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Research Article

Existence of Solutions for Nonlinear Mixed Type Integrodifferential Functional Evolution Equations with Nonlocal Conditions

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Using Mönch fixed point theorem, this paper proves the existence and controllability of mild solutions for nonlinear mixed type integrodifferential functional evolution equations with nonlocal conditions in Banach spaces, some restricted conditions on a priori estimation and measure of noncompactness estimation have been deleted, our results extend and improve many known results. As an application, we have given a controllability result of the system.

1. Introduction

This paper related to the existence and controllability of mild solutions for the following nonlinear mixed type integrodifferential functional evolution equations with nonlocal conditions in Banach space X:

$$\begin{gathered} x'(t) = A(t)x(t) + f\left(t, x_t, \int_0^t K(t,s,x_s)ds, \int_0^b H(t,s,x_s)ds\right), \quad t \in J, \\ x_0 = \phi + g(x), \quad t \in [-q, 0], \end{gathered} \tag{1.1}$$

where $q > 0$, $J = [0, b]$, $A(t)$ is closed linear operator on X with a dense domain $D(A)$ which is independent of t, $x_0 \in X$, and $x_t : [-q, 0] \to X$ defined by $x_t(\theta) = x(t+\theta)$ for $\theta \in [-q, 0]$ and $x \in C(J, X)$, $f : J \times C([-q, 0], X) \times X \times X \to X$, $K : \Delta \times C([-q, 0], X) \to X$, $\Delta = \{(t, s) \in J \times J : s \leq t\}$, $H : J \times J \times C([-q, 0], X) \to X$, $g : C([0, b], X) \to C([-q, 0], X)$, $\phi : [-q, 0] \to X$ are given functions.

For the existence and controllability of solutions of nonlinear integrodifferential functional evolution equations in abstract spaces, there are many research results, see [1–13] and their references. However, in order to obtain existence and controllability of mild solutions in these study papers, usually, some restricted conditions on a priori estimation and compactness conditions of evolution operator are used. Recently, Xu [6] studied existence of mild solutions of the following nonlinear integrodifferential evolution system with equicontinuous semigroup:

$$\begin{gathered}(Ex(t))' + Ax(t) = f\left(t, x(\sigma_1(t)), \int_0^t k(t,s)h(s, x(\sigma_2(s))ds)\right), \quad t \in [0,b], \\ x(0) + g(x) = x_0.\end{gathered} \tag{1.2}$$

Some restricted conditions on a priori estimation and measure of noncompactness estimation:

$$\begin{gathered}\frac{(1-\alpha\beta Mc)N}{\alpha\beta M(d + \|x_0\|) + \alpha M\|\theta_1\|_{L^1}\Omega_1(N + K\|\theta_2\|_{L^1}\Omega_2(N))} > 1, \\ 2\alpha\|\theta_1\|_{L^1} M(1 + 2K\|\theta_2\|_{L^1}) < 1\end{gathered} \tag{1.3}$$

are used, and some similar restricted conditions are used in [14, 15]. But estimations (3.15) and (3.21) in [15] seem to be incorrect. Since spectral radius $\sigma(B) = 0$ of linear Volterra integral operator $(Bx)(t) = \int_0^t k(t,s)x(s)ds$, in order to obtain the existence of solutions for nonlinear Volterra integrodifferential equations in abstract spaces by using fixed point theory, usually, some restricted conditions on a priori estimation and measure of noncompactness estimation will not be used even if the infinitesimal generator $A = 0$.

In this paper, using Mönch fixed point theorem, we investigate the existence and controllability of mild solution of nonlinear Volterra-Fredholm integrodifferential system (1.1), some restricted conditions on a priori estimation and measure of noncompactness estimation have been deleted, our results extend and improve the corresponding results in papers [2–20].

2. Preliminaries

Let $(X, \|\cdot\|)$ be a real Banach space and let $C([a,b],X)$ be a Banach space of all continuous X-valued functions defined on $[a,b]$ with norm $\|x\|_{[a,b]} = \sup_{t\in[a,b]}\|x(t)\|$ for $x \in C([a,b],X)$. $B(X)$ denotes the Banach space of bounded linear operators from X into itself.

Definition 2.1. The family of linear bounded operators $\{R(t,s) : 0 \le s \le t < +\infty\}$ on X is said an evolution system, if the following properties are satisfied:

(i) $R(t,t) = I$, where I is the identity operator in X;

(ii) $R(t,s)R(s,\tau) = R(t,\tau)$ for $0 \le s \le t < +\infty$;

(iii) $R(t,s) \in B(X)$ the space of bounded linear operator on X, where for every $(t,s) \in \{(t,s) : 0 \le s \le t < +\infty\}$ and for each $x \in X$, the mapping $(t,s) \to R(t,s)x$ is continuous.

The evolution system $R(t,s)$ is said to be equicontinuous if for all bounded set $Q \subset X$, $\{s \to R(t,s)x : x \in Q\}$ is equicontinuous for $t > 0$. $x \in C([-q,b],X)$ is said to be a mild solution of the nonlocal problem (1.1), if $x(t) = \phi(t) + g(x)(t)$ for $t \in [-q,0]$, and, for $t \in J$, it satisfies the following integral equation:

$$x(t) = R(t,0)\left[\phi(0) + g(x)(0)\right] + \int_0^t R(t,s)f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)ds. \tag{2.1}$$

The following lemma is obvious.

Lemma 2.2. *Let the evolution system $R(t,s)$ be equicontinuous. If there exists a $\rho \in L^1[J,\mathbb{R}^+]$ such that $\|x(t)\| \le \rho(t)$ for a.e. $t \in J$, then the set $\{\int_0^t R(t,s)x(s)ds\}$ is equicontinuous.*

Lemma 2.3 (see [21]). *Let $V = \{x_n\} \subset L^1([a,b],X)$. If there exists $\sigma \in L^1([a,b],\mathbb{R}^+)$ such that $\|x_n(t)\| \le \sigma(t)$ for any $x_n \in V$ and a.e. $t \in [a,b]$, then $\alpha(V(t)) \in L^1([a,b],\mathbb{R}^+)$ and*

$$\alpha\left(\left\{\int_0^t x_n(s)ds : n \in \mathbb{N}\right\}\right) \le 2\int_0^t \alpha(V(s))ds, \quad t \in [a,b]. \tag{2.2}$$

Lemma 2.4 (see [22]). *Let $V \subset C([a,b],X)$ be an equicontinuous bounded subset. Then $\alpha(V(t)) \in C([a,b],\mathbb{R}^+)$ ($\mathbb{R}^+ = [0,\infty)$), $\alpha(V) = \max_{t\in[a,b]}\alpha(V(t))$.*

Lemma 2.5 (see [23]). *Let X be a Banach space, Ω a closed convex subset in X, and $y_0 \in \Omega$. Suppose that the operator $F : \Omega \to \Omega$ is continuous and has the following property:*

$$V \subset \Omega \text{ countable}, \quad V \subset \overline{\mathrm{co}}\left(\{y_0\} \cup F(V)\right) \Longrightarrow V \text{ is relatively compact}. \tag{2.3}$$

Then F has a fixed point in Ω.

Let $V(t) = \{x(t) : x \in C([-q,b],X)\} \subset X$ $(t \in J)$, $V_t = \{x_t : x \in C([-q,b],X)\} \subset C([-q,0],X)$, $\alpha(\cdot)$ and $\alpha_C(\cdot)$ denote the Kuratowski measure of noncompactness in X and $C([-q,b],X)$, respectively. For details on properties of noncompact measure, see [22].

3. Existence Result

We make the following hypotheses for convenience.

(H_1) $g : C([0,b],X) \to C([-q,0],X)$ is continuous, compact and there exists a constant N such that $\|g(x)\|_{[-q,0]} \le N$.

(H_2) (1) $f : J \times C([-q,0],X) \times X \times X \to X$ satisfies the Carathodory conditions, that is, $f(\cdot,x,y,z)$ is measurable for each $x \in C([-q,b],X)$, $y,z \in X$, $f(t,\cdot,\cdot,\cdot)$ is continuous for a.e. $t \in J$.

(2) There is a bounded measure function $p : J \to \mathbb{R}^+$ such that

$$\left\|f(t,x,y,z)\right\| \le p(t)\left(\|x\|_{[-q,0]} + \|y\| + \|z\|\right), \quad \text{a.e. } t \in J,\ x \in C\left([-q,0],X\right),\ y,z \in X. \tag{3.1}$$

(H_3) (1) For each $x \in C([-q,0],X), K(\cdot,\cdot,x), H(\cdot,\cdot,x) : J \times J \to X$ are measurable and $K(t,s,\cdot), H(t,s,\cdot) : C([-q,0],X) \to X$ is continuous for a.e. $t,s \in J$.

(2) For each $t \in (0,b]$, there are nonnegative measure functions $k(t,\cdot), h(t,\cdot)$ on $[0,b]$ such that

$$\begin{aligned} \|K(t,s,x)\| &\le k(t,s)\|x\|_{[-q,0]}, \quad (t,s) \in \Delta,\ x \in C([-q,0],X), \\ \|H(t,s,x)\| &\le h(t,s)\|x\|_{[-q,0]}, \quad t,s \in J,\ x \in C([-q,0],X), \end{aligned} \tag{3.2}$$

and $\int_0^t k(t,s)ds, \int_0^b h(t,s)ds$ are bounded on $[0,b]$.

(H_4) For any bounded set $V_1 \subset C([-q,0],X), V_2, V_3 \subset X$, there is bounded measure function $l_i \in C[J,\mathbb{R}^+]$ $(i = 1,2,3)$ such that

$$\begin{gathered} \alpha(f(t,V_1,V_2,V_3)) \le l_1(t) \sup_{-q\le\theta\le0} \alpha(V_1(\theta)) + l_2(t)\alpha(V_2) + l_3(t)\alpha(V_3), \quad a.e.\ t \in J, \\ \alpha\left(\int_0^t K(t,s,V_1)ds\right) \le k(t,s) \sup_{-q\le\theta\le0} \alpha(V_1(\theta)), \quad t \in J, \\ \alpha\left(\int_0^b H(t,s,V_1)ds\right) \le h(t,s) \sup_{-q\le\theta\le0} \alpha(V_1(\theta)),\ t \in J. \end{gathered} \tag{3.3}$$

(H_5) The resolvent operator $R(t,s)$ is equicontinuous and there are positive numbers $M \ge 1$ and

$$w = \max\left\{Mp_0(1 + k_0 + h_0) + 1,\ 2M\left(l_1^0 + 2l_2^0k_0 + 2l_3^0h_0\right) + 1\right\}, \tag{3.4}$$

such that $\|R(t,s)\| \le Me^{-w(t-s)}, 0 \le s \le t \le b$, where $k_0 = \sup_{(t,s)\in\Delta} \int_0^t k(t,s)ds$, $h_0 = \sup_{t,s\in J} \int_0^b h(t,s)ds, p_0 = \sup_{t\in J}p(t)$, $l_i^0 = \sup_{t\in J}l_i(t)$ $(i = 1,2,3)$.

Theorem 3.1. *Let conditions* (H_1)–(H_5) *be satisfied. Then the nonlocal problem* (1.1) *has at least one mild solution.*

Proof. Define an operator $F : C([-q,b],X) \to C([-q,b],X)$ by

$$(Fx)(t) = \begin{cases} \phi(t) + g(x)(t), & t\in[-q,0], \\ R(t,0)[\phi(0) + g(x)(0)] \\ \quad + \displaystyle\int_0^t R(t,s)f\left(s,x_s,\int_0^s K(s,r,x_r)dr,\int_0^b H(s,r,x_r)dr\right)ds, & t \in [0,b]. \end{cases} \tag{3.5}$$

We have by (H_1)–(H_3) and (H_5),

$$
\begin{aligned}
\|(Fx)(t)\| &\leq \|\phi\|_{[-q,0]} + N \leq M\Big(\|\phi\|_{[-q,0]} + N\Big) =: L, \quad t \in [-q,0], \\
\|(Fx)(t)\| &\leq L + M\int_0^t e^{w(s-t)} \left\| f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)\right\| ds \\
&\leq L + M\int_0^t e^{w(s-t)} p(s)\left(\|x_s\|_{[-q,0]} + \int_0^s \|K(s,r,x_r)\|dr + \int_0^b \|H(s,r,x_r)\|dr\right)ds \\
&\leq L + Mp_0\int_0^t e^{w(s-t)}\left(\|x_s\|_{[-q,0]} + \int_0^s k(s,r)\|x_r\|_{[-q,0]}dr + \int_0^b h(s,r)\|x_r\|_{[-q,0]}dr\right)ds \\
&\leq L + Mp_0(1+k_0+h_0)w^{-1}\|x\|_{[-q,b]}, \quad t \in [0,b].
\end{aligned}
\tag{3.6}
$$

Consequently,

$$
\|(Fx)(t)\| \leq L + Mp_0(1+k_0+h_0)w^{-1}\|x\|_{[-q,b]} = L + \eta\|x\|_{[-q,b]}, \quad t \in [-q,b], \tag{3.7}
$$

where $0 < \eta = Mp_0(1+k_0+h_0)w^{-1} < 1$. Taking $R > L(1-\eta)^{-1}$, let

$$
B_R = \left\{ x \in C([-q,b],X) : \|x\|_{[-q,b]} \leq R \right\}. \tag{3.8}
$$

Then B_R is a closed convex subset in $C([-q,b],X)$, $0 \in B_R$ and $F : B_R \to B_R$. Similar to the proof in [14, 24], it is easy to verify that F is a continuous operator from B_R into B_R. For $x \in B_R$, $s \in [0,b]$, (H_2) and (H_3) imply

$$
\left\| f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)\right\| \leq p(s)(1+k_0+h_0)R. \tag{3.9}
$$

We can show that from (H_5), (3.9) and Lemma 2.2 that $F(B_R)$ is an equicontinuous in $C([-q,b],X)$.

Let $V \subset B_R$ be a countable set and

$$
V \subset \overline{\text{co}}(\{0\} \cup (FV)). \tag{3.10}
$$

From equicontinuity of $F(B_R)$ and (3.10), we know that V is an equicontinuous subset in $C([-q,b],X)$. By (H_1), it is easy to see that $\alpha((FV)(t)) = 0$, $t \in [-q,0]$. By properties of non-compact measure, (H_4) and Lemma 2.3, we have

$$\begin{aligned}
\alpha((FV)(t)) &\le 2\int_0^t \|R(t-s)\|\alpha\left(f\left(s, V_s, \int_0^s K(s,r,V_r)dr, \int_0^b H(s,r,V_r)dr\right)\right)ds \\
&\le 2M\int_0^t e^{w(s-t)}\left[l_1(s)\sup_{-q\le\theta\le 0}\alpha(V_s(\theta)) + l_2(s)\alpha\left(\int_0^s K(s,r,V_r)dr\right)\right. \\
&\qquad \left. + l_3(s)\alpha\left(\int_0^b H(s,r,V_r)dr\right)\right]ds \\
&\le 2M\int_0^t e^{w(s-t)}\left[l_1^0\sup_{-q\le\theta\le 0}\alpha(V(s+\theta)) + 2l_2^0\int_0^s k(s,r)\sup_{-q\le\theta\le 0}\alpha(V(r+\theta))dr\right. \\
&\qquad \left. + 2l_3^0\int_0^b h(s,r)\sup_{-q\le\theta\le 0}\alpha(V(r+\theta))dr\right]ds \\
&\le 2M\left(l_1^0 + 2l_2^0k_0 + 2l_3^0h_0\right)\int_0^t e^{w(s-t)}ds\sup_{-q\le\tau\le b}\alpha(V(\tau)) \\
&\le 2M\left(l_1^0 + 2l_2^0k_0 + 2l_3^0h_0\right)w^{-1}\alpha_{C}(V), \quad t \in [0,b].
\end{aligned} \tag{3.11}$$

Consequently,

$$\alpha_C(FV) = \sup_{-q\le t\le b}\alpha((FV)(t)) \le 2M\left(l_1^0 + 2l_2^0k_0 + 2l_3^0h_0\right)w^{-1}\alpha_C(V). \tag{3.12}$$

Equations (3.10), (3.12), and Lemma 2.4 imply

$$\alpha_C(V) \le \alpha_C(FV) \le \delta\alpha_C(V), \tag{3.13}$$

where $\delta = 2M(l_1^0 + 2l_2^0k_0 + 2l_3^0h_0)w^{-1} < 1$. Hence $\alpha_C(V) = 0$ and V is relative compact in $C([-q,b],X)$. Lemma 2.5 implies that F has a fixed point in $C([-q,b],X)$, then the system (1.1), (1.2) has at least one mild solution. The proof is completed. □

4. An Example

In this section, we give an example to illustrate Theorem 3.1.

Let $X = L^2([0,\pi],\mathbb{R})$. Consider the following functional integrodifferential equation with nonlocal condition:

$$\begin{aligned}
u_t(t,y) &= a_1(t,y)u_{yy}(t,y) \\
&\quad + a_2(t)\left[\sin u(t+\theta,y)ds + \int_0^t\int_{-q}^s a_3(s+\tau)\frac{u(\tau,y)d\tau ds}{(1+t)} + \int_0^b \frac{u(s+\theta,y)ds}{(1+t)(1+s)^2}\right], \\
&\qquad 0 \le t \le b, \\
u(t,y) &= \phi(t,y) + \int_0^\pi\int_0^b F(r,y)\log\left(1+|u(r,s)|^{1/2}\right)dr\,ds, \quad -q \le t \le 0,\ 0 \le y \le \pi, \\
&u(t,0) = u(t,\pi) = 0, \quad 0 \le t \le b,
\end{aligned} \tag{4.1}$$

where functions $a_1(t,y)$ is continuous on $[0,b]\times[0,\pi]$ and uniformly Hölder continuous in t, $a_2(t)$ is bounded measure on $[0,b]$, $\phi : [-q,0]\times[0,\pi]$, $a_3 : [-q,b]$, and $F : [0,b]\times[0,\pi]$ are continuous, respectively. Taking $u(t,y) = u(t)(y)$, $\phi(t,y) = \phi(t)(y)$,

$$\begin{aligned}
&f\left(t,u_t,\int_0^t K(t,s,u_s)ds,\int_0^b H(t,s,u_s)ds\right)(y) \\
&\quad = a_2(t)\left[\sin u(t+\theta,y) + \int_0^t\int_{-q}^s a_3(s+\tau)\frac{u(\tau,y)d\tau ds}{(1+t)} + \int_0^b \frac{u(s+\theta,y)ds}{(1+t)(1+s)^2}\right], \\
&K(t,s,u_s)(y) = \int_{-q}^s a_3(s+\tau)\frac{u(\tau,y)d\tau}{(1+t)}, \quad H(t,s,u_s)(y) = \frac{u(s+\theta,y)}{(1+t)(1+s)^2}, \\
&g(u)(y) = \int_0^\pi\int_0^b F(r,y)\log\left(1+|u(r,s)|^{1/2}\right)dr\,ds.
\end{aligned} \tag{4.2}$$

The operator A defined by $A(t)w = a_1(t,y)w''$ with the domain

$$D(A) = \{w \in X : w, w' \text{ are absolutely continuous, } w'' \in X,\ w(0) = w(\pi) = 0\}. \tag{4.3}$$

Then $A(t)$ generates an evolution system, and $R(t,s)$ can be deduced from the evolution systems so that $R(t,s)$ is equicontinuous and $\|R(t,s)\| \le Me^{\beta(t-s)}$ for some constants M and β (see [24, 25]). The system (4.1) can be regarded as a form of the system (1.1), (1.2). We have by (4.2)

$$\begin{aligned}
&\|f(t,u,v,z)\| \le |a_2(t)|\left(\|u\|_{[-q,0]} + \|v\| + \|z\|\right), \\
&\|K(t,s,u)\| \le \int_{-q}^s |a_3(s+\tau)|d\tau\frac{\|u\|_{[-q,0]}}{(1+t)}, \quad \|H(t,s,u)\| \le \frac{\|u\|_{[-q,0]}}{(1+t)(1+s)}
\end{aligned} \tag{4.4}$$

for $u \in C([-q,0],X), v, z \in X$,

$$\|g(u)\|_{[-q,0]} \le b\pi \max_{(r,y)\in[0,b]\times[0,\pi]} |F(r,y)| \left(\|u\|_{[0,b]} + \sqrt{\pi}\right), \tag{4.5}$$

and $g : C([0,b],X) \to C([-q,b],X)$ is continuous and compact (see the example in [7]). $w > 0$ and $M \ge 1$ can be chosen such that $\|R(t,s)\| \le Me^{w(t-s)}, 0 \le s \le t \le b$. In addition, for any bounded set $V_1 \subset C([-q,0],X), V_2, V_3 \subset X$, we can show that by the diagonal method

$$\begin{gathered}
\alpha(f(t,V_1,V_2,V_3)) \le |a_2(t)| \left(\sup_{-q\le\theta\le 0} \alpha(V_1(\theta)) + \alpha(V_2) + \alpha(V_3) \right), \quad t \in J, \\
\alpha(K(t,s,V_1)) \le \frac{1}{1+t} \sup_{-q\le\theta\le 0} \alpha(V_1(\theta)), \quad t,s \in \Delta, \\
\alpha(H(t,s,V_1)) \le \frac{1}{(1+t)(1+s)^2} \sup_{-q\le\theta\le 0} \alpha(V_1(\theta)), \quad t,s \in [0,b].
\end{gathered} \tag{4.6}$$

It is easy to verify that all conditions of Theorem 3.1 are satisfied, so the system (5.1) has at least one mild solution.

5. An Application

As an application of Theorem 3.1, we shall consider the following system with control parameter:

$$\begin{gathered}
x'(t) = A(t)x(t) + f\left(t, x_t, \int_0^t K(t,s,x_s)ds, \int_0^b H(t,s,x_s)ds\right), \quad t \in J, \\
x_0 = \phi + g(x), \quad t \in [-q,0],
\end{gathered} \tag{5.1}$$

where C is a bounded linear operator from a Banach space U to X and $v \in L^2(J,U)$. Then the mild solution of systems (5.1) is given by

$$x(t) = \begin{cases} \phi(t) + g(x)(t), & t \in [-q,0], \\ R(t,0)[\phi(0) + g(x)(0)] + \displaystyle\int_0^t R(t,s)(Cv)(s)ds & \\ \quad + \displaystyle\int_0^t R(t,s) f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)ds, & t \in [0,b], \end{cases} \tag{5.2}$$

where the resolvent operator $R(t,s) \in B(X)$, f, K, H, g, and ϕ satisfy the conditions stated in Section 3.

Definition 5.1. The system (5.2) is said to be controllable on $J = [0,b]$, if for every initial function $\phi \in C([-q,0],X)$ and $x_1 \in X$ there is a control $v \in L^2(J,U)$ such that the mild solution $x(t)$ of the system (5.1) satisfies $x(b) = x_1$.

To obtain the controllability result, we need the following additional hypotheses.

(H_5') The resolvent operator $R(t,s)$ is equicontinuous and $\|R(t,s)\| \leq Me^{-w(t-s)}$, $0 \leq s \leq t \leq b$, for $M \geq 1$ and positive number

$$w = \max\left\{ Mp_0(1+k_0+h_0)(1+MM_1b)+1, 2M\left(l_1^0+2l_2^0k_0+2l_3^0h_0\right)(1+MM_1b)+1 \right\}, \quad (5.3)$$

where k_0, h_0, p_0, l_i^0 $(i=1,2,3)$ are as before.

(H_6) The linear operator W from $L^2(J,U)$ into X, defined by

$$Wv = \int_0^b R(b,s)(Cv)(s)ds, \quad (5.4)$$

has an inverse operator W^{-1}, which takes values in $L^2(J,U)/\ker W$ and there exists a positive constant M_1 such that $\|CW^{-1}\| \leq M_1$.

Theorem 5.2. *Let the conditions* (H_1)–(H_4), (H_5') *and* (H_6) *be satisfied. Then the nonlocal problem* (1.1), (1.2) *is controllable.*

Proof. Using hypothesis (H_6), for an arbitrary $x(\cdot)$, define the control

$$\begin{aligned} v(t) = W^{-1}\Bigg(& x_1 - R(b,0)[\phi(0)+g(x)(0)] \\ & + \int_0^b R(b,s) f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)ds\Bigg)(t), \quad t \in [0,b]. \end{aligned} \quad (5.5)$$

Define the operator $T : C([-q,b],X) \to C([-q,b],X)$ by

$$(Tx)(t) = \begin{cases} \phi(t) + g(x)(t), & t \in [-q,0], \\ R(t,0)[\phi(0)+g(x)(0)] + \displaystyle\int_0^t R(t,s)(Cv)(s)ds & \\ \quad + \displaystyle\int_0^t R(t,s) f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)ds, & t \in [0,b]. \end{cases} \quad (5.6)$$

Now we show that, when using this control, T has a fixed point. Then this fixed point is a solution of the system (5.1). Substituting $v(t)$ in (5.6), we get

$$(Tx)(t) = \begin{cases} \phi(t) + g(x)(t), & t \in [-q,0], \\ R(t,0)[\phi(0)+g(x)(0)] + \displaystyle\int_0^t R(t,s)CW^{-1}\big(x_1 - R(b,0)[\phi(0)+g(x)(0)] & \\ \quad + \displaystyle\int_0^b R(b,\tau) f\left(\tau, x_\tau, \int_0^\tau K(\tau,r,x_r)dr, \int_0^b H(\tau,r,x_r)dr\right)d\tau\Big)(s)ds & \\ \quad + \displaystyle\int_0^t R(t,s) f\left(s, x_s, \int_0^s K(s,r,x_r)dr, \int_0^b H(s,r,x_r)dr\right)ds, & t \in [0,b]. \end{cases} \quad (5.7)$$

Clearly, $(Tx)(b) = x_1$, which means that the control v steers the system (5.1) from the given initial function ϕ to the origin in time b, provided we can obtain a fixed point of nonlinear operator T. The remaining part of the proof is similar to Theorem 3.1, we omit it. □

Remark 5.3. Since the spectral radius of linear Fredholm type integral operator may be greater than 1, in order to obtain the existence of solutions for nonlinear Volterra-Fredholm type integrodifferential equations in abstract spaces by using fixed point theory, some restricted conditions on a priori estimation and measure of noncompactness estimation will not be used even if the generator $A = 0$. But, these restrictive conditions are not being used in Theorems 3.1 and 5.2.

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