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Nonlinear Study of Hydrodynamic Rayleigh-Taylor Instability in a Composite Fluid-Porous Layer

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Abstract

The nonlinear hydrodynamic Rayleigh-Taylor instability (RTI) bounded above by porous layer using Saffman [1] slip condition at the interface and below by a rigid surface using no-slip condition has been studied. The nonlinear problem is studied numerically in the present paper using Adams-Bashforth predictor and Adams-Moulton corrector numerical techniques. In the conclusion, the nonlinear problem discussed here is quite different from that of Babchin et al. [2] considering plane Couette flow. The present problem is greatly influenced by slip velocity at the interface between porous layer and thin film. Also, the effect of magnetic field to stabilize the system. It is not amenable to analytical treatment as that of Babchin et al.[2]. Therefore, numerical solutions have to be found. Fourth order accurate central differences are used for spatial discretization using predictor and corrector numerical technique.

Keywords: *Nonlinear hydrodynamic Rayleigh-Taylor instability, Predictor-Corrector numerical technique, Porous layer.*

1 Introduction

The phenomenon of instability of the interface between a heavy fluids supported by a lighter fluid known as Rayleigh-Taylor instability (RTI) has been extensively studied in a wide range of physical contexts both experimentally and theoretically. In spite of a long history of investigations there are many important motivations which still attract attention to different branches of physics, namely astrophysics (Arons et al. [3], Bernstein and Book [4], Rudraiah [5]), plasma fusion (Finn [6]), space (Amatucci et al. [7]; Penano et al. [8]), atmospheric (Sazonov [9]) and geophysics (Wilcock and Whitehead [10]), etc. The primary source by which this instability is triggered is the gravitational force acting on an inverted density gradient (e.g. a heavy fluid supported by a light fluid). The basic mechanism of this instability, an interchange of flux tube to tap the gravitational free energy, is the same mechanism that drives the Rayleigh-Benard instability in the thermal convection of a gravitationally unstable fluid. In this case the mean temperature gradient of the fluid plays a similar role as the density gradient and the buoyancy force acts similar to the gravity. Apart from fluid dynamics RT mode exists in magnetized plasmas in both collisional and collisionless regimes. It plays a crucial role in the areas of inertial confinement fusion (ICF) (see Mikelian [11]).

Rayleigh [12] initiated the study of hydrodynamic instability of fluid having a vertical density variation. He showed the equilibrium of a horizontal layer of incompressible, in viscid (ideal) fluid is stable or unstable according as the density increases or decreases anywhere in the vertically upward direction. Under various physical effects the Rayleigh-Taylor instability problem of a semi-infinite layer of a fluid has been studied by several authors in hydrodynamics and in MHD. The detailed account of the various assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [13]. Roberts [14] has extended the analysis to the case of two fluids of equal kinematic viscosities in the presence of a vertical magnetic field, while Gerwin [15] has studied the case of compressible streaming fluids. The influence of viscosity on the stability of the plane interface separating two incompressible superposed fluids of uniform densities, when the whole system is immersed in a uniform horizontal magnetic field, has been studied by Bhatia [16]. He carried out the stability analysis for two fluids of equal kinematic viscosities and different densities. A generalized theory of hydromagnetic stability of the interface between two infinitely conducting superposed fluids is given by Shivamoggi [17]. Rudraiah et al. [18] have pointed out that a magnetic field applied obliquely to the interface between two kinds of electrically conducting viscous fluids exerts a stabilizing influence on the configuration.

Although copious literature is available on linear RTI, the work on nonlinear RTI is very sparse. With the invention of high-speed computers the nonlinear approach has changed drastically. If the perturbed quantities are very small in comparison with the basic state then the product of the perturbed quantities can be neglected leading to linear theory. On the other hand if the perturbations are not small then product of the perturbed quantities can't be neglected leading to nonlinear theory. Before we proceed into details of this problem we briefly review the work done on nonlinear RTI. McCrory et al. [19]) have given the simulations of the RTI of ablatively accelerated thin-shell fusion targets and showed that the nonlinear evolution exhibits spike amplitude saturation. A simple model is derived heuristically for the nonlinear evolution of the RTI by Baker and Freeman [20]). Babchin et al. [2] have studied the nonlinear saturation of RTI in thin films. They have found that the combined action of flow shear and surface tension is the essence of the saturation mechanism. Shivamoggi [21]) has used the method of strained co-ordinates in investigating the nonlinear RTI problem. After incorporating the corrections pointed out by Malik and Singh [22] he obtained a revised expression for the nonlinear cut-off wave number which separates the region of stability from that of instability. Mohamed and Shehawey [23] have investigated nonlinear electrohydrodynamic RTI in the absence of surface charges and a charge free surface separating two semi infinite dielectric fluids influenced by a normal electric field subjected to nonlinear deformations. Allah and Yahia [24] have studied the nonlinear RTI in the presence of magnetic field and also mass and heat transfer using the simplified formulation. Later, Verma and Shukla [25] have studied the linear and nonlinear properties of Rayleigh-Taylor modes.

Rudraiah et al. [26] have studied the linear and nonlinear RTI in a viscous fluid layer bounded below by a rigid surface and above by a porous layer based on the approximations in effect which are similar to lubrication and Stokes approximations. The linear problem has been studied analytically, while the nonlinear problem is studied numerically. They have shown that the stability curve can be controlled by the porous-slip parameter. Mahmoud [27] has discussed the theoretical analysis of the nonlinear Rayleigh-Taylor instability of two fluids under the influence of a periodic radial magnetic field. A weakly nonlinear stability for magnetic fluid has been discussed by El-Dib [28]. The research of an interface between two strong viscous homogeneous incompressible fluids through porous medium is investigated theoretically and graphically. The effect of the vertical magnetic field has been demonstrated in this study. The kinematic viscosities play a stabilizing role when the fluid flows through a porous media, while a destabilizing influence is recorded when the fluid flows through non-porous media. The investigation has shown that the porous permeability plays a dual role in the stability behavior. Recently, Anjali Devi and Hemamalini [29] have analyzed the effects of rotation and magnetic field on nonlinear RTI of two superposed ferrofluids. More recently, Rudraiah et al. [30] has investigated the non-linear study of electrohydrodynamic Rayleigh–Taylor instability in a composite fluid–porous layer. This problem has greatly influenced by the slip velocity at the interface between porous layer and thin film.

Nevertheless, much attention has not been given in the literature on the study of nonlinear RTI in a poorly conducting fluid bounded above by a porous layer and below by a rigid surface in the presence of magnetic field in spite of its importance in varied problems. Therefore, in this paper we have investigated nonlinear hydrodynamic RTI in a composite fluid–porous layer. The evolution of the interface is analyzed numerically by employing fourth order Adams-Bashforth predictor and Adams-Moulton corrector methods. The control of instability of the interface is analyzed in detail.

To achieve this objective, this paper is planned as follows. The basic equations for poorly conducting fluid in the presence of magnetic field called MHD equations are given in section 2 with suitable approximations and boundary conditions. The dispersion relation of RTI in MHD in the presence of magnetic field in fluid layer bounded above by a porous layer is derived in section 3. The importance conclusions are drawn in the final section.

2 Mathematical Formulation

The physical configuration in this paper is shown in Fig.1. We consider a two-dimensional fluid-porous medium composite system with heavy fluid of constant density ρ_p in the porous region supported by a lighter fluid of density ρ_f in a region of height H bounded by a rigid surface at *y=0.* The interface between the two fluid –porous medium and the film is described by $\eta(x,t)$. Let u and v denote the velocity components in x and y directions respectively. The fluids are assumed to be viscous and incompressible. The fluid in the thin film is set in motion by acceleration normal to the interface whereas in the porous layer it is assumed to be static and small perturbations are amplified when acceleration is directed from the lighter fluid in the thin film to the heavier fluid in the porous layer. Between two fluids there exists a surface tension γ. The instability at the interface in the presence of magnetic field is known as hydrodynamic Rayleigh-Taylor instability (RTI). To investigate this RTI, we consider a rectangular coordinate system (x, y) with the x-axis parallel to the film and y-axis normal to it.

2.1 Basic Equations:

Following are the basic equations for film-porous layer composite system:

The conservation of mass:

$$\nabla\cdot\vec{q}=0 \qquad (1)$$

The conservation of momentum:

$$\rho\left(\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right)=-\nabla p+\mu\nabla^2\vec{q}+\mu_0\left(\vec{J}\times\vec{H}\right) \qquad (2)$$

where $\vec{q} = (u, v)$ the fluid velocity, p the pressure, μ the fluid viscosity, ρ the fluid density and $\vec{J}$ the current density.

2.2 Boundary and Surface Conditions:

i) The no-slip condition at the rigid surface :

$$u = 0 \quad \text{at} \quad y = 0 \tag{3}$$

ii) The Saffman[1] slip condition :

$$\frac{\partial u}{\partial y} = \frac{\alpha_p}{\sqrt{k}} u \quad at \quad y = h \tag{4}$$

iii) The kinematic condition :

$$v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{at} \quad y = h \tag{5}$$

iv) The dynamic condition :

$$p = -\delta\eta - \gamma \frac{\partial^2 \eta}{\partial x^2} + \mu \frac{\partial \eta}{\partial t} \quad \text{at} \quad y = h\,. \tag{6}$$

where μ is the fluid viscosity , k the permeability of the porous layer, α the slip parameter at the interface, γ the fluid surface tension and $\delta = g(\rho_p - \rho_f)$ the gravitational force.

In solving Eqs. (1) to (2), following Rudraiah et al. [18], we make use of the following approximations:

(i) The film thickness h is much smaller than the thickness H of the dense fluid above the film. That is

$$h << H$$

(ii) The surface elevation η is assumed to be small compared to film thickness h. That is

$$\eta << h$$

(iii) The Strauhal number S, a measure of the local acceleration to inertial acceleration in Eq.(2.2), is negligibly small. That is

$$S = \frac{L}{TU} << 1$$

where $U = \nu / L$ is the characteristic velocity, ν the kinematic viscosity, $L = \sqrt{\gamma / \delta}$ the characteristic length and $T = \mu\gamma / h^3\delta^2$ the characteristic time.

These approximations are usually called Stokes and lubrication approximations. Also we assume that the heavy fluid in the porous layer is almost static because of heavy creeping flow approximation in a densely packed porous medium, which is needed to use the Saffman [1] slip condition.

To understand the physics of the problem, it is simplified using the following dimensionless variables

$$x^* = \frac{x}{h}, y^* = \frac{y}{h}, u^* = \frac{u}{\delta h^2 / \mu_f}, v^* = \frac{v}{\delta h^2 / \mu_f}, p^* = \frac{p}{\delta h}, \sigma_p = \frac{h}{\sqrt{k}} \tag{7}$$

Equations (1) to (6) reduce to the following form:

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{8}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{9}$$

$$0 = \frac{\partial p}{\partial y} \quad . \tag{10}$$

Subject to the above boundary and surface conditions:

$$u = 0 \quad \text{at} \quad y = 0 \tag{11}$$

$$\frac{\partial u}{\partial y} = \alpha_p \sigma_p u \quad at \quad y = 1 \tag{12}$$

$$v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad at \ y = 1 \tag{13}$$

$$p = -\eta - \frac{1}{B} \frac{\partial^2 \eta}{\partial x^2} \quad at \ y = 1 \tag{14}$$

where $M = \mu_0 H_0 h \sqrt{\sigma_f / \mu_f}$ is the Hartmann number, $B = \delta h^2 / \gamma$ the Bond number, $\eta = \eta(x, y, t)$ the elevation of the interface and $\sigma_p = \frac{h}{\sqrt{k}}$ the porous parameter. It may be noted here that the kinematic condition given by Eq. (13) is nonlinear.

3 Dispersion Relation

To find the dispersion relation, first we have to find the velocity distribution from Eq. (9) using the above boundary and surface conditions.

The solution of (9) subject to the above conditions is

$$u = A_1 coshMy + A_2 sinhMy - \frac{1}{M^2}\frac{\partial p}{\partial x} \quad (15)$$

where
$$A_1 = \frac{1}{M^2}\frac{\partial p}{\partial x}$$
$$A_2 = \frac{K_1}{M^2}\frac{\partial p}{\partial x}$$
$$K_1 = \frac{[\beta(1-coshM) - MsinhM]}{[M\, coshM + \beta sinhM]}$$
$$\beta = \alpha_p \sigma_p .$$

After integrating Eq.(8) with respect to y between y = 0 and 1 and using Eq.(15), we get

$$\mathrm{v}(1) = v_1 = -\int_0^1 \frac{\partial u}{\partial x} dy = \Delta_1 \frac{\partial^2 p}{\partial x^2} . \quad (16)$$

$$\Delta_1 = \frac{M - sinhM - K_1(1-coshM)}{M^3}$$

To find the expression for the interface evolution η, using Eqs. (16) and (13), we get

$$\frac{\partial \eta}{\partial t} = \Delta_2 \left[\frac{\partial \eta}{\partial x} + \frac{1}{B}\frac{\partial^3 \eta}{\partial x^3}\right]\frac{\partial \eta}{\partial x} + \Delta_1 \left[\frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B}\frac{\partial^4 \eta}{\partial x^4}\right]. \quad (17)$$

where $\Delta_2 = \dfrac{coshM + K_1 sinhM - 1}{M^2}$.

Let us analyze the interface evolution by this equation.

Equation (17) in the limit of $M \to 0$ reduces to the one given by Rudraiah et al., (1998), where the effect of porous lining on the nonlinear evolution of the interface is studied. In this paper we study the combined effect of porous layer and applied magnetic field on the nonlinear evolution of the interface. The process described here is quite different from a process in which the film is bounded by a fluid with moving boundaries, instead of porous layer, discussed by Babchin et al. [2]. Therefore, we use Eq. (3.3) to study the nonlinear interface evolution. Eq. (17) is not amenable to analytical treatment and hence we solve it numerically using a 4^{th} order central differences in space and time as explained below. For time-integration of Eq.(17), Adams-Bashforth predictor and Adams-Moulton corrector steps of fourth order are used, as described in Chapra and Canale [31]. Spatial derivatives are described by the following central difference formulae of fourth order accuracy:

$$\frac{\partial\eta}{\partial x}\to\frac{1}{12\Delta x}\left[\eta(i-2)-8\eta(i-1)+8\eta(i+1)-\eta(i+2)\right] \tag{18}$$

$$\frac{\partial^2\eta}{\partial u^2}\to\frac{1}{12\Delta x^2}\left[-\eta(i-2)+16(i-1)-30\eta(i)+16\eta(i+1)-\eta(i+2)\right] \tag{19}$$

$$\frac{\partial^3\eta}{\partial x^3}\to\frac{1}{8\Delta x^3}\left[-\eta(i-3)-8\eta(i-2)+13\eta(i-1)-13(i+1)+8\eta(i+2)-\eta(i+3)\right] \tag{20}$$

$$\frac{\partial^4\eta}{\partial x^4}\to\frac{1}{6\Delta x^4}\left[-\eta(i-3)+12\eta(i-2)-39\eta(i-1)+56\eta(i)-39\eta(i+1)+12\eta(i+2)-\eta(i+3)\right]. \tag{21}$$

Here $\eta(i-2)$ stands for the value of η at the position $x-2\Delta x$. The integer i indicates the i^{th} grid point.

The initial condition used in the numerical integration is a sine-wave with wave number ℓ and is of the form

$$\eta(x,0)=\eta_0\sin(\ell x) \qquad \left(0<x\le\frac{\pi}{2}\ell\right). \tag{22}$$

Here the amplitude η_0 is assumed to be small. In our numerical computation we use $\eta_0=10^{-4}$ in non-dimensional form and periodic boundary conditions have been applied in the x-direction.

4 Results and Discussion

The nonlinear hydrodynamic RTI (ERTI) in a fluid layer bounded above by a thick porous layer and below by rigid surface in the presence of magnetic field is investigated. This equation is solved numerically using fourth order differences in space and time and the results are depicted in Figs. 2-9.

In Figs 2-9, we discuss the spatial structure of growth rate of the interface in terms of $n_{NL}\text{-}n_{NL,max} = \frac{1}{\eta}\frac{\partial \eta}{\partial t} - \frac{1}{\eta_{max}}\frac{\partial \eta_{max}}{\partial t}$ at an early stage (initial time) before instability occurs for two wave number $\ell = 0.75$ and $\ell = 2.0$ and other parameters defined earlier. In Figs. 2 and 3 six waves are contained in the interval 16π the case of small wave numbers. The peaks reflect the position of the wave modes. The case of large wave numbers $\ell = 2.0$ is presented in Figs. 4 and 5 and it contains 16 complete waves in the interval 16π. Figures 6 and 7 represent whether the full numerical solutions deviate from a simple harmonic behaviour of η, namely $\eta = \eta_0 sin(\ell x)$ for $\ell = 0.75$ where only part of the wave is shown and similarly in Figs. 8 and 9 discussion is on large wave number $\ell = 2.0$.

Figures 6(a)-(d) shows that the spatial structures of the interface for $t>0$ and Figs.2 describes the interface at $t=0$. Initial interface profile is symmetric in the interval $0 \le x \le 16\pi$. But for $t>0$ every point of the interface moves in the x direction with velocity proportional to that point evolution η.Thus the points where $\eta = 0$ do not move, while the points of maximal elevation move faster than all other points. In the subsequent evolution the symmetry is lost, since the maximum moves farther from one of the zeros and closer to the other one. However, this process of steepening of the forward faces of the profile doesn't result in the breakup of the interface, because of the effects of surface tension, slip and magnetic field. Initially these parameters may have negligible effects, but as time progresses those parameters play an important role in the stabilization. In Figs.6(a)-(d) only one part of the wave is shown(i.e., only one period 2π) as we move from top to bottom and clearly notice that the symmetry can obtained with the effect of slip due to porous layer and hence reduced the growth rate of RTI at the interface considerably for $t>0$.

From Figs. 3 it is clear that the interface profile is symmetric in the interval $0 \le x \le 16\pi$ for initial time $t=0$. As we move from top to bottom in Figs. 7(a)-(d), we observe that the symmetry can be recovered for increasing the magnetic field *M*. Therefore, the effect of magnetic field is to reduce the asymmetry of the system and hence stabilize the system.

Figures 8 and 9 are similar to Figs. 6 and 7 but they differ in the value of ℓ, that is $\ell = 2.0$. In each of these cases we notice that the shape of the curve remain the same but they vary in magnitude. The effect of these parameters is also to reduce the growth rate as in the earlier cases.

It may also be noted that the full numerical solution for all cases is not possible because of the limitation of the numerical scheme. In this problem computation of the influence for larger values of Hartmann number *M* is not possible because of the above limitation (when M>20). Also similar behaviour happens when β is very large. In addition to this full numerical solution becomes unstable for such cases.

5 Figures

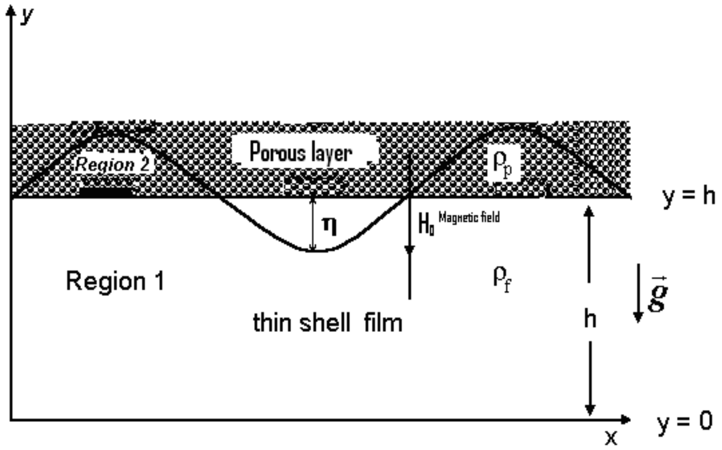


Fig. 1: Physical configuration

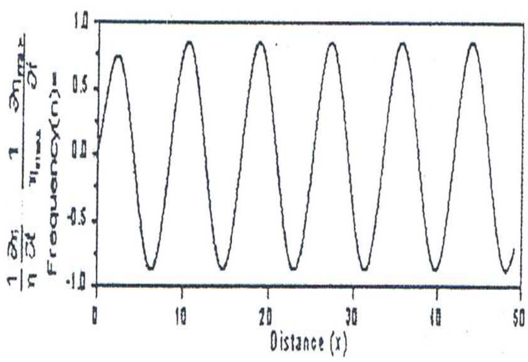


Fig. 2(a)

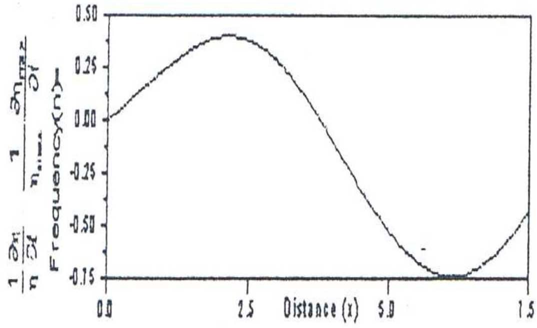


Fig. 6(a)

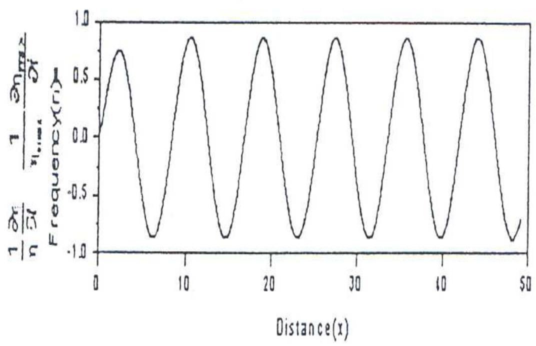


Fig. 2(b)

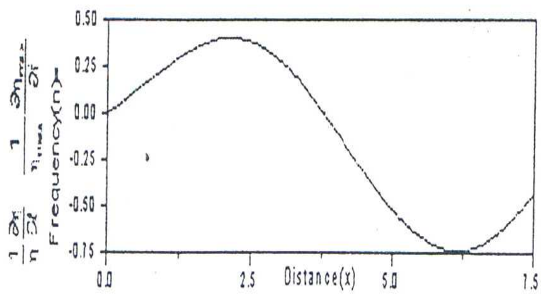


Fig. 6(b)

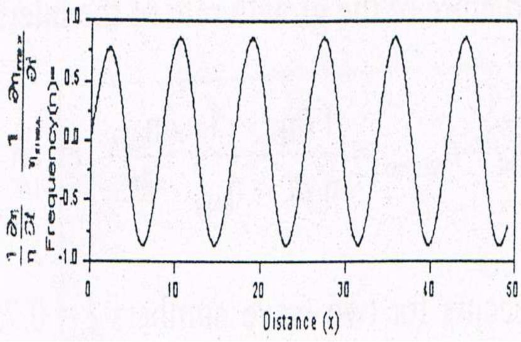


Fig. 2(c)

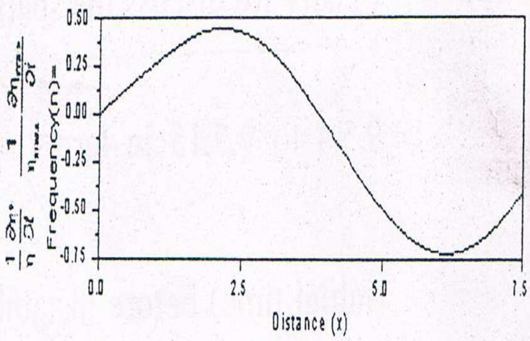


Fig.6 (c)

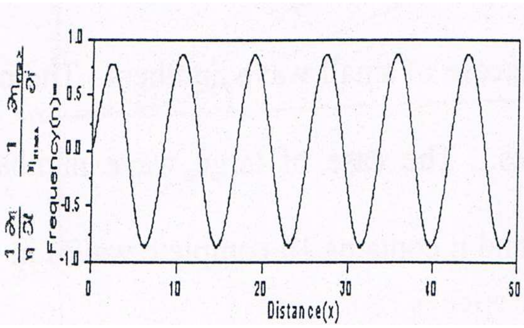


Fig. 2(d): for t=0

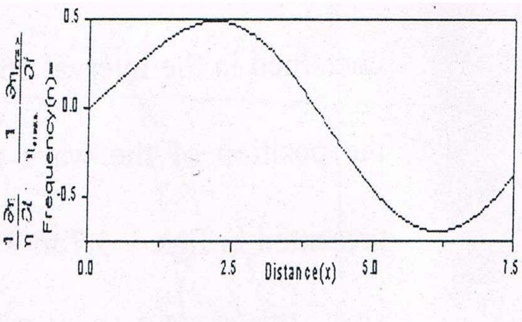


Fig. 6(d): for t >0

Shape of the interface for $\ell = 0.75$ with different values of $\beta = 0.0, 1.0, 10.0, 100.0$.

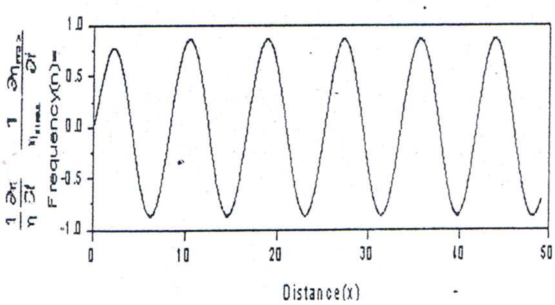


Fig. 3(a)

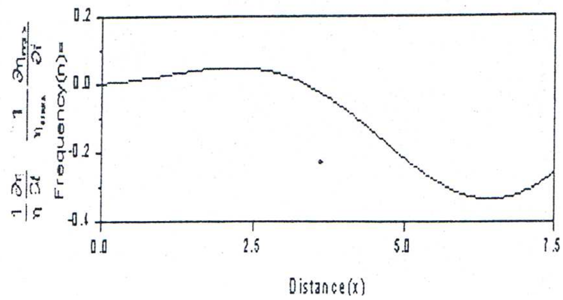


Fig. 7(a)

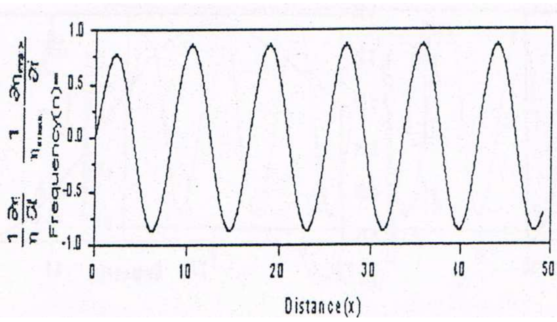


Fig. 3(b)

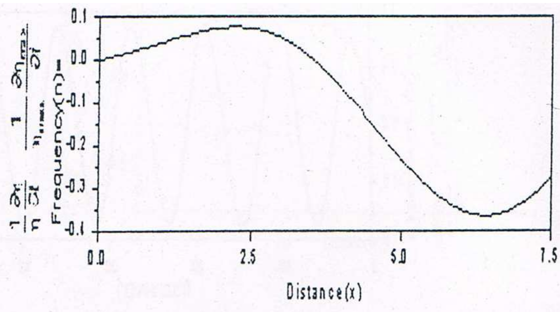


Fig. 7(b)

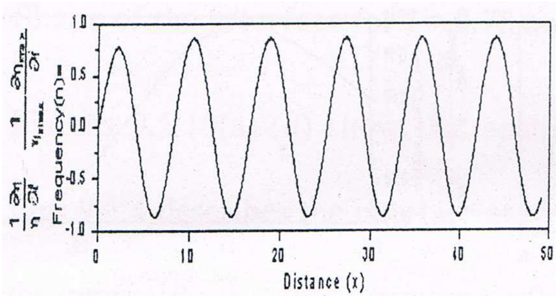


Fig. 3(c)

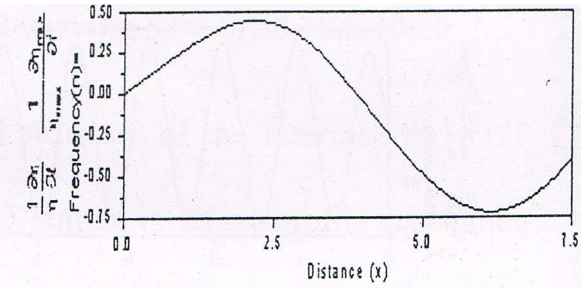


Fig.7(c)

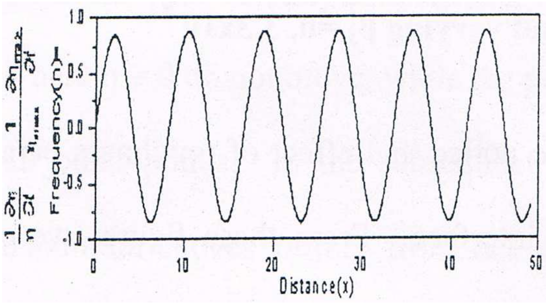


Fig. 3(d): For t=0

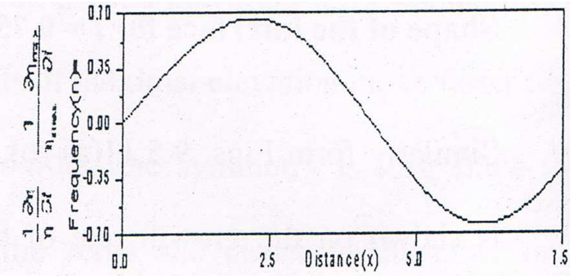


Fig.7(d): For t > 0.

Shape of the interface for $\ell = 0.75$ with different values of $M = 0.0001, 1.0, 10.0, 20.0.$

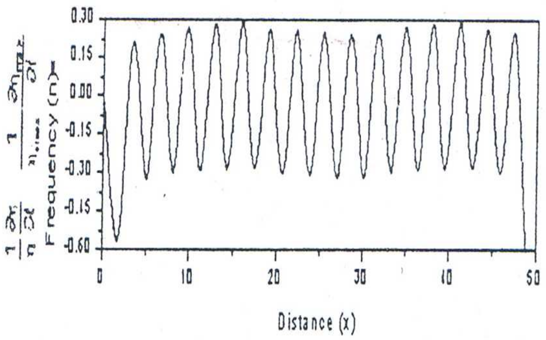


Fig. 4(a)

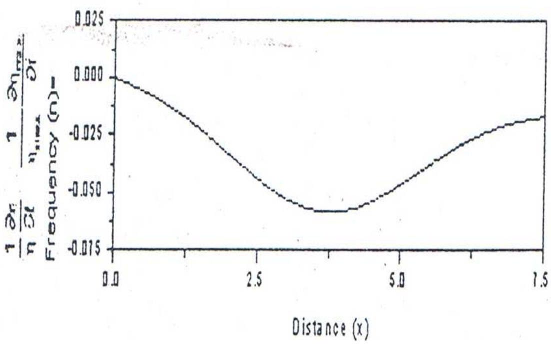


Fig. 8(a)

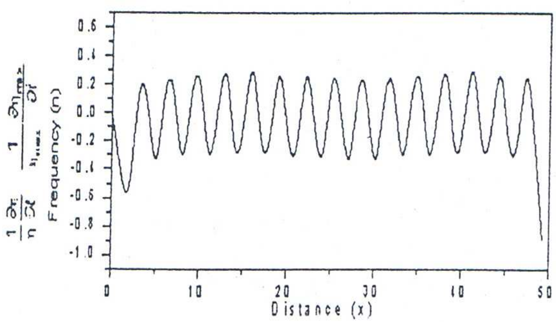


Fig. 4(b)

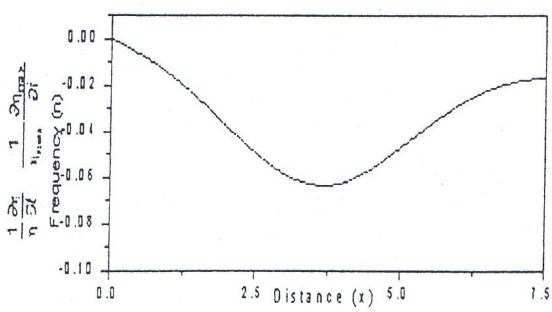


Fig. 8(b)

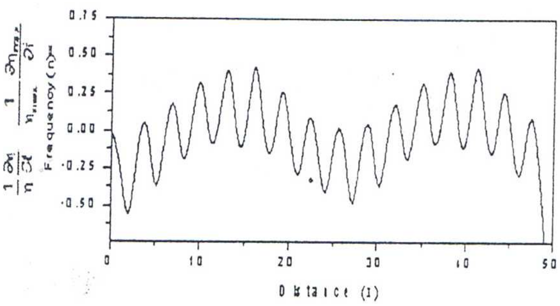


Fig. 4(c)

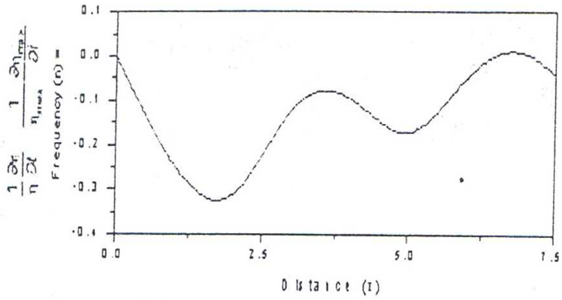


Fig. 8(c)

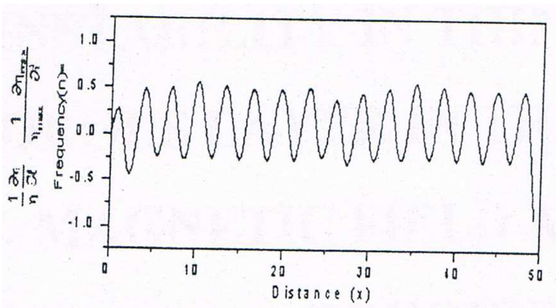


Fig. 4(d): For t=0

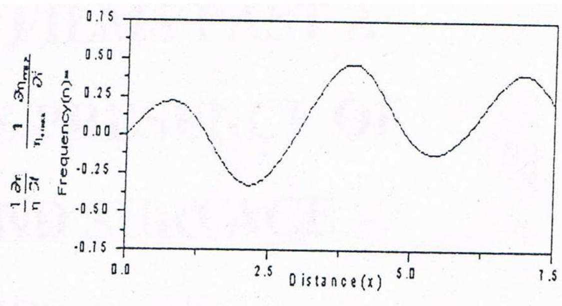


Fig. 8(d): For t>0

Shape of the interface for $\ell = 2.0$ **with different values of** $M = 0.0001, 1.0, 10.0, 20.0$ **.**

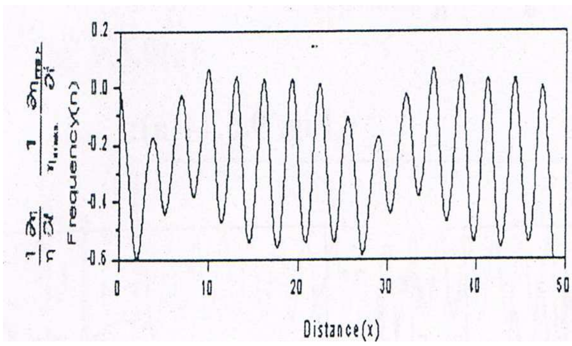


Fig. 5(a)

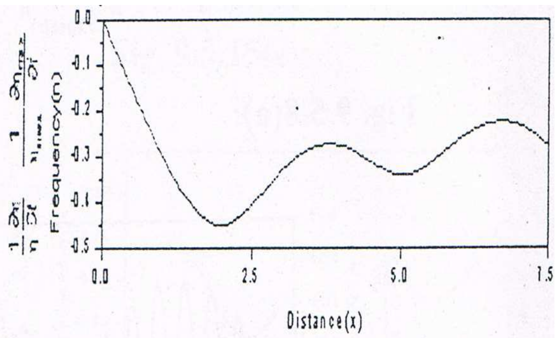


Fig. 9(a)

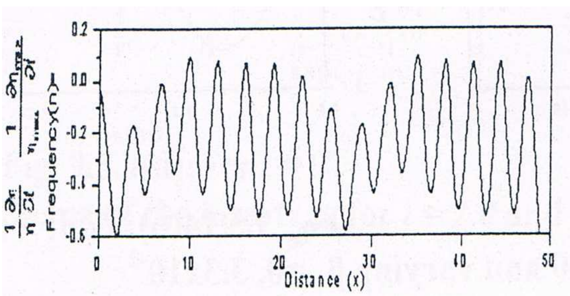


Fig. 5(b)

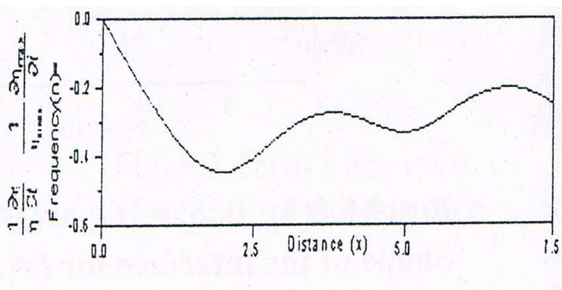


Fig. 9(b)

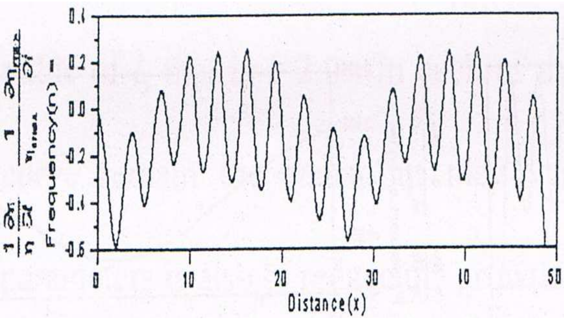


Fig.5(c)

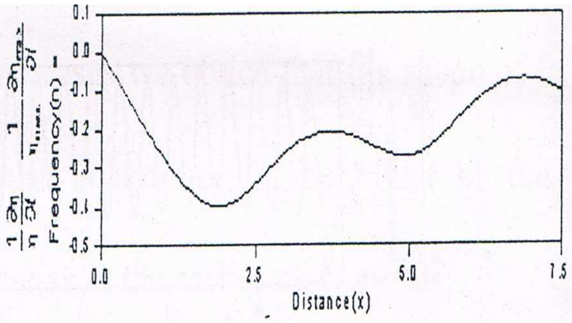
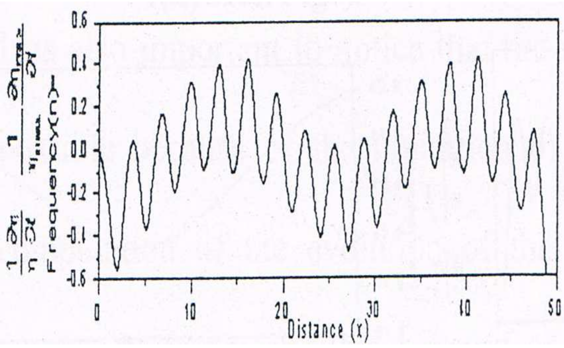


Fig. 9(c)



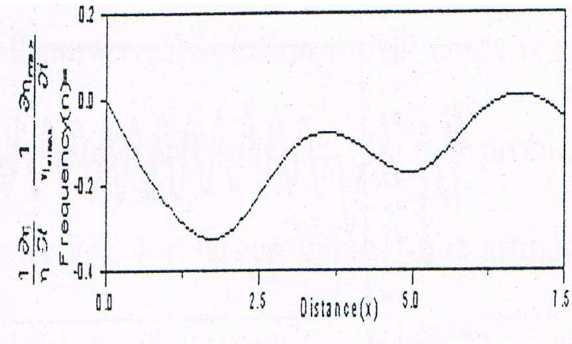


Fig. 5(d): For t=0

Fig. 9(d): For t>0

Shape of the interface for $\ell = 2.0$ **with different values of** $\beta = 0.0, 1.0, 10.0, 100.0$.

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