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Research Article

Stability of the n-Dimensional Mixed-Type Additive and Quadratic Functional Equation in Non-Archimedean Normed Spaces

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We will prove the stability of the functional equation $2f(\sum_{i=1}^{n} x_i) + \sum_{1\leq i,j\leq n, i\neq j} f(x_i - x_j) = (n+1)\sum_{i=1}^{n} f(x_i) + (n-1)\sum_{i=1}^{n} f(-x_i)$ in non-Archimedean normed spaces.

1. Introduction

A classical question in the theory of functional equations is "when is it true that a function, which approximately satisfies a functional equation, must be somehow close to an exact solution of the equation?" Such a problem, called *a stability problem* of the functional equation, was formulated by Ulam in 1940 (see [1]). In the following year, Hyers [2] gave a partial solution of Ulam's problem for the case of approximate additive functions. Subsequently, his result was generalized by Aoki [3] for additive functions and by Rassias [4] for linear functions. Indeed, they considered the stability problem for unbounded Cauchy differences. During the last decades, the stability problems of functional equations have been extensively investigated by a number of mathematicians (see [5–23]).

A non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot| : \mathbb{K} \to [0,\infty)$ such that

(F_1) $|r| = 0$ if and only if $r = 0$;

(F_2) $|rs| = |r||s|$;

(F_3) $|r + s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$.

Clearly, it holds that $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$.

Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean and nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(N_1) $\|x\| = 0$ if and only if $x = 0$;

(N_2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(N_3) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$.

Then $(X, \|\cdot\|)$ is called a non-Archimedean space. Due to the fact that

$$\|x_n - x_m\| \leq \max_{m \leq i < n} \|x_{i+1} - x_i\| \quad (n > m), \tag{1.1}$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean space. A complete non-Archimedean space is a non-Archimedean space in which every Cauchy sequence is convergent.

Recently, Moslehian and Rassias [24] proved the Hyers-Ulam stability of the Cauchy functional equation

$$f(x + y) = f(x) + f(y), \tag{1.2}$$

and the quadratic functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y) \tag{1.3}$$

in non-Archimedean normed spaces.

We now consider the *n-dimensional mixed-type quadratic and additive functional equation*

$$2f\left(\sum_{i=1}^{n} x_i\right) + \sum_{1 \leq i, j \leq n, i \neq j} f(x_i - x_j) = (n + 1)\sum_{i=1}^{n} f(x_i) + (n - 1)\sum_{i=1}^{n} f(-x_i), \tag{1.4}$$

whose solution is called *a quadratic-additive function*.

In 2009, Towanlong and Nakmahachalasint [25] obtained a stability result for the functional equation (1.4), in which they constructed a quadratic-additive function F by composing an additive function A and a quadratic function Q, where A and Q approximate the odd part and the even part of the given function f, respectively.

In this paper, we investigate a general stability problem for the n-dimensional mixed-type quadratic and additive functional equation (1.4) in non-Archimedean normed spaces.

2. Solutions of (1.4)

In this section, we prove the generalized Hyers-Ulam stability of the n-dimensional mixed-type quadratic and additive functional equation (1.4). Assume that H is an additive group and X is a complete non-Archimedean space.

For a given function $f : H \to X$, we use the abbreviations

$$\begin{aligned}
f_e(x) &:= \frac{f(x)+f(-x)}{2}, \\
f_o(x) &:= \frac{f(x)-f(-x)}{2}, \\
Af(x,y) &:= f(x+y)-f(x)-f(y), \\
Qf(x,y) &:= f(x+y)+f(x-y)-2f(x)-2f(y), \\
D_n f(x_1,x_2,\ldots,x_n) &:= 2f\left(\sum_{i=1}^{n} x_i\right)+\sum_{1\le i,j\le n, i\neq j} f(x_i-x_j) \\
&\quad - (n+1)\sum_{i=1}^{n} f(x_i)-(n-1)\sum_{i=1}^{n} f(-x_i)
\end{aligned} \tag{2.1}$$

for all $x, y, x_1, x_2, \ldots, x_n \in H$ and for an arbitrarily fixed $n \in \mathbb{N}$.

Theorem 2.1. *Assume that $n \ge 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. A function $f : H \to X$ is a solution of* (1.4) *if and only if f_e is quadratic, f_o is additive, and $f_e(0) = 0$.*

Proof. If a function $f : H \to X$ is a solution of (1.4), then we have $f_e(0) = 0$,

$$\begin{aligned}
Qf_e(x,y) &= f_e(x+y)+f_e(x-y)-2f_e(x)-2f_e(y) \\
&= \frac{1}{2}D_n f_e(x,y,0,\ldots,0)+\frac{1}{2}(n-2)(n+3)f_e(0) \\
&= 0, \\
Af_o(x,y) &= f_o(x+y)-f_o(x)-f_o(y) = \frac{1}{2}D_n f_o(x,y,0,\ldots,0) = 0
\end{aligned} \tag{2.2}$$

for all $x, y \in H$, that is, f_e is quadratic and f_o is additive.

Conversely, assume that f_e is quadratic, f_o is additive, and $f_e(0) = 0$. We apply an induction on j to prove $D_n f_e(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in H$. For $j = 2$, we have

$$\begin{aligned}
&D_n f_e(x_1,x_2,0,\ldots,0) \\
&\quad = 2f_e(x_1+x_2)+2f_e(x_1-x_2)-4f_e(x_1)-4f_e(x_2)-(n-2)(n+3)f_e(0) \\
&\quad = 0.
\end{aligned} \tag{2.3}$$

If $n > 2$ and $D_n f_e(x_1, x_2, \ldots, x_j, 0, \ldots, 0) = 0$ for some integer j $(2 \leq j < n)$ and for all $x_1, x_2, \ldots, x_j \in H$, then a routine calculation yields

$$\begin{aligned}
&D_n f_e(x_1, x_2, \ldots, x_{j+1}, 0, \ldots, 0)\\
&\quad = Qf_e(x_1 + \cdots + x_j, x_{j+1} - x_j) + \frac{1}{2} D_n f_e(x_1, \ldots, x_{j-1}, 2x_j, 0, \ldots, 0)\\
&\qquad + \frac{1}{2} D_n f_e(x_1, \ldots, x_{j-1}, 2x_{j+1}, 0, \ldots, 0) - \sum_{k=1}^{j-1} (Qf_e(x_k, x_j) + Qf_e(x_k, x_{j+1}))\\
&\qquad - \frac{j}{2} Qf_e(x_{j+1}, x_{j+1}) - \frac{j}{2} Qf_e(x_j, x_j)\\
&\quad = 0
\end{aligned} \tag{2.4}$$

for all $x_1, x_2, \ldots, x_{j+1} \in H$. Hence, we conclude that

$$D_n f_e(x_1, x_2, \ldots, x_n) = 0 \tag{2.5}$$

for all $x_1, x_2, \ldots, x_n \in H$.

Since f_o is additive, a long calculation yields

$$\begin{aligned}
&D_n f_o(x_1, x_2, \ldots, x_n)\\
&\quad = \sum_{1 \leq i, j \leq n, i \neq j} Af_o(x_i, -x_j) + 2 \sum_{i=1}^{n-1} Af_o\left(\sum_{j=1}^{i} x_j, x_{i+1} \right)\\
&\quad = 0.
\end{aligned} \tag{2.6}$$

Hence, it follows from (2.5) and (2.6) that

$$D_n f(x_1, x_2, \ldots, x_n) = D_n f_e(x_1, x_2, \ldots, x_n) + D_n f_o(x_1, x_2, \ldots, x_n) = 0 \tag{2.7}$$

for all $x_1, x_2, \ldots, x_n \in H$; that is, f is a solution of (1.4). □

3. Generalized Hyers-Ulam Stability of (1.4)

In the following theorem, we will investigate the stability problem of the functional equation (1.4).

Theorem 3.1. *Assume that $n \geq 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. Assume that $\varphi : H^n \to [0, \infty)$ is a function such that*

$$\lim_{m \to \infty} \frac{\varphi(n^m x_1, n^m x_2, \ldots, n^m x_n)}{|n|^{2m}} = 0 \tag{3.1}$$

for all $x_1, x_2, \ldots, x_n \in H$*. Moreover, assume that the limit*

$$\tilde{\varphi}(x) := \lim_{m\to\infty} \max_{0\le i<m} \left\{ \frac{\varphi(n^i x, \ldots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \ldots, -n^i x)}{|4||n|^{2i+2}} \right\} \tag{3.2}$$

exists for each $x \in H$*. If a function* $f : H \to X$ *satisfies the inequality*

$$\|D_n f(x_1, x_2, \ldots, x_n)\| \le \varphi(x_1, x_2, \ldots, x_n) \tag{3.3}$$

for any $x_1, x_2, \ldots, x_n \in H$*, then there exists a unique quadratic-additive function* $T : H \to X$ *such that*

$$\|f(x) - T(x)\| \le \tilde{\varphi}(x) \tag{3.4}$$

for each $x \in H$*. In particular,* T *is given by*

$$T(x) = \lim_{m\to\infty} \left(\frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \right) \tag{3.5}$$

for all $x \in H$*.*

Proof. If we replace x_i in (3.1) with 0 for each $i \in \{1, 2, \ldots, n\}$, then we have

$$\lim_{m\to\infty} \frac{\varphi(0, 0, \ldots, 0)}{|n|^{2m}} = 0. \tag{3.6}$$

Since $|n| \le 1$, it holds that $\varphi(0, 0, \ldots, 0) = 0$ and

$$\|(n-1)(n+2)f(0)\| = \|D_n f(0, 0, \ldots, 0)\| \le \varphi(0, 0, \ldots, 0) = 0. \tag{3.7}$$

Hence, we conclude that $f(0) = 0$.

Let $J_m f : H \to Y$ be a function defined by

$$J_m f(x) = \frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \tag{3.8}$$

for all $x \in H$ and $m \in \{0,1,2,\ldots\}$. A tedious calculation, together with (F_2), (N_3), and (3.3), yields

$$\begin{aligned}\left\|J_i f(x) - J_{i+1} f(x)\right\| &= \left\| -\frac{D_n f(n^i x,\ldots,n^i x)}{4n^{2i+2}} - \frac{D_n f(-n^i x,\ldots,-n^i x)}{4n^{2i+2}} \right.\\ &\quad \left. - \frac{D_n f(n^i x,\ldots,n^i x)}{4n^{i+1}} + \frac{D_n f(-n^i x,\ldots,-n^i x)}{4n^{i+1}} \right\| \\ &\le \max\left\{ \frac{\|D_n f(n^i x,\ldots,n^i x)\|}{|4||n|^{2i+2}}, \frac{\|D_n f(-n^i x,\ldots,-n^i x)\|}{|4||n|^{2i+2}}, \right. \\ &\quad \left. \frac{\|D_n f(n^i x,\ldots,n^i x)\|}{|4||n|^{i+1}}, \frac{\|D_n f(-n^i x,\ldots,-n^i x)\|}{|4||n|^{i+1}} \right\} \\ &\le \max\left\{ \frac{\varphi(n^i x,\ldots,n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x,\ldots,-n^i x)}{|4||n|^{2i+2}} \right\}\end{aligned} \tag{3.9}$$

for all $x \in H$ and $i \in \{0,1,2,\ldots\}$. It follows from (3.1) and (3.9) that the sequence $\{J_m f(x)\}$ is Cauchy. Since X is complete, we conclude that $\{J_m f(x)\}$ is convergent.

Let us define

$$T(x) := \lim_{m\to\infty} J_m f(x) \tag{3.10}$$

for any $x \in H$. It follows from (N_3) and (3.9) that

$$\begin{aligned}\left\|f(x) - J_m f(x)\right\| &= \left\| \sum_{i=0}^{m-1} \left(J_i f(x) - J_{i+1} f(x)\right) \right\| \\ &\le \max_{0\le i<m} \left\|J_i f(x) - J_{i+1} f(x)\right\| \\ &\le \max_{0\le i<m} \left\{ \frac{\varphi(n^i x,\ldots,n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x,\ldots,-n^i x)}{|4||n|^{2i+2}} \right\}\end{aligned} \tag{3.11}$$

for all $m \in \{0,1,2,\ldots\}$ and $x \in H$. In view of (3.2), if we let $m \to \infty$ in (3.11), then we obtain the inequality (3.4).

Replacing x_i in (3.3) with $n^m x_i$ for $i \in \{1,2,\ldots,n\}$ and considering (F_2) and (N_3), we get

$$\begin{aligned}\left\|D_n J_m f(x_1, x_2, \ldots, x_n)\right\| &= \left\| \frac{D_n f(n^m x_1,\ldots,n^m x_n) - D_n f(-n^m x_1,\ldots,-n^m x_n)}{2n^m} \right. \\ &\quad \left. + \frac{D_n f(n^m x_1,\ldots,n^m x_n) + D_n f(-n^m x_1,\ldots,-n^m x_n)}{2n^{2m}} \right\|\end{aligned}$$

$$\leq \max\left\{\frac{\varphi(n^m x_1,\ldots,n^m x_n)}{|2||n|^m}, \frac{\varphi(-n^m x_1,\ldots,-n^m x_n)}{|2||n|^m},\right.$$
$$\left.\frac{\varphi(n^m x_1,\ldots,n^m x_n)}{|2||n|^{2m}}, \frac{\varphi(-2^m x_1,\ldots,-2^m x_n)}{|2||n|^{2m}}\right\} \tag{3.12}$$

for all $m \in \{0,1,2,\ldots\}$ and $x_1, x_2, \ldots, x_n \in H$. If we let $m \to \infty$ in the last inequality, then it follows from the condition (3.1) that $D_n T(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in H$; that is, T is a quadratic-additive function.

Assume that $T' : H \to X$ is another quadratic-additive function satisfying (3.4). By the definition of D_n, a routine calculation yields

$$\begin{aligned}
&-\frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{2i+2}} - \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{2i+2}} - \frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{i+1}} + \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{i+1}} \\
&\quad = -\frac{1}{2n^{2(i+1)}}\left(T'\left(n^{i+1}x\right) + T'\left(-n^{i+1}x\right)\right) + \frac{1}{2n^{2i}}\left(T'\left(n^i x\right) + T'\left(-n^i x\right)\right) \\
&\qquad - \frac{1}{2n^{i+1}}\left(T'\left(n^{i+1}x\right) - T'\left(-n^{i+1}x\right)\right) + \frac{1}{2n^i}\left(T'\left(n^i x\right) - T'\left(-n^i x\right)\right)
\end{aligned} \tag{3.13}$$

for each $i \in \{0,1,2,\ldots\}$ and $x \in H$. Hence, it follows from (3.8) that

$$\begin{aligned}
&\sum_{i=0}^{k-1}\left(-\frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{2i+2}} - \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{2i+2}}\right. \\
&\qquad \left. - \frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{i+1}} + \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{i+1}}\right) = T'(x) - J_kT'(x)
\end{aligned} \tag{3.14}$$

for any $k \in \mathbb{N}$ and $x \in H$. Since T' is a solution of (1.4), it follows from the last equality that

$$T'(x) = J_kT'(x) \tag{3.15}$$

for any $k \in \mathbb{N}$ and $x \in H$. Obviously, this equality also holds for T.

Consequently, by considering that $|n| \leq 1$, it follows from (N_3), (3.1), (3.4), and (3.8) that

$$\begin{aligned}
&\left\|T(x) - T'(x)\right\| \\
&\quad = \lim_{k\to\infty}\left\|J_kT(x) - J_kT'(x)\right\| \\
&\quad \leq \lim_{k\to\infty}\max\left\{\left\|J_kT(x) - J_kf(x)\right\|, \left\|J_kf(x) - J_kT'(x)\right\|\right\}
\end{aligned}$$

$$
\begin{aligned}
&\leq \lim_{k\to\infty} |2|^{-1}|n|^{-2k} \max\Big\{ \left\| T\left(n^k x\right) - f\left(n^k x\right) \right\|, \left\| T\left(-n^k x\right) - f\left(-n^k x\right) \right\|, \\
&\qquad\qquad \left\| f\left(n^k x\right) - T'\left(n^k x\right) \right\|, \left\| f\left(-n^k x\right) - T'\left(-n^k x\right) \right\| \Big\} \\
&\leq \lim_{k\to\infty} \lim_{m\to\infty} \max_{k\leq i<m+k} \left\{ \frac{\varphi(n^i x, \ldots, n^i x)}{|8||n|^{2i+2}}, \frac{\varphi(-n^i x, \ldots, -n^i x)}{|8||n|^{2i+2}} \right\} \\
&= 0
\end{aligned}
\tag{3.16}
$$

for all $x \in H$. Therefore, $T = T'$, which proves the uniqueness of T. □

Corollary 3.2. *Let X and Y be non-Archimedean normed spaces over $\mathbb{K}$ with $|n| < 1$. If Y is complete and $f : X \to Y$ satisfies the inequality*

$$
\left\| Df(x_1, x_2, \ldots, x_n) \right\| \leq \theta \sum_{i=1}^{n} \|x_i\|^r \tag{3.17}
$$

for all $x_1, x_2, \ldots, x_n \in X$ and for some $r > 2$, then there exists a unique quadratic-additive function $T : X \to Y$ such that

$$
\left\| f(x) - T(x) \right\| \leq \frac{n\theta}{|4||n|^2} \|x\|^r \tag{3.18}
$$

for all $x \in X$.

Proof. Let $\varphi(x_1, x_2, \ldots, x_n) = \theta \sum_{i=1}^{n} \|x_i\|^r$. Since $|n| < 1$ and $r - 2 > 0$, we get

$$
\lim_{m\to\infty} |n|^{-2m} \varphi(n^m x_1, n^m x_2, \ldots, n^m x_n) = \lim_{m\to\infty} |n|^{m(r-2)} \varphi(x_1, x_2, \ldots, x_n) = 0 \tag{3.19}
$$

for all $x_1, x_2, \ldots, x_n \in X$. Therefore, the conditions of Theorem 3.1 are satisfied. Indeed, it is easy to see that $\tilde{\varphi}(x) = n\theta(|4|^{-1}|n|^{-2})\|x\|^r$. By Theorem 3.1, there exists a unique quadratic-additive function $T : X \to Y$ such that (3.18) holds. □

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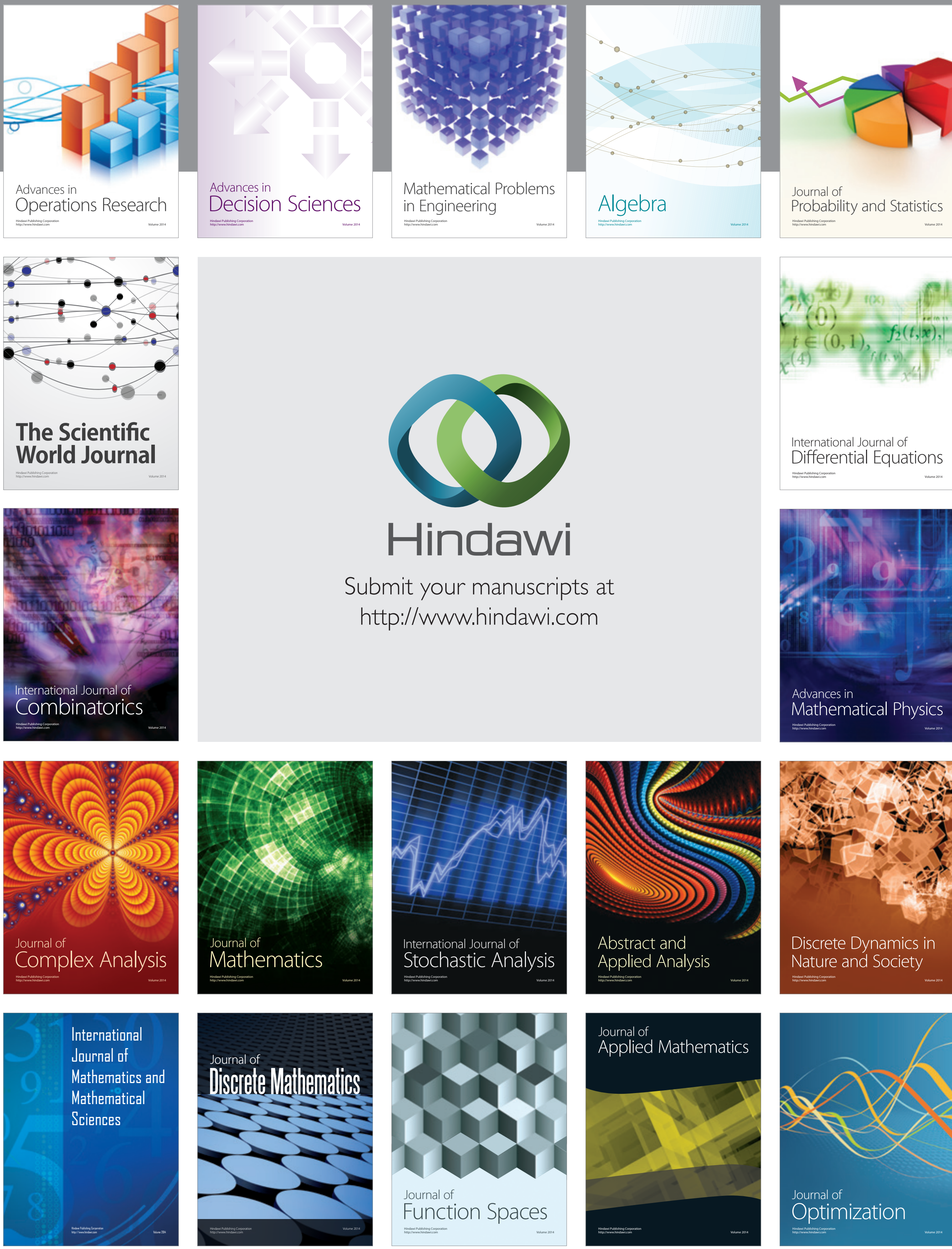
References

[1] S. M. Ulam, *A Collection of Mathematical Problems*, Interscience Publishers, London, UK, 1960.
[2] D. H. Hyers, "On the stability of the linear functional equation," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 27, pp. 222–224, 1941.
[3] T. Aoki, "On the stability of the linear transformation in Banach spaces," *Journal of the Mathematical Society of Japan*, vol. 2, pp. 64–66, 1950.

[4] T. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.
[5] L. M. Arriola and W. A. Beyer, "Stability of the Cauchy functional equation over p-adic fields," *Real Analysis Exchange*, vol. 31, no. 1, pp. 125–132, 2006.
[6] I.-S. Chang, E. H. Lee, and H.-M. Kim, "On Hyers-Ulam-Rassias stability of a quadratic functional equation," *Mathematical Inequalities & Applications*, vol. 6, no. 1, pp. 87–95, 2003.
[7] S. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific Publishing, River Edge, NJ, USA, 2002.
[8] A. Ebadian, N. Ghobadipour, Th. M. Rassias, and M. Eshaghi Gordji, "Functional inequalities associated with Cauchy additive functional equation in non-Archimedean spaces," *Discrete Dynamics in Nature and Society*, vol. 2011, Article ID 929824, 14 pages, 2011.
[9] P. Găvruţa, "A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings," *Journal of Mathematical Analysis and Applications*, vol. 184, no. 3, pp. 431–436, 1994.
[10] M. Eshaghi Gordji, "Nearly ring homomorphisms and nearly ring derivations on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2010, Article ID 393247, 12 pages, 2010.
[11] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1198–1202, 2010.
[12] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of cubic and quartic functional equations in non-Archimedean spaces," *Acta Applicandae Mathematicae*, vol. 110, no. 3, pp. 1321–1329, 2010.
[13] D. H. Hyers, G. Isac, and T. M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser Boston Inc., Boston, Mass, USA, 1998.
[14] K.-W. Jun and Y.-H. Lee, "A generalization of the Hyers-Ulam-Rassias stability of the Pexiderized quadratic equations—II," *Kyungpook Mathematical Journal*, vol. 47, no. 1, pp. 91–103, 2007.
[15] S.-M. Jung, "Hyers-Ulam-Rassias stability of Jensen's equation and its application," *Proceedings of the American Mathematical Society*, vol. 126, no. 11, pp. 3137–3143, 1998.
[16] S.-M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*, vol. 48, Springer, New York, NY, USA, 2011.
[17] G. H. Kim, "On the stability of functional equations with square-symmetric operation," *Mathematical Inequalities & Applications*, vol. 4, no. 2, pp. 257–266, 2001.
[18] H.-M. Kim, "On the stability problem for a mixed type of quartic and quadratic functional equation," *Journal of Mathematical Analysis and Applications*, vol. 324, no. 1, pp. 358–372, 2006.
[19] Y.-H. Lee, "On the stability of the monomial functional equation," *Bulletin of the Korean Mathematical Society*, vol. 45, no. 2, pp. 397–403, 2008.
[20] Y.-H. Lee and K.-W. Jun, "A generalization of the Hyers-Ulam-Rassias stability of Jensen's equation," *Journal of Mathematical Analysis and Applications*, vol. 238, no. 1, pp. 305–315, 1999.
[21] Y.-H. Lee and K.-W. Jun, "A generalization of the Hyers-Ulam-Rassias stability of the Pexider equation," *Journal of Mathematical Analysis and Applications*, vol. 246, no. 2, pp. 627–638, 2000.
[22] A. K. Mirmostafaee, "Approximately additive mappings in non-Archimedean normed spaces," *Bulletin of the Korean Mathematical Society*, vol. 46, no. 2, pp. 387–400, 2009.
[23] F. Skof, "Local properties and approximation of operators," *Rendiconti del Seminario Matematico e Fisico di Milano*, vol. 53, pp. 113–129, 1983.
[24] M. S. Moslehian and T. M. Rassias, "Stability of functional equations in non-Archimedean spaces," *Applicable Analysis and Discrete Mathematics*, vol. 1, no. 2, pp. 325–334, 2007.
[25] W. Towanlong and P. Nakmahachalasint, "An n-dimensional mixed-type additive and quadratic functional equation and its stability," *Science Asia*, vol. 35, pp. 381–385, 2009.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