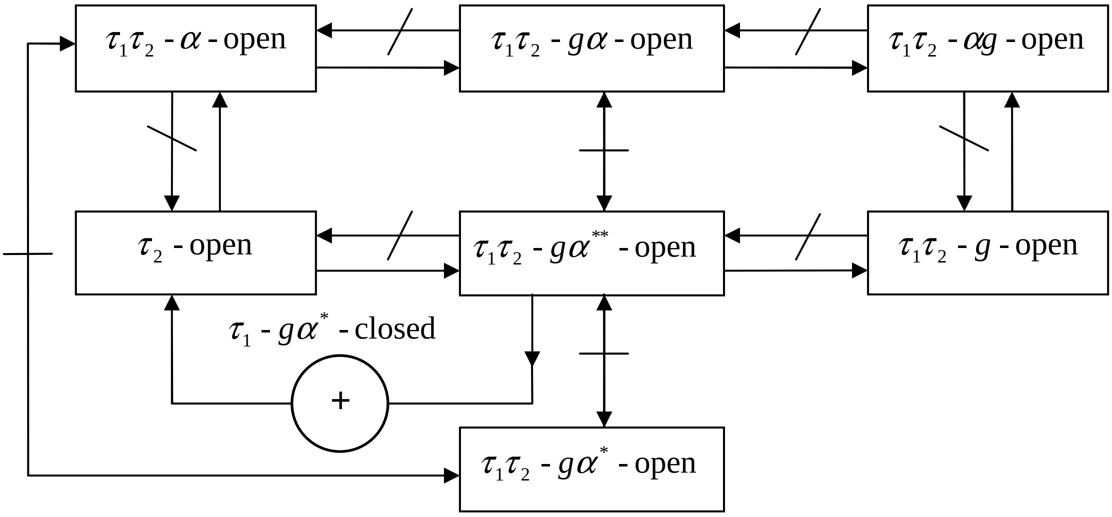
Example 4.17: In example (3.2), if we take $A = \{c,d\}$, then $\tau_2\text{-}\mathrm{int}(A) \cup A^c \subseteq X$, X is $\tau_1\text{-}g\alpha^{*}$-open, but A is not $\tau_1\tau_2\text{-}g\alpha^{**}$-open.

Lemma 4.18: *The intersection of* $\tau_1\tau_2\text{-}g\alpha^{**}$*-open set and* τ_2*-open set is always* $\tau_1\tau_2\text{-}g\alpha^{**}$*-open.*

Proof: Suppose that A is $\tau_1\tau_2\text{-}g\alpha^{**}$-open and B is τ_2-open. Since B is τ_2-open, we have B^c is τ_2-closed. Then B^c is $\tau_1\tau_2\text{-}g\alpha^{**}$-closed {by theorem

(3.3) (i)}. Hence, B is $\tau_1\tau_2$ - $g\alpha^{**}$ -open. Hence $A \cap B$ is $\tau_1\tau_2$ - $g\alpha^{**}$ -open {by theorem (4.11)}.

Remark 4.19: The following diagram shows the relations among the different types of weakly open sets that were studied in this section:



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