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Research Article

Nonlinear Waveforms for Ion-Acoustic Waves in Weakly Relativistic Plasma of Warm Ion-Fluid and Isothermal Electrons

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The reductive perturbation method has been employed to derive the Korteweg-de Vries (KdV) equation for small- but finite-amplitude electrostatic ion-acoustic waves in weakly relativistic plasma consisting of warm ions and isothermal electrons. An algebraic method with computerized symbolic computation is applied in obtaining a series of exact solutions of the KdV equation. Numerical studies have been made using plasma parameters which reveal different solutions, that is, bell-shaped solitary pulses, rational pulses, and solutions with singularity at finite points, which called "blowup" solutions in addition to the propagation of an explosive pulses. The weakly relativistic effect is found to significantly change the basic properties (namely, the amplitude and the width) of the ion-acoustic waves. The result of the present investigation may be applicable to some plasma environments, such as ionosphere region.

1. Introduction

Nonlinear evolution equations are widely used as models to describe complex physical phenomena and have a significant role in several scientific and engineering fields [1, 2]. The propagation of solitary waves is important as it describes characteristic nature of the interaction of the waves and the plasmas. In the case where the velocity of particles is much smaller than that of light, ion-acoustic waves present the nonrelativistic behaviors, but in the case where the velocity of particles approaches that of light, the relativistic effect becomes dominant [3]. Actually high-speed and energetic streaming ions with the energy from 0.1 to 100 MeV are frequently observed in solar atmosphere and interplanetary space. Nevertheless,

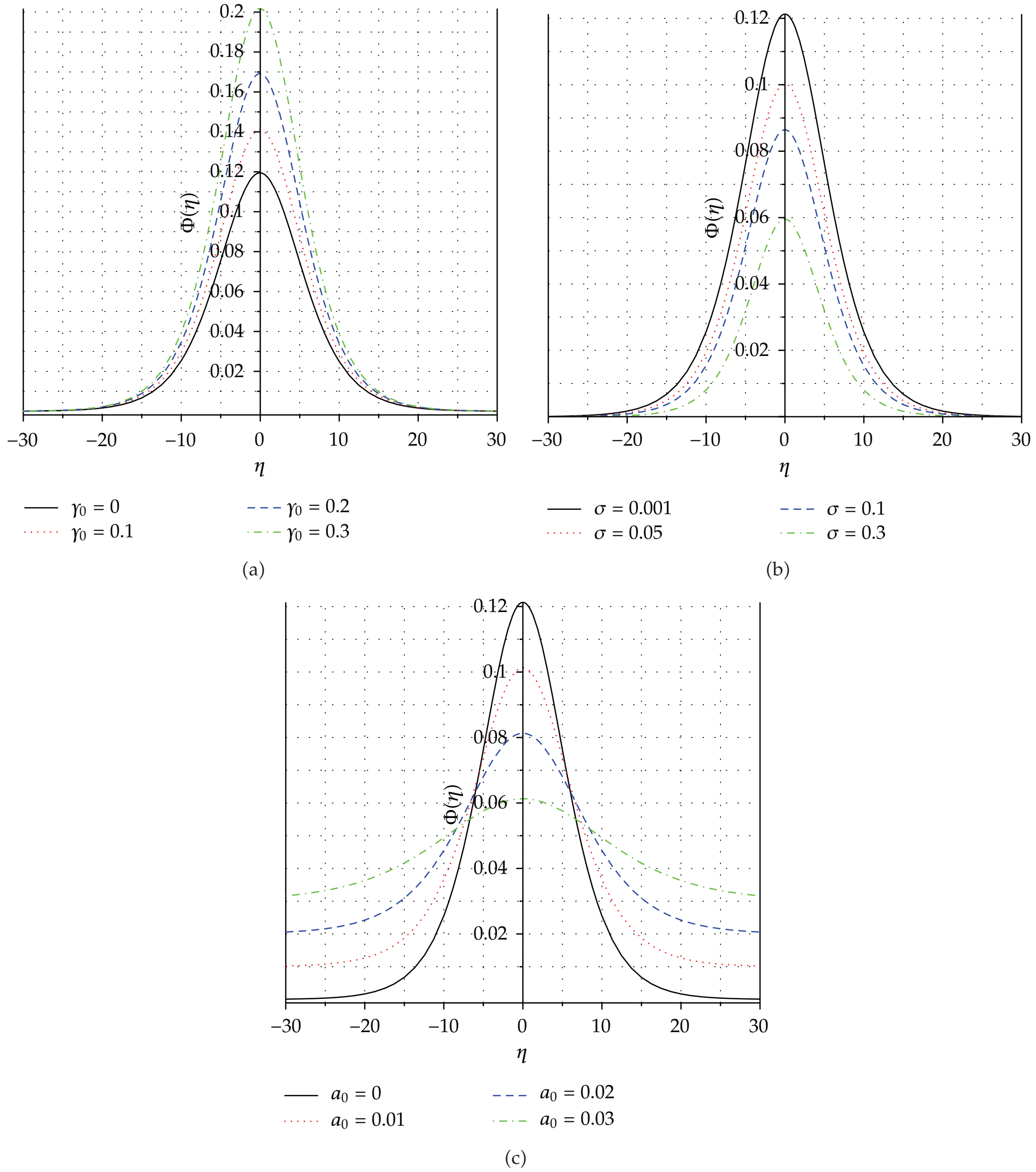
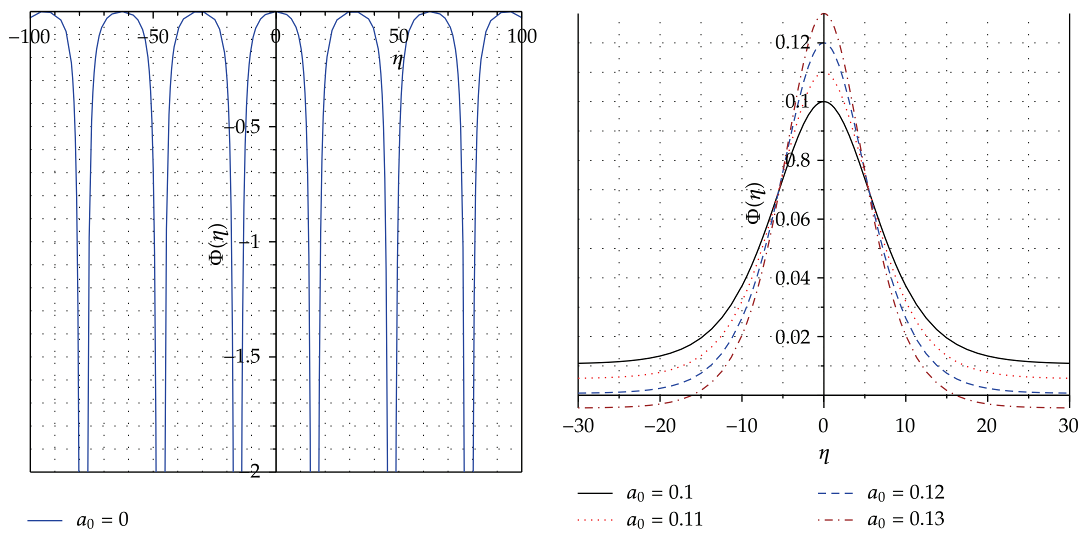


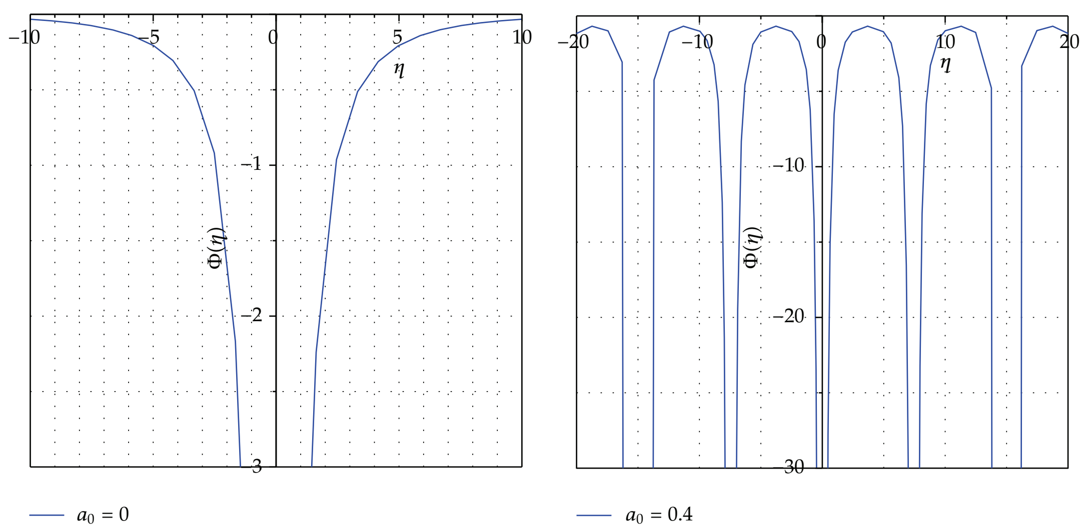
Figure 1: Profile of localized pulses for expression (4.3).

relativistic ion-acoustic waves have not been well investigated. When we assume that the ion energy depends only on the kinetic energy, such plasma ions have to attain very high velocity of relativistic order. Thus, by considering the weakly relativistic effect where the ion velocity is about $1/10$ of the velocity of light, we can describe the relativistic motion of such ions in the study of nonlinear interaction of the waves and the plasmas [4]. It appears that the weakly relativistic and ion temperature effects play an important role in energetic ion-acoustic waves propagating in interplanetary space [5, 6].



(a) (b)

Figure 2: Profile of pulses for expression (4.5) for $v = 0.04$, $\sigma = 0.001$, and $\gamma_0 = 0.01$.



(a) (b)

Figure 3: The explosive solution for expression (4.7) for $v = 0.04$, $\sigma = 0.001$, and $\gamma_0 = 0.01$.

Washimi and Taniuti [7] were the first to use reductive perturbation method to study the propagation of a slow modulation of quasi-monochromatic waves through plasma. And then the attention has been focused by many authors [8–11].

The evolution of small-but finite-amplitude solitary waves, studied by means of the Korteweg-de Vries (KdV) equation, is of considerable interest in plasma dynamics.

Many powerful methods have been established and developed to study nonlinear evolution equations (NLEEs). These methods include the inverse scatting method [12],

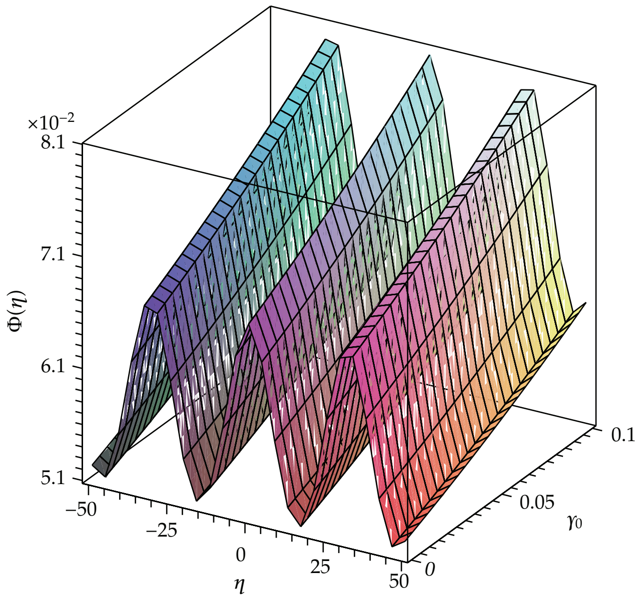


Figure 4: The asymptotical property of Jacobi elliptic doubly periodic wave for expression (4.10) with $v = 0.04$, $\sigma = 0.001$, $a_0 = 0$, and $m = 2$.

the tanh function method [13], extended tanh method [14–16], the Exp-function method [17], the extended F-expansion method [18], the Jacobi elliptic function expansion method [19], the homogeneous balance method [20], sech-function method [21], F-expansion method [22], and the multiple exp-function method [23]. There is no unified method that can be used to deal with all types of NLEEs. Fan [24] developed a new algebraic method with computerized symbolic computation, which greatly exceeds the applicability of the existing tanh, extended tanh methods, and Jacobi function expansion method in obtaining a series of exact solutions of nonlinear differential equations.

Recently, the ion-acoustic solitary wave in collisionless unmagnetized plasma consisting of warm ions fluid and isothermal electrons is studied using the time-fractional KdV equation by El-Wakil et al. [25]. They showed that the time fractional can be used to modulate the electrostatic potential wave.

The major topic of this work is to study the ion-acoustic solitary and other type waves in relativistic warm plasma. This paper is organized as follows. In Section 2, we present the basic set of fluid equations governing our plasma model. In Section 3, an algorithm describing the computerized symbolic computation method is presented. In Section 4, explicit solutions for KdV equation are obtained. Finally, some discussions and conclusions are given in Section 5.

2. Basic Equations and KdV Equation

Consider collisionless ionization-free unmagnetized plasma consisting of a mixture of warm ion-fluid and isothermal electrons. Assume that the ion flow velocity has a weak relativistic effect, and therefore there exist streaming ions in an equilibrium state when sufficiently

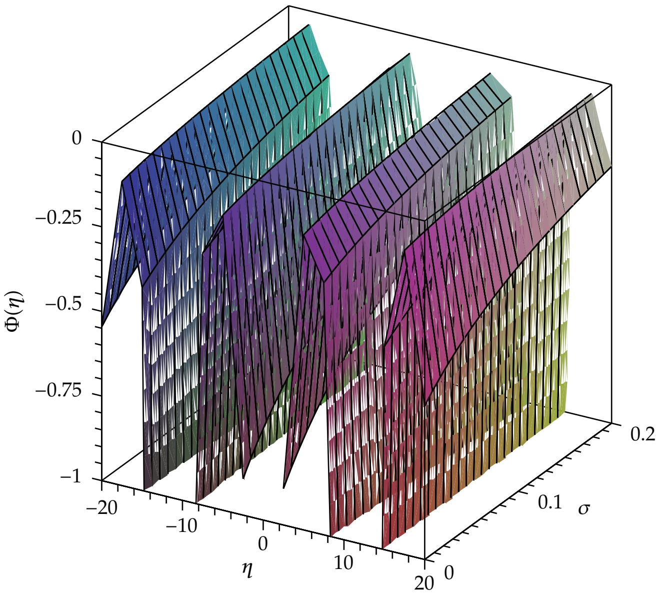


Figure 5: Weierstrass elliptic doubly periodic type solution (4.13) for $v = 0.04$, $\gamma_0 = 0.01$, $c_0 = -1$, $c_1 = 1$, and $c_3 = 0.1$.

small- but finite-amplitude waves propagate one-dimensionally. Such a system is governed by the following normalized equations [26]:

$$\frac{\partial}{\partial t}n(x,t) + \frac{\partial}{\partial x}[n(x,t)\ u(x,t)] = 0, \tag{2.1a}$$

$$\left[\frac{\partial}{\partial t} + u(x,t)\frac{\partial}{\partial x}\right][\gamma(x,t)\ u(x,t)] + \frac{\sigma}{n(x,t)}\frac{\partial}{\partial x}p(x,t) + \frac{\partial}{\partial x}\phi(x,t) = 0, \tag{2.1b}$$

$$\left[\frac{\partial}{\partial t} + u(x,t)\frac{\partial}{\partial x}\right]p(x,t) + 3p(x,t)\frac{\partial}{\partial x}[\gamma(x,t)\ u(x,t)] = 0, \tag{2.1c}$$

$$\frac{\partial^2}{\partial x^2}\phi(x,t) + n(x,t) - n_e(x,t) = 0. \tag{2.1d}$$

The electron temperature T_e is much larger than the ion temperature T_i and in this case for simplicity one can neglect the inertia of the electrons relative to that of the ions, that is, the high-frequency plasma oscillations are neglected. Since it is interested with the regime of density and velocity fluctuations near the ion plasma frequency, so the isothermal electrons density is given by

$$n_e(x,t) = \exp[\phi(x,t)]. \tag{2.1e}$$

For weakly relativistic effects, the relativistic factor $\gamma(x,t) = 1/\sqrt{1-u^2(x,t)/c^2}$ is approximated by

$$\gamma(x,t) \approx 1 + \frac{u^2(x,t)}{(2c^2)}, \tag{2.1f}$$

where c is the velocity of light.

In (2.1a)–(2.1f), $n(x,t)$ and $n_e(x,t)$ are the densities of ions and electrons, respectively, $u(x,t)$ is the ion flow velocity, $p(x,t)$ is the ion pressure, $\phi(x,t)$ is the electric potential, x is the space coordinate, and t is the time variable. $\sigma = T_i/T_e \ll 1$ is the ratio of the ion temperature to the electron temperature. All of these quantities are dimensionless and are normalized in terms of the following characteristic quantities: $n(x,t)$ and $n_e(x,t)$ by the unperturbed electron density n_0, $u(x,t)$ and c by the sound velocity $\sqrt{k_B T_e/m_i}$, $p(x,t)$ and $\phi(x,t)$ by $n_0 k_B T_i$ and $k_B T_e/e$, respectively; t and x by the inverse of the plasma frequency $\omega_{pi}^{-1} = 1/\sqrt{4\pi\, e^2 n_0/m_i}$ and the electron Debye length $\lambda_D = \sqrt{k_B T_e/(4\pi\, e^2 n_0)}$, respectively, k_B is Boltzmann's constant, and m_i is the mass of plasma ion.

According to the general method of reductive perturbation theory, the stretched variables are introduced as [7]

$$\tau = \varepsilon^{3/2} t, \qquad \xi = \varepsilon^{1/2}(x - \lambda t), \tag{2.2}$$

where λ is the phase velocity and ε represents the amplitude of the perturbation. All the physical quantities that appeared in (2.1a)–(2.1f) are expanded as power series in ε about the equilibrium values as

$$n(\xi,\tau) = 1 + \varepsilon n_1(\xi,\tau) + \varepsilon^2 n_2(\xi,\tau) + \cdots, \tag{2.3a}$$

$$u(\xi,\tau) = u_0 + \varepsilon u_1(\xi,\tau) + \varepsilon^2 u_2(\xi,\tau) + \cdots, \tag{2.3b}$$

$$p(\xi,\tau) = 1 + \varepsilon p_1(\xi,\tau) + \varepsilon^2 p_2(\xi,\tau) + \cdots, \tag{2.3c}$$

$$\phi(\xi,\tau) = \varepsilon \phi_1(\xi,\tau) + \varepsilon^2 \phi_2(\xi,\tau) + \cdots. \tag{2.3d}$$

The boundary conditions of this problem are imposed as $|\xi| \to \infty$, $n = n_e = p = 1$, $u = u_0$, and $\phi = 0$.

Substituting (2.2) and (2.3a)–(2.3d) into the system of (2.1a)–(2.1f) and equating the coefficients of like powers of ε, then from the lowest order

$$n_1(\xi,\tau) = \phi_1(\xi,\tau), \tag{2.4a}$$

$$u_1(\xi,\tau) = (\lambda - u_0)\phi_1(\xi,\tau), \tag{2.4b}$$

$$p_1(\xi,\tau) = 3\gamma_1 \phi_1(\xi,\tau), \tag{2.4c}$$

with the transcendental equation of λ that is given as

$$1 - \left(\lambda^2 - 3\sigma\right)\gamma_1 = 0 \Longrightarrow \lambda = \pm\sqrt{\frac{(3\sigma\gamma_1 + 1)}{\gamma_1}}, \tag{2.4d}$$

where

$$\gamma_1 = 1 + \frac{3\gamma_0^2}{2}, \qquad \gamma_0 = \frac{u_0}{c}. \tag{2.4e}$$

Using second-order equations in ε and eliminating the second-order perturbed quantities n_2, u_2, p_2, and ϕ_2, the following KdV equation for the first-order perturbed potential is obtained:

$$\frac{\partial}{\partial\tau}\phi_1(\xi,\tau) + A\phi_1(\xi,\tau)\frac{\partial}{\partial\xi}\phi_1(\xi,\tau) + B\frac{\partial^3}{\partial\xi^3}\phi_1(\xi,\tau) = 0. \tag{2.5a}$$

Therefore, the nonlinear coefficient A and the dispersion coefficient B are represented by

$$A = B\left(g_1 - \frac{g_2\gamma_2}{\sqrt{\gamma_1^3}}\right), \qquad B = \frac{1}{g_2\sqrt{\gamma_1}}, \tag{2.5b}$$

with

$$g_1 = 3\sigma\gamma_1(3\gamma_1 + 1) + 2, \qquad g_2 = 2\sqrt{3\sigma\gamma_1 + 1}, \qquad \gamma_2 = \frac{3\gamma_0}{(2c)}. \tag{2.5c}$$

3. Computerized Symbolic Computation Method

An algebraic method with computerized symbolic computation has been developed by Fan [24], which can be used to solve a given partial differential equation in $\phi_1(\xi,\tau)$ of the form

$$H\left(\phi_1, \frac{\partial\phi_1}{\partial\tau}, \frac{\partial\phi_1}{\partial\xi}, \frac{\partial^2\phi_1}{\partial\xi^2}, \ldots\right) = 0. \tag{3.1a}$$

This equation may be transformed into an ordinary differential equation of the form

$$H\left(\Phi, \frac{d\Phi}{d\eta}, \frac{d^2\Phi}{d\eta^2}, \ldots\right) = 0 \tag{3.1b}$$

using a traveling frame of reference

$$\phi_1(\xi,\tau) = \Phi(\eta), \quad \eta = \xi - v\tau, \tag{3.1c}$$

where v is the traveling wave propagation velocity.

The computational technique used to solve the NLEE is described as follows [24].

Step 1. Reduce partial differential equation (3.1a) to the ordinary differential equation (3.1b) by considering the traveling wave transformation (3.1c).

Step 2. Expand the solution of (3.1b) in the form

$$\Phi(\eta) = \sum_{i=0}^{n} a_i \varphi^i(\eta), \tag{3.2a}$$

where the new variable $\varphi(\eta)$ is a solution of the following ordinary differential equation:

$$\frac{d\varphi(\eta)}{d\eta} = \in \sqrt{\sum_{j=0}^{r} c_j \varphi^j(\eta)}, \quad \in = \pm 1. \tag{3.2b}$$

Step 3. Substituting (3.2b) into (3.1b) and balancing the highest derivative term with the highest nonlinear term lead to a relation between n and r, from which the different possible values of n and r can be obtained. These values lead to the different series expansions of the solutions.

Step 4. Substituting the expansions (3.2a)–(3.2b) into (3.1b) and setting the coefficients of all powers of φ^i and $\varphi^i d\varphi/d\eta$ to zero will give a system of algebraic equations, from which the parameters a_i $(i = 0, 1, \ldots, n)$ and c_j $(j = 0, 1, \ldots, r)$ can be found explicitly.

Step 5. Substituting the parameters c_j $(j = 0, 1, \ldots, r)$ obtained in Step 4 into (3.2b) gives all the possible solutions $\varphi(\eta)$.

It is remarked here that the solutions of (3.1a) depend on the explicit solvability of (3.2b). The solutions of (3.2b) will get a series of fundamental solutions such as polynomial, exponential, soliton, rational, triangular periodic, Jacobi, and Weierstrass elliptic doubly periodic solutions.

4. Explicit Solutions for the KdV Equation

For KdV equation (2.5a), the traveling wave transformation (3.1c) leads to

$$-v\frac{d}{d\eta}\Phi(\eta) + A\Phi(\eta)\frac{d}{d\eta}\Phi(\eta) + B\frac{d^3}{d\eta^3}\Phi(\eta) = 0. \tag{4.1}$$

Balancing the highest derivative term with the highest nonlinear term leads to a relation between n and r as $r = n + 2$. Taking $n = 2$ gives $r = 4$ and leads to

$$\Phi(\eta) = a_0 + a_1\varphi(\eta) + a_2\varphi^2(\eta), \tag{4.2a}$$

$$\frac{d\varphi}{d\eta} = \in\sqrt{c_0 + c_1\varphi(\eta) + c_2\varphi^2(\eta) + c_3\varphi^3(\eta) + c_4\varphi^4(\eta)}. \tag{4.2b}$$

Substituting (4.2a)–(4.2b) into (4.1) and equating coefficients of all powers of φ^i and $\varphi^i d\varphi/d\eta$ to zero will get a system of algebraic equations, from which the parameters a_i $(i = 0, 1, 2)$ and c_j $(j = 0, 1, \ldots, 4)$ can be found explicitly. Substituting c_j into (4.2b) and using the symbolic software package *Maple* give explicit solutions of (4.2b).

Substituting the coefficients a_i and (4.2a) into (4.1) and using the symbolic software package *Maple*, we obtain, for KdV equation (4.1), the following solutions:

$$\Phi(\eta) = a_0 - 3\frac{(Aa_0 - v)}{A}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\frac{(v - Aa_0)}{B}}\eta\right), \qquad c_0 = c_1 = c_4 = 0, \qquad \frac{(v - Aa_0)}{B} > 0, \tag{4.3}$$

$$\Phi(\eta) = a_0 - 3\frac{(Aa_0 - v)}{A}\sec^2\left(\frac{1}{2}\sqrt{\frac{(Aa_0 - v)}{B}}\eta\right), \qquad c_0 = c_1 = c_4 = 0, \qquad \frac{(v - Aa_0)}{B} < 0, \tag{4.4}$$

$$\Phi(\eta) = a_0 + \frac{3}{2}\frac{(Aa_0 - v)}{A}\tan^2\left(\frac{1}{2}\sqrt{\frac{(v - Aa_0)}{2B}}\eta\right), \qquad c_0 \neq 0,$$
$$c_1 = c_3 = 0, \qquad \frac{(v - Aa_0)}{B} > 0, \tag{4.5}$$

$$\Phi(\eta) = a_0 - \frac{3}{2}\frac{(Aa_0 - v)}{A}\tanh^2\left(\frac{1}{2}\sqrt{\frac{(Aa_0 - v)}{2B}}\eta\right), \qquad c_0 \neq 0,$$
$$c_1 = c_3 = 0, \qquad \frac{(v - Aa_0)}{B} < 0, \tag{4.6}$$

$$\Phi(\eta) = a_0 + 3\frac{(Aa_0 - v)}{A}\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\frac{(v - Aa_0)}{B}}\eta\right),$$
$$c_0 = c_1 = c_3 = 0, \qquad \frac{(v - Aa_0)}{B} > 0, \tag{4.7}$$

$$\Phi(\eta) = a_0 - 3\frac{(Aa_0 - v)}{A}\csc^2\left(\frac{1}{2}\sqrt{\frac{(Aa_0 - v)}{B}}\eta\right),$$
$$c_0 = c_1 = c_3 = 0, \qquad \frac{(v - Aa_0)}{B} < 0, \tag{4.8}$$

$$\Phi(\eta) = \frac{v}{A} - \frac{12B}{A\eta^2}, \qquad c_0 = c_1 = c_3 = 0, \qquad \frac{(v - Aa_0)}{B} = 0, \tag{4.9}$$

$$\Phi(\eta) = a_0 - 3\frac{(Aa_0 - v)}{A}\frac{m^2}{(2m^2 - 1)}\operatorname{cn}^2\left(\frac{1}{2}\sqrt{\frac{Aa_0 - v}{B(1 - 2m^2)}}\eta\right), \qquad c_0 \neq 0,$$
$$c_1 = c_3 = 0, \qquad \frac{v - a_0A}{B} > 0, \tag{4.10}$$

$$\Phi(\eta) = a_0 + 3\frac{(Aa_0 - v)}{A}\frac{1}{(m^2 - 2)}\operatorname{dn}^2\left(\frac{1}{2}\sqrt{\frac{Aa_0 - v}{B\ (m^2 - 2)}}\eta\right), \qquad c_0 \neq 0,$$
$$c_1 = c_3 = 0, \qquad \frac{v - a_0A}{B} > 0, \tag{4.11}$$

$$\Phi(\eta) = a_0 - 3\frac{(Aa_0 - v)}{A}\frac{m^2}{(m^2 + 1)}\operatorname{sn}^2\left(\frac{1}{2}\sqrt{\frac{Aa_0 - v}{B(m^2 + 1)}}\eta\right), \qquad c_0 \neq 0,$$
$$c_1 = c_3 = 0, \qquad \frac{v - a_0A}{B} < 0, \tag{4.12}$$

$$\Phi(\eta) = \frac{v}{A} - 3\sqrt[3]{4c_3^2}\,\frac{B}{A}\wp\left(\sqrt[3]{\frac{c_3}{4}}\eta; -\sqrt[3]{\frac{4}{c_3}}c_1, -c_0\right), \qquad c_2 = c_4 = 0, \qquad c_3 > 0. \tag{4.13}$$

where m is the modulus of the Jacobi elliptic functions, a_0, c_0, c_1, and c_3 are arbitrary constants, and $\wp$ is the Weierstrass elliptic doubly periodic function.

5. Results and Discussion

Solutions (4.3) and (4.6) are hyperbolic wave solutions. In Figure 1, a profile of the bell-shaped solitary pulse is obtained for solution (4.3). Figures (1(a) and 1(b)) show that the soliton amplitude and width are sensitive to the relativistic factor γ_0 and the temperatures ratio σ. Also, the arbitrary value a_0 gives the same effect of adding a higher-order perturbation correction in increasing the amplitude and decreasing the width as shown in Figure 1(c).

Solutions (4.4) and (4.5) are triangular solutions that develop solitons with singularity at finite points, which are called "blowup" solutions [27] as in Figure 2(a). In Figure 2(b), for some values of a_0 the pulse does not vanish to infinity, so it has a localized form. Therefore, the arbitrary value a_0 plays a role in obtaining a pulse-shaped localized solution [28]. Solutions (4.7) and (4.8) lead to the propagation of an explosive pulses [29]. In Figure 3, the profiles of explosive (divergent) pulses are depicted for expression (4.7). For $a_0 = 0$, the solution gives a localized explosive pulse, as in Figure (2.3a) while a periodic solution is obtained for $a_0 = 0.4$, as shown in Figure (2.3b). The rational solution (4.9) may be helpful to explain certain physical phenomena. Because a rational solution is a disjoint union of manifolds, particle systems describing the motion of a pole of rational solutions for a KdV equation were analyzed [30]. Equations (4.10)–(4.12) are three Jacobi elliptic doubly periodic wave solutions. When $m \to 1$, solutions (4.10) and (4.11) reduce to (4.3) while (4.11) reduces to (4.6) [31]. In Figure 4, a profile of triangular periodic wave solution for expression (4.10) for $m = 2$ is shown. On the other hand, (4.13) gives the Weierstrass elliptic doubly periodic type solution as depicted in Figure 5.

In summary, it has been found that the amplitude and the width of the ion-acoustic waves as well as parametric regime where the solitons can exist are sensitive to the relativistic factor γ_0 and the ratio of the ion to the electron temperatures σ. Moreover, solutions for KdV equation have been obtained. It may be important to explain some physical phenomena in some plasma environments, such as ionosphere region.

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