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On Ideal Convergent Sequences in p-Adic Linear 2-Normed Spaces

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Abstract

In this paper, we introduce $I-$limit operation for sequences in p-adic linear 2-normed space ($X, N(\bullet,\bullet)_p$) is linear with respect to summation and scalar multiplication and we investigate the relation between $I-$cluster points and ordinary limit points of p-adic linear 2-normed spaces.

Keywords: *2-normed spaces, p-adic 2-norm, p-adic linear 2-normed space, $I-$cluster points.*

1 Introduction

Kummer, in 1850, first introduced to *p*-adic numbers. Then the German Mathematician, Kurt Hensel (1861-1941) developed the *p* -adic numbers in a paper which was concerned with the development of algebraic numbers in power series, around the end of the nineteenth century, in 1897. Then *p*-adic numbers were generalized to ordinals (or valuation) by Kürschak in 1913, and Minkowski (1884), Tate (1960), Kubota-Leopoldt (1964), Iwasawa, Serre, Mazur, Manin, Katz, and the others. There are numbers of all kinds such as rational, real, complex, *p*-adic numbers. Hensel's *p*-adic's numbers are now widely used in many fields such as analysis, physics and computer science. The *p*-adic numbers are less well known than the others, but they play a fundamental role in number theory in other parts of mathematics. Although, they have penatrated several mathematical fields, among them, number theory, algebraic geometry, algebraic topology and analysis. These numbers are now well-established in mathematical world and used more and more by physicists as well. Over the last century *p*-adic numbers and *p*-adic analysis have come to play an important role in number theory. They have many applications in mathematics, for example: Representation theory, algebraic geometry, and modern number theory and many applications in mathematical physics since 1897, for example; String theory, QFT, quantum mechanics, dynamical systems, complex systems, etc. Recently, Branko Dragovich in his study ([4]) he constructed *p*-adic approach to the genetic code and the genome and gave a new approach between *p*-adic fields and biology with chemistry, especially organic chemistry. The other researchers gave the different approach with *p*-adic on various disciplines of mathematics and its allied subjects (see ([1], [2], [3], [9], [10], [11], [12], [15], [21], [24], [25]) for more details).

The concept of linear 2-normed spaces has been investigated by Gähler in 1965 ([5]) and has been developed extensively in different subjects by others. Lewandowska published a series of papers on 2-normed sets and generalized 2-normed spaces, convergent sequences, 2-Banach spaces,...etc. (see ([8], [16], [17], [18], [20], for more details) . The notion of ideal convergence was introduced first by P.Kostyrko et al [14] as an interesting generalization of statistical convergence. The concept of an $I-$cluster point and $I-$limit point of a sequence in metric space was introduced and some results for the set of I-cluster points and I-limit points obtained in [13]. A.Sahiner et al [22] introduced $I-$cluster points convergence sequences in 2-normed linear spaces and and Gurdal [7] investigated the relation between $I-$cluster points and ordinary limit points of sequences in 2-normed spaces.

Mehmet Acikgoz ([17]) introduced a very understandable and readable connection between the concepts in *p*-adic numbers, *p*-adic analysis and linear 2-normed spaces. Recently B.Surender Reddy [23] introduced some properties of *p*-adic linear 2-normed spaces and obtain necessary and sufficient conditions for *p*-adic 2-norms to be equivalent on *p*-adic linear 2-normed spaces.

The main aim of this paper is to we introduce $I-$cluster points limit operation for sequences in *p*-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is linear with respect to summation and scalar multiplication and we investigate the relation between $I-$cluster points and ordinary limit points of p-adic linear 2-normed spaces.

2 Preliminaries

In this paper, we will use the notations; p for a prime number, Z - the ring of rational integers, Z^+ - the positive integers, Q - the field of rational numbers, R - the field of real numbers, R^+ - the positive real numbers, Z_p - the ring of *p*-adic rational integers, Q_p - the field of *p*-adic rational numbers, C - the field of complex numbers and C_p - the *p*-adic completion of the algebraic closure of Q_p.

Definition 2.1:

(i) *The p-adic ordinal (valuation) of x and y, for* $0 \neq x, y \in Z$ *is*

$$ord_p(x, y) = \max\{r : p^r / x \ and \ p^r / y\} \geq 0.$$

(ii) *For* $\frac{a}{b}, \frac{c}{d} \in Q$, *The p-adic value of* $\frac{a}{b}$ *and* $\frac{c}{d}$ *is*

$$ord_p(\frac{a}{b}, \frac{c}{d}) = ord_p(a, c) - ord_p(a, d) - ord_p(b, c) + ord_p(b, d)$$

(iii) *For* $\frac{a}{b}$, $c \in Q$, *with* $d = 1$, *The p-adic value of* $\frac{a}{b}$ *and* c *is*

$$ord_p(\frac{a}{b}, c) = ord_p(a, c) - ord_p(b, c).$$

Notice that in all cases, ord_p, in 2-norm, gives an integer and that for rational number $\frac{a}{b}$ and $\frac{c}{d}$ the value of $ord_p(\frac{a}{b}, \frac{c}{d})$ is well defined. i.e., if $\frac{a}{b} = \frac{a'}{b'}$ and $\frac{c}{d} = \frac{c'}{d'}$ then $ord_p(\frac{a}{b}, \frac{c}{d}) = ord_p(\frac{a'}{b'}, \frac{c'}{d'})$.

We also introduce the convention that $ord_p(0, y) = ord_p(x, 0) = \infty$.

The *p*-adic valuation has the following properties:

Proposition 2.2: *For all* $x, y \in Q$, *we have for* ord_p;

(i) $ord_p(x, y) = \infty$ *iff* $x = 0$ *or* $y = 0$,

(ii) $ord_p(xz, y) = ord_p(x, y) + ord_p(z, y)$,

(iii) $ord_p(x + z, y) \geq \min\{ord_p(x, y), ord_p(z, y)\}$ *and with equality when* $ord_p(x, y) \neq ord_p(z, y)$.

Definition 2.3: *Let X be a linear space of dimension greater than 1 over K, where K is the real or complex numbers field. Suppose* $N(\bullet,\bullet)$ *be a non-negative real valued function on* $X \times X$ *satisfying the following conditions:*

$(2-N_1)$: $N(x,y) > 0$ and $N(x,y) = 0$ if and only if x and y are linearly dependent vectors,
$(2-N_2)$: $N(x,y) = N(y,x)$ for all $x, y \in X$,
$(2-N_3)$: $N(\lambda x, y) = |\lambda| N(x,y)$ for all $\lambda \in K$ and $x, y \in X$,
$(2-N_4)$: $N(x+y,z) \le N(x,z) + N(y,z)$ for all $x, y, z \in X$.

Then $N(\bullet,\bullet)$ is called a 2-norm on X and the pair $(X, N(\bullet,\bullet))$ is called a linear 2-normed space.

Definition 2.4: *For all* $x, y \in Q$ *, let the p-adic norm of x , y be given by*

$$N(x,y)_p = p^{-ord_p(x,y)} \text{ , for } x, y \neq 0$$
$$= p^{-\infty} = 0 \text{ , for } x = 0 \text{ or } y = 0$$

Where $ord_p(x,y) = \max\{r : p^r / x \text{ and } p^r / y\}$.

Proposition 2.5: *Let the function* $N(\bullet,\bullet)_p$ *be a non-negative real valued function on* $Q \times Q$ *satisfying the following conditions:*

$$N(\bullet,\bullet)_p : Q \times Q \to R^+ \cup \{0\} = \{r : r \ge 0\}$$

(i) $N(x,z)_p = 0$ if and only if $x = 0$ or $z = 0$,

(ii) $N(xy,z)_p = N(x,z)_p . N(y,z)_p$ for all x , y and $z \in Q$,

(iii) $N(x+y,z)_p \le \max\{N(x,z)_p, N(y,z)_p\}$ and with equality when $N(x,z)_p \neq N(y,z)_p$, where $N(\bullet,\bullet)_p$ is a non-Archimedean norm on Q.

Let $N(x,z)_p$ be a non-negative real valued function defined on the rational numbers $Q \times Q$ such that $N(x,z)_p = 0$ for $x = 0$ or $z = 0$, $N(x,z)_p > 0$ when $x \neq 0, z \neq 0$. $N(xy,z)_p = N(x,z)_p . N(y,z)_p$ for all $x, y, z \in Q$ and

(2.6) $$N(x+y,z)_p \le K\left(N(x,z)_p + N(y,z)_p\right)$$

for some $K \ge 1$ and all $x, y, z \in Q$. For the usual triangle inequality one ask that this condition holds with $K = 1$, i.e.,

(2.7) $$N(x+y,z)_p \le N(x,z)_p + N(y,z)_p$$

for all $x, y, z \in Q$.

The ultrametric version of the triangle inequality is stronger still and asks that

(2.8) $$N(x+y,z)_p \leq \max(N(x,z)_p, N(y,z)_p)$$

for all $x, y, z \in Q$. If $N(\bullet,\bullet)_p$ satisfies (2.6), n is a positive integer and $x_1, x_2, x_3, ..., x_{2^n} \neq 0, z \in Q$, then

(2.9) $$N(\sum_{k=1}^{2^n} x_k, z)_p \leq K^n \sum_{k=1}^{2^n} N(x_k, z)_p$$

as one can check using induction on n. For all $a > 0$, $N(x,z)_p^a$ is a non-negative real valued function on $Q \times Q$ which vanished at 0, is positive at all nonzero $x \in Q$ and sends products to products. If $N(x,z)_p$ satisfies (2.6), then

(2.10) $$N(x+y,z)_p^a \leq K^a \left(N(x,z)_p^a + N(y,z)_p^a \right)$$

when $0 < a \leq 1$ and

(2.11) $$N(x+y,z)_p^a \leq 2^{a-1} K^a \left(N(x,z)_p^a + N(y,z)_p^a \right)$$

when $a \geq 1$.

In particular, if $N(x,z)_p$ satisfies the well-known triangle inequality (2.7) and $0 < a \leq 1$, then $N(x,z)_p^a$ also satisfies the the well-known triangle inequality.

If $N(x,z)_p$ satisfies the ultrametric version (2.8) of the triangle inequality, then $N(x,z)_p^a$ satisfies the ultrametric version of the triangle inequality for all $a \geq 0$.

3 p-Adic 2-Metric Space:

Suppose a mapping $d_p : X \times X \times X \to R$ on a non-empty set X satisfying the following conditions, for all $x, y, z \in X$

D_1) For any two different elements x and y in X there is an element z in X such that $d_p(x, y, z) \neq 0$

D_2) $d_p(x, y, z) = 0$ when two of three elements are equal

D_3) $d_p(x, y, z) = d_p(x, z, y) = d_p(y, z, x)$

D_4) $d_p(x, y, z) \leq d_p(x, y, w) + d_p(x, w, z) + d_p(w, y, z)$ for any w in X. Then d_p is called p-adic 2-metric on X and the pair (X, d_p) is called p-adic 2-metric space. If p-adic 2-metric also satisfies the condition $d_p(x, y, z) \leq \max\{d_p(x, y, w), d_p(x, w, z), d_p(y, w, z)\}$ for $x, y, z, w \in X$, then d_p is called a p-adic ultra 2-metric and the pair (X, d_p) is called a p-adic ultra 2-metric space.

Definition 3.1: *Let X be a linear space of dimension greater than 1 over K, where K is the real or complex numbers field. Suppose* $N(\bullet,\bullet)_p$ *be a non-negative real valued function on* $X \times X$ *satisfying the following conditions:*

$(2-pN_1)$: $N(x,z)_p = 0$ if and only if x and z are linearly dependent vectors.
$(2-pN_2)$: $N(xy,z)_p = N(x,z)_p . N(y,z)_p$ for all $x,y,z \in X$,
$(2-pN_3)$: $N(x+y,z)_p \le N(x,z)_p + N(y,z)_p$ for all $x,y,z \in X$,
$(2-pN_4)$: $N(\lambda x,z)_p = |\lambda|\ N(x,z)_p$ for all $\lambda \in K$ and $x,z \in X$.

Then $N(\bullet,\bullet)_p$ is called a p-adic 2-norm on X and the pair $(X, N(\bullet,\bullet)_p)$ is called p-adic linear 2-normed space.

For every p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ the function defined on $X \times X \times X$ by $d_p(x,y,z) = N(x-z, y-z)_p$ is a p-adic 2-metric.Thus every p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ will be considered to be a p-adic 2-metric space with this 2-metric.

A sequence $\{x_n\}_{n=1}^{\infty}$ of p-adic 2-metric space (X, d_p) converges to $x \in X$ in p-adic 2-metric if for every $\varepsilon > 0$, there is an $l \ge 1$ such that $d_p(x_n, x, z) = N(x_n - z, x - z)_p < \varepsilon$ for every $n \ge l$.

For the given two sequences of p-adic 2-metric space (X, d_p) which are $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ converges to $x, y \in X$ in the p-adic 2-metric respectively, then the sequence of sums $x_n + y_n$ and the product $x_n y_n$ converges to the sum $x + y$ and to the product xy of the limits of initial sequences.

A sequence $\{x_n\}_{n=1}^{\infty}$ of p-adic 2-metric space (X, d_p) is a Cauchy sequence with respect to the p-adic 2-metric if for each $\varepsilon > 0$, there is an $l \ge 1$ such that $d_p(x_n, x_m, z) = N(x_n - z, x_m - z)_p < \varepsilon$, for every $n, m \ge l$.

Definition 3.2: *A sequence* $\{x_n\}_{n\ge 1}$ *in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is called convergent if there exists an* $x \in X$ *such that* $\lim_{n\to\infty} N(x_n - x, z)_p = 0$ *for all* $z \in X$.

Definition 3.3: *A sequence* $\{x_n\}_{n\ge 1}$ *in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is called Cauchy sequence if for each* $\varepsilon > 0$*, there is an* $l \ge 1$ *such that* $N(x_n - x_m, z)_p < \varepsilon$*, for all* $n, m \ge l$ *and for all* $z \in X$.

Proposition 3.4: *If a sequence* $\{x_n\}$ *in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is convergent to* $x \in X$, *then* $\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ *for each* $z \in X$.

Definition 3.5: *A p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is called complete if every Cauchy sequence is convergent in p-adic linear 2-normed space.*

Definition 3.6: *A p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is called p-adic 2-Banach space if p-adic linear 2-normed space is complete.*

Proposition 3.7: *If* $\lim_{n\to\infty} N(x_n, z)_p$ *exists then we say that* $\{x_n\}_{n\geq 1}$ *is a Cauchy sequence with respect to* $N(\bullet,\bullet)_p$.

Proof: Let us suppose that $\lim_{n\to\infty} N(x_n, z)_p = x$. Then we can obtain a constant M_1 such that $n > M_1 \Rightarrow N(x - x_n, z)_p < \frac{\varepsilon}{2}$.

If $m, n > M_1$ then $N(x - x_n, z)_p < \frac{\varepsilon}{2}$ and $N(x - x_m, z)_p < \frac{\varepsilon}{2}$,

Hence by using the triangle inequality, we have

$$N(x_m - x_n, z)_p = N(x_m - x + x - x_n, z)_p \leq N(x_m - x, z)_p + N(x - x_n, z)_p < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Definition 3.8: *A sequence* $\{x_n\}_{n\geq 1}$ *is called a null sequence in p-adic linear 2-normed space if* $\lim_{n\to\infty} N(x_n, z)_p = 0$ *for all* $z \in X$.

Example 3.9: Let $x_n = p^n$ and $z = p^r$ with $r < n$ in the p-adic 2-norm over $X = Q$.

Then, $N(p^n, p^r)_p = p^{-ord_p(p^n, p^r)}$, if $p^n \neq 0$

and $p^r \neq 0 = p^{-\infty}$, if $p^n = 0$ or $p^r = 0$.

In this case $N(p^n, p^r)_p = p^{-n} = \frac{1}{p^n} = 0$, as $n \to \infty$. Therefore, $\lim_{n\to\infty} N(x_n, z)_p = 0$ for all $z \in X$. Hence this sequence is a null sequence with respect to the p-adic 2-norm.

Definition 3.10: *A p-adic number* (α, β) *can be uniquely written in the form*

$$(\alpha, \beta) = \sum_{i=n, j=m}^{\infty} (a_i p^i, b_j p^j)$$

Where each $0 \le a_i, b_j \le p-1$ *and p-adic 2-norm of the number* (α, β) *is defined as* $N(\alpha,\beta)_p = n$, $(n \in R)$ *and the double series*
$(1+p+p^2+p^3+...,1+p+p^2+p^3+...)$ *converges to* $\dfrac{1}{1-p}$ *in the p-adic 2-norm.*

4 Main Results:

In this section, we prove that $I-$limit operation for sequences in p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is liner with respect to summation and scalar multiplication. We investigate the relation between I-cluster points and ordinary limit points of p-adic linear 2-normed spaces.

A family of sets $I \subseteq 2^Y$ (power set of Y) is said to be an ideal if $\Phi \in I$, I is additive i.e., $A, B \in I \Rightarrow A \cup B \in I$ and hereditary i.e., $A \in I$, $B \subseteq A \Rightarrow B \in I$.

A non empty family of sets $F \subset 2^Y$ is a filter on Y if and only if $\Phi \notin F$, $A \cap B \in F$ for each $A, B \in F$, and any subset of an element of F is in F. An ideal I is called non-trivial if $I \ne \Phi$ and $Y \notin I$. Clearly I is a non-trivial ideal if and only if $F = F(I) = \{Y - A : A \in I\}$ is a filter in Y, called the filter associated with the ideal I.

A non-trivial ideal I is called admissible if and only if $\{\{n\} : n \in Y\} \subset I$.

An admissible ideal $I \subset 2^Y$ is said to have the property (AP) if for any sequence $\{A_1, A_2, A_3, ...\}$ of mutually disjoint sets of I there is a sequence $\{B_1, B_2, B_3, ...\}$ of sets such that each symmetric difference $A_i \Delta B_i$, $i = 1,2,3...$ is finite and $B = \bigcup_{i=1}^{\infty} B_i \in I$.

Definition 4.1: *Let* $(X, N(\bullet,\bullet))$ *be a linear 2-normed space. A sequence* $\{x_n\}$ *of elements of* X *is called to be statistically convergent to* $x \in X$ *if the set* $A(\varepsilon) = \{n \in N : N(x_n - x, z) \ge \varepsilon\}$ *has natural density zero for each* $\varepsilon > 0$.

Definition 4.2: *Let* $I \subset 2^N$ *be a non trivial ideal in* N *and* $(X, N(\bullet,\bullet))$ *be a linear 2-normed space. The sequence* $\{x_n\}$ *of elements of* X *is said to be* $I-$*convergent to* $x \in X$ *if for each* $\varepsilon > 0$ *the set*
$A(\varepsilon) = \{n \in N : N(x_n - x, z) \ge \varepsilon\} \in I$.

Definition 4.3: *Let* $I \subset 2^N$ *be a non trivial ideal in* N. *A sequence* $\{x_n\}$ *in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *is said to be* $I-$*convergent to x if for each* $\varepsilon > 0$ *and non- zero* z *in* X *the set* $A(\varepsilon) = \{n \in N : N(x_n - x, z)_p \ge \varepsilon\} \in I$.

If $\{x_n\}$ *is* $I-$*convergent to* $x\in X$, *then we write* $I-\lim_{n\to\infty} N(x_n - x, z)_p = 0$ *or* $I-\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ *for each non-zero* $z\in X$. *The number* x *is called* $I-$*limit of the sequence* $\{x_n\}$.

Theorem 4.4: *Let* I *be an admissible ideal and* $(X, N(\bullet,\bullet)_p)$ *be a p-adic linear 2-normed space. For each* $z\in X$

(i) If $I-\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ and $I-\lim_{n\to\infty} N(y_n, z)_p = N(y,z)_p$ then $I-\lim_{n\to\infty} N(x_n + y_n, z)_p = N(x+y,z)_p$

(ii) $I-\lim_{n\to\infty} N(ax_n, z)_p = N(ax,z)_p$, $a\in R$ or K.

Proof:

(i) Let $I-\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$ and $I-\lim_{n\to\infty} N(y_n, z)_p = N(y,z)_p$ then $I-\lim_{n\to\infty} N(x_n - x, z)_p = 0$ and $I-\lim_{n\to\infty} N(y_n - y, z)_p = 0$.

Let $\varepsilon > 0$ then $A_1, A_2 \in I$, where $A_1 = A_1(\varepsilon) = \{n\in N : N(x_n - x, z)_p \ge \frac{\varepsilon}{2}\}$ and

$A_2 = A_2(\varepsilon) = \{n\in N : N(y_n - y, z)_p \ge \frac{\varepsilon}{2}\}$, for each $z\in X$.

Let $A = A(\varepsilon) = \{n\in N : N((x_n + y_n) - (x+y), z)_p \ge \varepsilon\}$, for each $z\in X$.

Let $n\in (A_1\cup A_2)^C$ then $n\in A_1^C \cap A_2^C \Rightarrow n\in A_1^C$ and $n\in A_2^C$

$\Rightarrow N(x_n - x, z)_p < \frac{\varepsilon}{2}$ and $N(y_n - y, z)_p < \frac{\varepsilon}{2}$.

Now,
$$\begin{aligned} N((x_n + y_n) - (x+y), z)_p &= N((x_n - x) + (y_n - y), z)_p \\ &\le N(x_n - x, z)_p + N(y_n - y, z)_p \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \end{aligned}$$

$\Rightarrow n\in A^C$.

Thus, $(A_1\cup A_2)^C \subset A^c \Rightarrow A\subset A_1\cup A_2$.

Since $A_1, A_2 \in I$, therefore $A_1\cup A_2 \in I$ and $A\in I$.

Hence, $A = A(\varepsilon) = \{n\in N : N((x_n + y_n) - (x+y), z)_p \ge \varepsilon\}\in I$

$\Rightarrow I-\lim_{n\to\infty} N((x_n + y_n) - (x+y), z)_p = 0$

$\Rightarrow I-\lim_{n\to\infty} N(x_n + y_n, z)_p = N(x+y,z)_p$, for each $z \in X$. Thus $I-$limit operation for sequences in p-adic linear 2-normed spaces is linear with respect to summation.

(ii) Let $I-\lim_{n\to\infty} N(x_n, z)_p = N(x,z)_p$, $a \in K$, $a \neq 0$. Then for each $\varepsilon > 0$, the set

$$\{n \in N : N(x_n - x, z)_p \geq \frac{\varepsilon}{|a|}\} \in I \Rightarrow \text{ the set } \{n \in N : |a| N(x_n - x, z)_p \geq \varepsilon\} \in I$$

$$\Rightarrow \text{ the set } \{n \in N : N(ax_n - ax, z)_p \geq \varepsilon\} \in I$$

$$\Rightarrow I-\lim_{n\to\infty} N(ax_n - ax, z)_p = 0$$

$$\Rightarrow I-\lim_{n\to\infty} N(ax_n, z)_p = N(ax, z)_p$$

for each $z \in X$ .Thus I-limit operation for sequences in p-adic linear 2-normed spaces is linear with respect to scalar multiplication.

Lemma 4.5: *Let I be an admissible ideal and $u = \{u_1, u_2, ..., u_d\}$ be a basis for a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$. A sequence $\{x_n\}$ in X is $I-$convergent to x in X if and only if $I-\lim_{n\to\infty} N(x_n - x, u_i)_p = 0$ for $i = 1,2,...,d$.*

Proof: Suppose that a sequence $\{x_n\}$ in a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is $I-$convergent to x in X .

Then, $I-\lim_{n\to\infty} N(x_n - x, z)_p = 0$ for each non zero $z \in X$

$\Rightarrow I-\lim_{n\to\infty} N(x_n - x, u_i)_p = 0$, for $i = 1,2,...,d$.

Suppose that $I-\lim_{n\to\infty} N(x_n - x, u_i)_p = 0$, for $i = 1,2,...,d$.

Then $A_i \in I$, where $A_i = \{n \in N : N(x_n - x, u_i)_p \geq \frac{\varepsilon}{|\alpha_i| d}\}$, $i = 1,2,...,d$.

Now we have to prove that the sequence $\{x_n\}$ in X is $I-$convergent to x in X .
It suffices to prove that $I-\lim_{n\to\infty} N(x_n - x, z)_p = 0$ for each non zero $z \in X$.

Since every z in X can be written as $z = \alpha_1 u_1 + \alpha_2 u_2 + ... + \alpha_d u_d$, for $\alpha_{1,}\alpha_2,....\alpha_d \in K$. Let $A(\varepsilon) = \{n \in N : N(x_n - x, z)_p \geq \varepsilon\}$, for each non zero z∈ X.

Let $n \in (A_1 \cup A_2 \cup .... \cup A_d)^C$ then $n \in (A_1^c \cap A_2^c \cap ..... \cap A_d^c) \Rightarrow n \in A_1^C$ and $n \in A_2^C$ and , ……, and $n \in A_d^C$

$$\Rightarrow N(x_n - x, u_1)_p < \frac{\varepsilon}{|\alpha_1| d}, N(x_n - x, u_2)_p < \frac{\varepsilon}{|\alpha_2| d} ...., N(x_n - x, u_d)_p < \frac{\varepsilon}{|\alpha_d| d}$$

$\Rightarrow |\alpha_1| N(x_n - x, u_1)_p < \frac{\varepsilon}{d}, |\alpha_2| N(x_n - x, u_2)_p < \frac{\varepsilon}{d}$ ……, and

$$|\alpha_d| N(x_n - x, u_d)_p < \frac{\varepsilon}{d}$$

$\Rightarrow |\alpha_1| N(u_1, x_n - x)_p < \dfrac{\varepsilon}{d}, |\alpha_2| N(u_2, x_n - x)_p < \dfrac{\varepsilon}{d}$,……., and

$|\alpha_d| N(u_d, x_n - x)_p < \dfrac{\varepsilon}{d}$

$\Rightarrow N(\alpha_1 u_1, x_n - x)_p < \dfrac{\varepsilon}{d}$, $N(\alpha_2 u_2, x_n - x)_p < \dfrac{\varepsilon}{d}$,………, $N(\alpha_d u_d, x_n - x)_p < \dfrac{\varepsilon}{d}$

$\Rightarrow N(x_n - x, \alpha_1 u_1)_p < \dfrac{\varepsilon}{d}$, $N(x_n - x, \alpha_2 u_2)_p < \dfrac{\varepsilon}{d}$, ……., $N(x_n - x, \alpha_d u_d)_p < \dfrac{\varepsilon}{d}$

Now $N(x_n - x, z)_p = N(z, x_n - x)_p = N(\alpha_1 u_1 + \alpha_2 u_2 + ... + \alpha_d u_d, x_n - x)_p$

$\leq N(\alpha_1 u_1, x_n - x)_p + N(\alpha_2 u_2, x_n - x)_p + \ldots\ldots\ldots + N(\alpha_d u_d, x_n - x)_p$

$< \dfrac{\varepsilon}{d} + \dfrac{\varepsilon}{d} + \ldots\ldots\ldots + \dfrac{\varepsilon}{d} = \varepsilon$.

Therefore $n \in A^C$

$\Rightarrow (A_1 \cup A_2 \cup ..... \cup A_d)^C \subset A^c \Rightarrow A \subset (A_1 \cup A_2 \cup .... \cup A_d)$.

Since A_1, A_2 , … , $A_d \in$ I, therefore $(A_1 \cup A_2 \cup .... \cup A_d) \in I$ and hence $A \in I$

$\Rightarrow I - \lim_{n\to\infty} N(x_n - x, z)_p = 0$ for each non zero $z \in X$. Thus the sequence $\{x_n\}$ in X is $I-$convergent to x in X .

Remark 4.6: With respect to the basis $\{u_1, u_2, ..., u_d\}$, we can define a norm on X , which we shall denote it by $[N(x)_p]_\infty = \max\{N(x, u_i)_p : i = 1,2,..., d\}$.

Lemma 4.7: *Let I be an admissible ideal. A sequence $\{x_n\}$ in a p-adic linear 2-normed space ($X, N(\bullet,\bullet)_p$) is $I-$convergent to x in X if and only if $I - \lim_{n\to\infty} \max\{N(x_n - x, u_i)_p : i = 1,2,..., d\} = 0$, where $\{u_1, u_2, ..., u_d\}$ be a basis for X .*

Proof: Suppose a sequence $\{x_n\}$ is $I-$convergent to x in *a p-adic linear 2-normed space ($X, N(\bullet,\bullet)_p$).* Then by Lemma 4.5, $I - \lim_{n\to\infty} N(x_n - x, u_i)_p = 0$,

for i=1, 2,……, d , where $\{u_1, u_2, ...u_d\}$ be a basis for X

$\Rightarrow I - \lim_{n\to\infty} \max.\{N(x_n - x, u_i)_p : i = 1,2,......, d\} = 0$.

Conversely suppose that I-$\lim_{n\to\infty} \max.\{N(x_n - x, u_i)_p : i = 1,2,......, d\} = 0$.

Then $I - \lim_{n\to\infty} N(x_n - x, u_i)_p = 0$, for $i = 1,2,..., d$.

By Lemma 4.5, we have I-$\lim_{n\to\infty} N(x_n - x, z)_p = 0$, for each $z \in X$

$\Rightarrow \{x_n\}$ is $I-$convergent to x in X .

Lemma 4.8: *Let I be an admissible ideal. A sequence $\{x_n\}$ in a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is $I-$convergent to x in X if and only if* $I-\lim_{n\to\infty}[N(x)_p]_\infty = 0$.

We can define the open balls $B_u(x,\varepsilon)$ centered at x having radius r by $B_u(x,\varepsilon) = \{y : [N(x-y)_p]_\infty < \varepsilon\}$, where $u = \{u_1, u_2, ..., u_d\}$ be a basis for X. By using open balls definition, Lemma 4.8 becomes as follows,

Lemma 4.9: *Let I be an admissible ideal. A sequence $\{x_n\}$ in a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is $I-$convergent to x in X if and only if $A(\varepsilon) = \{n \in N : x_n \notin B_u(x,\varepsilon)\}$ belong to ideal, where $\{u_1, u_2, ..., u_d\}$ be a basis for X.*

Definition 4.10: Let I be a non trivial ideal in N. *A sequence $\{x_n\}$ in a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is called $I-$cauchy sequence if for each $\varepsilon > 0$, there is a $l \geq 1$ such that $\{k \in N : N(x_k - x_m, z)_p \geq \varepsilon\} \in I$, for all $k, m \geq l$, for each non zero $z \in X$.*

Theorem 4.11: *Let I be an admissible ideal. A p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$ is p-adic 2-Banach space if and only if $(X, [N(x)_p]_\infty)$ is a p-adic Banach space.*

Proof: By Lemma 4.8, $I-$convergence in the *p*-adic 2-norm is equivalent to that in the derived norm. To prove above Theorem, it suffices to show that $\{x_n\}$ is Cauchy sequence with respect to the *p*-adic 2-norm if and only if it is Cauchy sequence with respect to the derived norm. $\{x_n\}$ is Cauchy sequence with respect to the *p*-adic 2-norm if and only if for each $\varepsilon > 0$ and non zero $z \in X$, there is a $l \geq 1$ such that $\{k \in N : N(x_k - x_m, z)_p \geq \varepsilon\} \in I$, for all $k, m \geq l$, for each non zero $z \in X$, if and only if $I-\lim_{k,m\to\infty} N(x_k - x_m, z)_p = 0$ for each non zero $z \in X$, if and only if $I-\lim_{k,m\to\infty} N(x_k - x_m, u_i)_p = 0$ for $i = 1,2,...,d$ where $\{u_1, u_2, ..., u_d\}$ be a basis for X (by Lemma(4.7)), if and only if $I-\lim_{k,m\to\infty}[N(x_k - x_m)_p]_\infty = 0$ (by Lemma (4.8)), if and only if $\{x_n\}$ is $I-$Cauchy sequence with respect to derived norm ,if and only if $(X, [N(x)_p]_\infty)$ is a *p*-adic Banach space.

Definition 4.12: *Let $I \subset 2^N$ be an admissible ideal and $x = \{x_n\}$ be sequence in a p-adic linear 2-normed space $(X, N(\bullet,\bullet)_p)$. A number ξ is called to be a $I-$limit point of $x = \{x_n\}$ if there is a set $M = \{m_1, m_2, ...\} \subset N$ such that $M \notin I$*

and $\lim_{k\to\infty} N(x_{m_k}-\xi,z)_p = 0$ *for each non zero* z *in* X *. The set of all* $I-$*limit points of* $x=\{x_n\}$ *is denoted by* $I(\Lambda_x^2)$*. A number* ξ *is called to be a* $I-$*cluster point of* $x=\{x_n\}$ *if* $\{n\in N : N(x_n-\xi,z)_p<\varepsilon\}\notin I$ *, for each* $\varepsilon>0$ *and non zero* $z\in X$ *. The set of all* $I-$*cluster points of* $x=\{x_n\}$ *is denoted by* $I(\Gamma_x^2)$*.*

Proposition 4.13: *Let* $I\subset 2^N$ *be an admissible ideal and* $(X,N(\bullet,\bullet)_p)$ *be a p-adic linear 2-normed space. Then for each sequence* $\{x_n\}$ *of* X *,* $I(\Lambda_x^2)\subset I(\Gamma_x^2)$ *and the set* $I(\Gamma_x^2)$ *is a closed set.*

Proof: Let $\xi\in I(\Lambda_x^2)$ .Then there exists a set $M=\{m_1<m_2<....\}\notin I$ such that

$$\lim_{k\to\infty} N(x_{m_k}-\xi,z)_p = 0, \text{ for each non zero } z\in X . \tag{4.14}$$

Let $\delta>0$. According to (4.14) there exists $k_0\in N$ such that $k>k_0$ and for each non zero $z\in X$, We have, $N(x_{m_k}-\xi,z)_p<\delta$.

Then $\{n\in N : N(x_n-\xi,z)_p<\delta\}\supset M-\{m_1,m_2,....,m_{k_0}\}$

$\Rightarrow \{n\in N : N(x_n-\xi,z)_p<\delta\}\notin I$

$\Rightarrow \xi\in I(\Gamma_x^2)$. Thus $I(\Lambda_x^2)\subset I(\Gamma_x^2)$.

Clearly, $I(\Gamma_x^2)\subset \overline{I(\Gamma_x^2)}$ (4.15)

Let $y\in \overline{I(\Gamma_x^2)}$. For $\varepsilon>0$, there exists $\xi_0\in I(\Gamma_x^2)\cap B_u(y,\varepsilon)$. Choose $\delta>0$ such that $B_u(\xi_0,\delta)\subset B_u(y,\varepsilon)$.

Obviously $\{n\in N : N(y-x_n,z)_p<\varepsilon\}\supset\{n\in N : N(\xi_0-x_n,z)_p<\delta\}$.

Hence $\{n\in N : N(y-x_n,z)_p<\varepsilon\}\notin I$ and $y\in I(\Gamma_x^2)$.

This implies that

$$\overline{I(\Gamma_x^2)}\subset I(\Gamma_x^2) \tag{4.16}$$

from (4.15) and (4.16) we get $I(\Gamma_x^2)=\overline{I(\Gamma_x^2)}$ and hence $I(\Gamma_x^2)$ is a closed set.

Definition 4.17: *Let* $I\subset 2^N$ *be an admissible ideal and* $x=\{x_n\}$ *be sequence in a p-adic linear 2-normed space* $(X,N(\bullet,\bullet)_p)$*. If* $K=\{k_1<k_2<,.....\}\in I$ *, then the subsequence* $x_K=\{x_k\}$ *is called* $I-$*thin subsequence of the sequence* x *. If* $M=\{m_1<m_2<....\}\notin I$ *, then the subsequence* $x_M=\{x_m\}$ *is called* $I-$*non thin subsequence of the sequence* x*.*

We can easily verify that, if ξ *is a* $I-$*limit point of* $x=\{x_n\}$*, then there is a* $I-$*non thin subsequence* x_M *that converges to* ξ*. Let* L_x^2 *be the set of all ordinary limit points of sequence* $x=\{x_n\}$*. It is obvious that* $I(\Lambda_x^2)\subseteq L_x^2$ *and* $I(\Gamma_x^2)\subseteq L_x^2$ *.*

Lemma 4.18: *Let* $I\subset 2^N$ *be an admissible ideal and* $x=\{x_n\}$ *be sequence in a p-adic linear 2-normed space* $(X,N(\bullet,\bullet)_p)$*. If* x *is* $I-$*convergent in* X*, then* $I(\Lambda_x^2)$ *and* $I(\Gamma_x^2)$ *are both equal to the singleton set* $\{I-\lim_{n\to\infty}N(x_n,z)_p\}$ *for each non zero* $z\in X$ *.*

Proof: Suppose that $x=\{x_n\}$ is $I-$convergent to ξ in X .
Then $I-\lim_{n\to\infty}N(x_n-\xi,z)_p=0$ for each non zero $z\in X$
$\Rightarrow A(\varepsilon)=\{n\in N:N(x_n-\xi,z)_p\geq\varepsilon\}\in I$, for each $\varepsilon>0$ and non zero $z\in X$.

Since I is an admissible ideal, we can choose the set $M=\{n_1<n_2<....\}\subset N$ such that $n_k\notin A(\frac{1}{k})$ and $N(x_{n_k}-\xi,z)_p<\frac{1}{k}$, for all $k\in N$ and non zero $z\in X$
$\Rightarrow \lim_{k\to\infty}N(x_{n_k}-\xi,z)_p=0$.

Suppose $M\in I$. Since $M\subset\{n\in N:N(x_n-\xi,z)_p<1\}$ for each non zero $z\in X$,
Then $(N-M)\cap\{n\in N:N(x_n-\xi,z)_p<1\}=\phi$, but $N-M\in F(I)$ and
$\{n\in N:N(x_n-\xi,z)_p<1\}\in F(I)$ for each non zero $z\in X$ .This is a contradiction.
Therefore $M\notin I$.

Hence we get $M=\{n_1<n_2<,.....\}\subset N$ and $M\notin I$ such that
$\lim_{k\to\infty}N(x_{n_k}-\xi,z)_p=0$. This implies $\xi\in I(\Lambda_x^2)$. Since $I(\Lambda_x^2)\subset I(\Gamma_x^2)$,
Therefore $\xi\in I(\Gamma_x^2)$.

Now suppose that $\eta\in I(\Gamma_x^2)$ such that $\eta\neq\xi$. Then

$$A=\{n\in N:N(x_n-\xi,z)_p\geq\frac{|\eta-\xi|}{2}\}\in I \text{ and}$$

$$B=\{n\in N:N(x_n-\xi,z)_p<\frac{|\eta-\xi|}{2}\}\notin I\text{, for each non zero } z\in X .$$

On the other hand, since

$$N(x_n-\xi,z)_p=N(x_n-\eta+\eta-\xi,z)_p\geq N(|x_n-\eta|-|\eta-\xi|,z)_p>\frac{|\eta-\xi|}{2}$$

for each $n \in B$ and non zero $z \in X$, we have $B \subset A \in I$. This contradiction shows that $I(\Gamma_x^2) = \{\xi\}$. Thus $I(\Lambda_x^2) = I(\Gamma_x^2) = \{\xi\}$.

Theorem 4.19: *Let* $I \subset 2^N$ *be an admissible ideal and* $x = \{x_n\}$, $y = \{y_n\}$ *are sequences in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$ *such that* $M = \{n \in N : x_n \neq y_n\} \in I$ then
$I(\Lambda_x^2) = I(\Lambda_y^2)$ and $I(\Gamma_x^2) = I(\Gamma_y^2)$.

Proof: Let $\xi \in I(\Lambda_x^2)$ then there is a set $K = \{k_1 < k_2 <,.....\} \notin I$ such that
$I - \lim_{n\to\infty} N(x_{k_n} - \xi, z)_p = 0$. Let $K_1 = \{n \in N : n \in K$ and $x_n \neq y_n\}$ then $K_1 \subset M$ and hence, $K_1 \in I$. Let $K_2 = \{n \in N : n \in K$ and $x_n = y_n\}$ then $K_2 \notin I$.
(If $K_2 \in I$ Then, $K = K_1 \cup K_2 \in I$, but $K \notin I$).
Now the sequence $y_{K_2} = \{y_n\}_{n\in K_2}$ is a $I-$non thin subsequence of $y = \{y_n\}_{n\in N}$ and y_{K_2} converges to ξ in X. This implies that $\xi \in I(\Lambda_y^2)$

$$\Rightarrow \qquad I(\Lambda_x^2) \subset I(\Lambda_y^2) \tag{4.20}$$

In the similar way, we can prove that $$I(\Lambda_y^2) \subset I(\Lambda_x^2) \tag{4.21}$$

From (4.20) and (4.21) we get $I(\Lambda_x^2) = I(\Lambda_y^2)$.

Let $\xi \in I(\Gamma_x^2)$ then $K_3 = \{n \in N : N(x_n - \xi, z)_p < \varepsilon\} \notin I$ for each $\varepsilon > 0$ and non zero $z \in X$ and $K_4 = \{n \in N : n \in K_3$ and $x_n = y_n\} \notin I$.
This implies that $K_4 \subset \{n \in N : N(y_n - \xi, z)_p < \varepsilon\}$ for each non zero $z \in X$. This shows that, for each $\varepsilon > 0$ and non zero $z \in X$, $\{n \in N : N(y_n - \xi, z)_p < \varepsilon\} \notin I$.
$\Rightarrow \xi$ is a $I-$cluster point of $y = \{y_n\}$ in X $\Rightarrow$ $\xi \in I(\Gamma_y^2)$.

Therefore $$I(\Gamma_x^2) \subset I(\Gamma_y^2) \tag{4.22}$$

In the similar way, we can easily show that $$I(\Gamma_y^2) \subset I(\Gamma_x^2) \tag{4.23}$$

From (4.22) and (4.23) we get $I(\Gamma_x^2) = I(\Gamma_y^2)$.

Theorem 4.24: *Let* $I \subset 2^N$ *be an admissible ideal with the property (AP) and* $x = \{x_n\}$ *is sequence in a p-adic linear 2-normed space* $(X, N(\bullet,\bullet)_p)$*. Then there is a sequence* $y = \{y_n\}$ *such that* $L_y^2 = I(\Gamma_x^2)$ *and* $\{n \in N : x_n \neq y_n\} \in I$*, where* L_y^2 *is ordinary limit points set of the sequence* $y = \{y_n\}$*. Moreover* $\{y_n : n \in N\} \subset \{x_n : n \in N\}$.

Proof: If $L_y^2 = I(\Gamma_x^2)$ then $y = x$ and this case is trivial.
Let $I(\Gamma_x^2)$ is a proper subset of L_x^2. Then $L_x^2 - I(\Gamma_x^2) \neq \phi$ and for each $\xi \in L_x^2 - I(\Gamma_x^2)$ there is an open interval $E_\xi = (\xi - \delta, \xi + \delta)$ such that $I - \lim_{k\to\infty} N(x_{j_k} - \xi, z)_p = 0$ .Hence, there is an open interval $E_\xi = (\xi - \delta, \xi + \delta)$ such that $\{k \in N : x_k \in E_\xi\} \in I$. It is obvious that the collection of all the intervals E_ξ is an open cover of $L_x^2 - I(\Gamma_x^2)$, so by covering Theorem there is a countable and mutually disjoint subcover $\{E_j\}_{j=1}^\infty$ such that each E_j contains an $I-$thin subsequence of $\{x_n\}$.

Now, let $A_j = \{n \in N : x_n \in E_j, j \in N\}$. It is clear that $A_j \in I$, $j = 1,2,3,...$ and $A_i \cap A_j = \phi$. Then by (AP) property of I there is a countable collection $\{B_j\}_{j=1}^\infty$ of subsets of N such that $B = \bigcup_{j=1}^{\infty} B_j$ and $A_j - B$ is a finite set for each $j \in N$. Let $M = N - B = \{m_1 < m_2 <,.....\} \subset N$.

Now the sequence $y = \{y_k\}$ is defined by $y = y_k$ if $k \in B$ and $y_k = x_k$ if $k \in M$. Obviously, $\{k \in N : x_k \neq y_k\} \subset B \in I$
So by Theorem (4.19) we have $I(\Gamma_x^2) = I(\Gamma_y^2)$.

Since $A_j - B$ is a finite set then the subsequence $y_B = \{y_k\}_{k\in B}$ has no limit point that is not also an $I-$limit point of $y \Rightarrow L_y^2 = I(\Gamma_y^2)$.

Therefore, we have proved $L_y^2 = I(\Gamma_x^2)$. Moreover, the construction of the sequence $y = \{y_n\}$ shows that $\{y_n : n \in N\} \subset \{x_n : n \in N\}$.

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