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Research Article

Harvesting Control for a Stage-Structured Predator-Prey Model with Ivlev's Functional Response and Impulsive Stocking on Prey

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We investigate a delayed stage-structured Ivlev's functional response predator-prey model with impulsive stocking on prey and continuous harvesting on predator. Sufficient conditions of the global attractivity of predator-extinction periodic solution and the permanence of the system are obtained. These results show that the behavior of impulsive stocking on prey plays an important role for the permanence of the system. We also prove that all solutions of the system are uniformly ultimately bounded. Our results provide reliable tactical basis for the biological resource management and enrich the theory of impulsive delay differential equations.

1. Introduction

Biological resources are renewable resources. Economic and biological aspects of renewable resources management have been considered by Clark [1]. In recent years, the optimal management of renewable resources, which has a direct relationship to sustainable development, has been studied extensively by many authors [1, 2]. Generally speaking, the exploitation of a species should be determined by the economic and biological values of the population. It is the purpose of this paper to analyze the exploitation of the stage-structured predator-prey model with harvesting on mature predator population.

In the natural world, there are many species whose individual members have a life history that takes them through two stages—immature and mature. In [3], a stage-structured model of population growth consisting of immature and mature individuals was analyzed, where the stage-structured was modeled by introduction of a constant time delay. Other population growth and infectious disease models with time delays were considered in [3–7]. For the above discussion, we investigate a delayed stage-structured

Ivlev's functional response predator-prey model with impulsive stocking on the prey and continuous harvesting on the predator. It may be more appropriate to the biological resource management. We will obtain the sufficient conditions for the global attractivity of the predator-extinction periodic solution and the permanence of the system. Our results provide reliable tactical basis for the biological resource management, and enrich the theory of impulsive differential equations.

2. Model formulation

There were many works concerning predator-prey system, and many good results are obtained [3, 8–11]. Especially, Kooij and Zegeling [12] investigated the predator-prey model with Ivlev's functional response. The basic predator-prey model is

$$\begin{aligned} x_1'(t) &= x_1(t)\big(r - ax_1 - bx_2(t)\big), \\ x_2'(t) &= x_2(t)\big(-d + cx_1(t)\big), \end{aligned} \tag{2.1}$$

where $x_1(t)$ and $x_2(t)$ are densities of the prey and the predator, respectively, $r > 0$ is the intrinsic growth rate of the prey, $a > 0$ is the coefficient of intraspecific competition, $b > 0$ is the per capita rate of predation of the predator, $d > 0$ is the death rate of the predator, $c > 0$ denotes the product of the per capita rate of predation and the rate of conversing prey into the predator. If $rc - da < 0$, system (2.1) do not have any positive equilibrium point, and the only unique equilibrium point $(r/a, 0)$ is globally asymptotically stable, which implies that the predator population will go extinction. If the prey is stocked at constant rate, then system (2.1) becomes the following differential equation:

$$\begin{aligned} x_1'(t) &= x_1(t)\big(r - ax_1 - bx_2(t)\big) + \mu, \\ x_2'(t) &= x_2(t)\big(-d + cx_1(t)\big). \end{aligned} \tag{2.2}$$

It can be easily derived that if $\mu > d(ad - rc)/c^2$, system (2.2) has a unique globally asymptotically stable positive equilibrium $(d/c, (rdc - ad^2 + \mu c)/bcd)$. This implies that the behavior of stocking prey assures the permanence of system (2.2).

While stage-structured models were analyzed in many literatures [3, 8, 9, 13–19], the following single-species stage-structured model was introduced by Aiello and Freedman [9]:

$$\begin{aligned} x'(t) &= \beta y(t) - rx(t) - \beta e^{-r\tau} y(t-\tau), \\ y'(t) &= \beta e^{-r\tau} y(t-\tau) - \eta_2 y^2(t), \end{aligned} \tag{2.3}$$

where $x(t)$, $y(t)$ represent the immature and mature populations densities, respectively. τ represents a constant time to maturity, and β, r, and η_2 are positive constants. This model is derived as follows. We assume that at any time $t > 0$, birth into the immature population is proportional to the existing mature population with proportionality constant β. We assume that the death rate of immature population is proportional to the existing immature population with proportionality constant r. We also assume that the death rate of mature population is of a logistic nature, that is, proportional to the square of the population with proportionality constant η_2.

According to the nature of biological resource management, developing (2.2) with (2.3) by introducing the stocking on prey at fixed moments and harvesting mature predator population throughout the whole year or continuously, and considering Ivlev's functional response, we consider the following impulsive delay differential equations:

$$\begin{aligned}
x_1'(t) &= x_1(t)(a - bx_1(t)) - \beta(1 - e^{-\theta x_1(t)})x_3(t), \quad t \neq n\tau, \\
x_2'(t) &= rx_3(t) - re^{-w\tau_1}x_3(t-\tau_1) - wx_2(t), \quad t \neq n\tau, \\
x_3'(t) &= re^{-w\tau_1}x_3(t-\tau_1) + k\beta(1 - e^{-\theta x_1(t)})x_3(t) - d_3x_3(t) - Ex_3(t) - d_4x_3^2(t), \quad t \neq n\tau, \\
\Delta x_1(t) &= \mu, \quad t = n\tau,\ n = 1,2,\dots, \\
\Delta x_2(t) &= 0, \quad t = n\tau,\ n = 1,2,\dots, \\
\Delta x_3(t) &= 0, \quad t = n\tau,\ n = 1,2,\dots, \\
(\varphi_1(\zeta),\varphi_2(\zeta),\varphi_3(\zeta)) &\in C_+ = C([-\tau_1,0],\mathbb{R}_+^3), \quad \varphi_i(0) > 0,\ i = 1,2,3,
\end{aligned} \tag{2.4}$$

where $x_1(t)$ denotes the density of the prey, $x_2(t)$, $x_3(t)$ represent the immature and mature predator densities, respectively. τ_1 represents a constant time to maturity, $a > 0$ is the intrinsic growth rate of the prey, $b > 0$ is the coefficient of intraspecific competition, r, w, θ, d_3, d_4, k, c, and β are positive constants, and $0 < E < 1$ is the effect of continuous harvesting on the predator. This model is derived as follows. We assume that at any time $t > 0$, birth into the immature predator population is proportional to the existing mature predator population with proportionality constant r. We then assume that the death rate of immature predator population is proportional to the existing immature predator population with proportionality constant w. $w(w > d)$, d_3 are called the death coefficient of $x_2(t)$, $x_3(t)$, respectively. We assume that the death rate of mature predator populations are of a logistic nature, that is, proportional to the square of the population with proportionality constant d_4. $k > 0$ is the rate of conversing the prey into the predator. $\Delta x_1(t) = x_1(t^+) - x_1(t)$, $\mu \geq 0$ is the stocking amount of the prey at $t = n\tau$, $n \in \mathbb{Z}_+$ and $\mathbb{Z}_+ = \{1,2,\dots\}$, τ is the period of the impulsive stocking on the prey. We will prove that the system (2.4) has a predator-extinction periodic solution. Further, it is globally attractive. Due to the stocking on the prey, the mature predator population will not go extinction for the continuous harvesting of mature predator population, that is, system (2.4) is permanent. In this paper, we always assume that the immature predator population cannot predate the prey population.

Because the first and third equations of (2.4) do not contain $x_2(t)$, we can simplify model (2.4) and restrict our attention to the following model:

$$\begin{aligned}
x_1'(t) &= x_1(t)(a - bx_1(t)) - \beta(1 - e^{-\theta x_1(t)})x_3(t), \quad t \neq n\tau, \\
x_3'(t) &= re^{-w\tau_1}x_3(t-\tau_1) + k\beta(1 - e^{-\theta x_1(t)})x_3(t) - d_3x_3(t) - Ex_3(t) - d_4x_3^2(t), \quad t \neq n\tau, \\
\Delta x_1(t) &= \mu, \quad t = n\tau,\ n = 1,2,\dots, \\
\Delta x_3(t) &= 0, \quad t = n\tau,\ n = 1,2,\dots.
\end{aligned} \tag{2.5}$$

The initial conditions for (2.5) are

$$(\varphi_1(\zeta),\varphi_3(\zeta)) \in C'_+ = C([-\tau_1,0],\mathbb{R}^2_+), \quad \varphi_i(0)>0,\ i=1,3. \tag{2.6}$$

3. Some important lemmas

The solution of (2.4), denoted by $x(t)=(x_1(t),x_2(t),x_3(t))^T$, is a piecewise continuous function $x:\mathbb{R}_+\to\mathbb{R}^3_+$, $x(t)$ is continuous on $(n\tau,(n+1)\tau]$, $n\in\mathbb{Z}_+$ and $x(n\tau^+)=\lim_{t\to n\tau^+}x(t)$ exists. Obviously the global existence and uniqueness of the solutions of (2.4) are guaranteed by the smoothness properties of f, which denote the mapping defined by right-hand side of system (2.4) (see Lakshmikantham et al. [20] and Baĭnov and Simeonov [21]). For the continuity of the initial conditions, we require

$$\varphi_2(0)=\int_{-\tau_1}^{0} re^{ws}\varphi_3(s)ds. \tag{3.1}$$

Before we have the the main results, we need to give some lemmas which will be used in the next.

Lemma 3.1. *Let* $(\varphi_1(t),\varphi_2(t),\varphi_3(t))>0$ *for* $-\tau_1<t<0$. *Then any solution of system (2.4) is strictly positive.*

Proof. First, we show that $x_3(t)\ge 0$ for all $t>0$. Notice $x_3(t)\ge 0$, hence if there exists t_0 such that $x_3(t_0)=0$, then $t_0>0$. Assume that t_0 is the first time such that $x_3(t)=0$, that is,

$$t_0=\inf\{t>0: x_3(t)=0\}, \tag{3.2}$$

then $x_3'(t_0)=re^{-w\tau_1}x_3(t_0-\tau_1)>0$. Hence for sufficiently small $\varepsilon>0$, $x_3'(t_0-\varepsilon)>0$. But by the definition of t_0, $x_3'(t_0-\varepsilon)\le 0$. This contradiction shows that $x_3(t)>0$ for all $t>0$.

By the uniqueness of the solutions of system (2.4) and $x_1'(t)=0$ whenever $x_1(t)=0$, $t\ne n\tau$, and $x_1(n\tau^+)=x_1(n\tau)+\mu$, $\mu\ge 0$, it is easy to see that $x_1(t)>0$ for all $t>0$.

Finally, we consider the following equation:

$$s'(t)=-re^{-w\tau_1}x_3(t-\tau_1)-ws(t). \tag{3.3}$$

Comparing with (2.4), we note that if $s(t)$ is the solution of (3.3) and if $x_2(t)$ can solve (2.4), then $x_2(t)>s(t)$ on $0<t<\tau_1$. Integrating (3.3) gives

$$s(t)=e^{-wt}\left[x_2(0)-\int_0^t re^{w(u-\tau_1)}x_3(u-\tau_1)du\right]. \tag{3.4}$$

From (3.1) one can obtain

$$s(\tau_1)=e^{-w\tau_1}\left[\int_{-\tau_1}^{0} re^{ws}\varphi_3(s)ds-\int_0^{\tau_1} re^{w(u-\tau_1)}x_3(u-\tau_1)du\right]. \tag{3.5}$$

By making transformation and $x_3(t)=\varphi_3(t)$, $t\in[-\tau_1,0]$, we know that $\int_{-\tau_1}^{0} re^{ws}\varphi_3(s)ds$ is equivalent to $\int_0^{\tau_1} re^{w(s-\tau_1)}x_3(s-\tau_1)ds$. Thus we obtain $s(\tau_1)=0$. Hence $x_2(t)>0$. Since $s(t)$ is strictly decreasing, then $x_2(t)>s(t)>0$ for $t\in(0,\tau_1)$. So $x_2(t)>0$ on $0\le t\le\tau_1$.

By induction and similar method to the proof of [22, Theorem 1], we can show that $x_2(t) > 0$ for all $t \geq 0$. This completes the proof. □

Lemma 3.2 (see [20, Lemma 2.2, page 23]). *Let the function* $m \in PC'[\mathbb{R}^+, \mathbb{R}]$ *satisfies the inequalities*

$$\begin{aligned} m'(t) &\leq p(t)m(t) + q(t), \quad t \neq t_k,\ k = 1,2,\dots, \\ m(t_k^+) &\leq d_k m(t_k) + b_k, \quad t = t_k,\ t \geq t_0, \end{aligned} \tag{3.6}$$

where $p, q \in PC[\mathbb{R}^+, \mathbb{R}]$ *and* $d_k \geq 0$, b_k *are constants, then*

$$\begin{aligned} m(t) \leq{}& m(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^{t} p(s)ds\right) + \sum_{t_0<t_k<t}\left(\prod_{t_k<t_j<t} d_j \exp\left(\int_{t_k}^{t} p(s)ds\right)\right) b_k \\ &+ \int_{t_0}^{t} \prod_{s<t_k<t} d_k \exp\left(\int_{s}^{t} p(\sigma)d\sigma\right) q(s)ds, \quad t \geq t_0. \end{aligned} \tag{3.7}$$

Now, we show that all solutions of (2.4) are uniformly ultimately bounded.

Lemma 3.3. *There exists a constant* $M > 0$ *such that* $x_1(t) \leq M/k$, $x_2(t) \leq M$, $x_3(t) \leq M$ *for each solution* $(x_1(t), x_2(t), x_3(t))$ *of (2.4) with all t large enough.*

Proof. Define $V(t) = kx_1(t) + x_2(t) + x_3(t)$, and because of $w > d$, when $t \neq n\tau$ we have

$$D^+V(t) + wV(t) = k(w+a)x_1 - kbx_1^2(t) + (r + w - d_3 - E)x_3(t) - d_4x_3^2(t) \leq M_0, \tag{3.8}$$

where $M_0 = k(a+w)^2/4b + (r+w-d_3-E)^2/4d_4$. When $t = n\tau$, $V(n\tau^+) = V(n\tau) + \mu$. By Lemma 3.2, for $t \in (n\tau, (n+1)\tau]$, we have

$$\begin{aligned} V(t) &\leq V(0)\exp(-dt) + \int_0^t M_0 \exp(-d(t-s))ds + \sum_{0<n\tau<t} \mu \exp(-d(t-n\tau)) \\ &= V(0)\exp(-dt) + \frac{M_0}{d}(1-\exp(-dt)) + \mu\frac{\exp(-d(t-\tau)) - \exp(-d(t-(n+1)\tau))}{1-\exp(d\tau)} \\ &< V(0)\exp(-dt) + \frac{M_0}{d}(1-\exp(-dt)) + \frac{\mu\exp(-d(t-\tau))}{1-\exp(d\tau)} + \frac{\mu\exp(d\tau)}{\exp(d\tau)-1} \\ &\longrightarrow \frac{M_0}{d} + \frac{\mu\exp(d\tau)}{\exp(d\tau)-1}, \quad \text{as } t \longrightarrow \infty. \end{aligned} \tag{3.9}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, there exists a constant $M = M_0/d + \mu\exp(d\tau)/(\exp(d\tau)-1) > 0$ such that $x(t) \leq M/k$, $x_2(t) \leq M$, $x_3(t) \leq M$ for t large enough. The proof is complete. □

Consider the following delay equation:

$$x'(t) = a_1x(t-\tau) - a_2x(t), \tag{3.10}$$

we assume that $a_1, a_2, \tau > 0$; $x(t) > 0$ for $-\tau \leq t \leq 0$. The following result for system (3.12) can be easily obtained from Lemma 3.4.

Lemma 3.4 [23]. *For system (3.10), assume that* $a_1 < a_2$. *Then*

$$\lim_{t\to\infty} x(t) = 0. \tag{3.11}$$

Lemma 3.5 [24]. *Consider the following impulsive system:*

$$\begin{aligned} v'(t) &= v(t)(a - bv(t)), \quad t \neq n\tau, \\ v(n\tau^+) &= v(n\tau) + \mu, \quad t = n\tau,\ n = 1,2,\ldots, \end{aligned} \tag{3.12}$$

where $a > 0$, $b > 0$, $\mu > 0$. *Then there exists a unique positive periodic solution of system (3.12)*

$$\widetilde{v(t)} = \frac{av^* \exp(a(t-n\tau))}{a - bv^* + bv^* \exp(a(t-n\tau))}, \quad t \in (n\tau, (n+1)\tau],\ n \in \mathbb{Z}_+, \tag{3.13}$$

which is globally asymptotically stable, where $v^* = (((a+b\mu)+\sqrt{(a+b\mu)^2 + 4ab\mu/(e^{a\tau}-1)})/2b)(> a/b)$.

According to the system (2.4), we can easily know that there exists $t_1 \in \mathbb{Z}_+$, $t > t_1$, such that $x_3(t-\tau_1) = 0$ and $x_3(t) = 0$. Then

$$\begin{aligned} x_1'(t) &= x_1(t)(a - bx_1(t)), \quad t \neq n\tau, \\ \Delta x_1(t) &= \mu, \quad t = n\tau,\ n = 1,2,\ldots. \end{aligned} \tag{3.14}$$

From (3.14) and Lemma 3.5, we know that (2.4) has a predator-extinction periodic solution

$$(\widetilde{x_1(t)}, 0, 0) = \left(\frac{ax_1^* \exp(a(t-n\tau))}{a - bx_1^* + bx_1^* \exp(a(t-n\tau))}, 0, 0\right), \quad t \in (n\tau, (n+1)\tau],\ n \in \mathbb{Z}_+, \tag{3.15}$$

or (2.5) has a predator-extinction periodic solution

$$(\widetilde{x_1(t)}, 0) = \left(\frac{ax_1^* \exp(a(t-n\tau))}{a - bx_1^* + bx_1^* \exp(a(t-n\tau))}, 0\right), \quad t \in (n\tau, (n+1)\tau],\ n \in \mathbb{Z}_+, \tag{3.16}$$

which is globally asymptotically stable, where $x_1^* = (((a+b\mu)+\sqrt{(a+b\mu)^2 + 4ab\mu/(e^{a\tau}-1)})/2b)(> a/b)$.

Similarly, we can obtain the following important lemma for our next work.

Lemma 3.6. *Consider the following impulsive system:*

$$\begin{aligned} u'(t) &= u(t)(a - bu(t)) - \beta\varepsilon, \quad t \neq n\tau, \\ u(n\tau^+) &= u(n\tau) + \mu, \quad t = n\tau,\ n = 1,2,\ldots, \end{aligned} \tag{3.17}$$

where $a > 0$, $b > 0$, $\mu > 0$, and $\varepsilon > 0$ are sufficiently small. Then there exists a unique globally asymptotically stable positive periodic solution of system (3.17):

$$\widetilde{u(t)} = \frac{k_1\left[(k_1+b_1(u^*-a/2b))e^{2k_1b_1(t-n\tau)}-(k_1-b_1(u^*-a/2b))\right]}{b_1\left[k_1-b_1(u^*-a/2b)+(k_1+b_1(u^*-a/2b))e^{2k_1b_1(t-n\tau)}\right]} \times\left(>\frac{\sqrt{a^2/4b-\beta\varepsilon}}{b}+\frac{a}{2b}\right), \quad t\in(n\tau,(n+1)\tau],\ n\in\mathbb{Z}_+, \tag{3.18}$$

where $u^* = a/2b + (b_1\mu + \sqrt{(2k_1+b_1\mu)^2+4k_1b_1\mu/(e^{2k_1b_1\tau}-1)})/2b_1$, $k_1 = \sqrt{a^2/4b-\beta\varepsilon}$, $b_1 = \sqrt{b}$.

Remark 3.7. From Lemmas 3.5 and 3.6, let $\varepsilon \to 0$, we can easily obtain that $\widetilde{u(t)} \to \widetilde{v(t)}$ and $u^* \to v^*$.

4. Global attractivity

In this section, we will obtain the sufficient condition of the global attractivity of the predator-extinction periodic solution of system (2.4).

Theorem 4.1. *Let $(x_1(t), x_2(t), x_3(t))$ be any solution of (2.4). If*

$$E > re^{-w\tau_1} + k\beta\left(1-\exp\left\{-\frac{\theta a x_1^* e^{a\tau}}{a-bx_1^*+bx_1^*e^{a\tau}}\right\}\right)-d_3 \tag{4.1}$$

holds, where $x_1^* = (((a+b\mu)+\sqrt{(a+b\mu)^2+4ab\mu/(e^{a\tau}-1)})/2b)(>a/b)$, then the predator-extinction periodic solution $(\widetilde{x_1(t)},0,0)$ of (2.4) is globally attractive.

Proof. It is clear that the global attraction of the predator-extinction periodic solution $(\widetilde{x_3(t)},0,0)$ of system (2.4) is equivalent to the global attraction of the predator-extinction periodic solution $(\widetilde{x_3(t)},0)$ of system (2.5). So we only devote to system (2.5). Since $E > re^{-w\tau_1} + k\beta(1-\exp\{-\theta a x_1^* e^{a\tau}/(a-bx_1^*+bx_1^*e^{a\tau})\})-d_3$, we can choose ε_0 sufficiently small such that

$$re^{-w\tau_1} + k\beta\left[1-\exp\left\{-\theta\left(\frac{a x_1^* e^{a\tau}}{a-bx_1^*+bx_1^*e^{a\tau}}+\varepsilon_0\right)\right\}\right] < d_3+E, \tag{4.2}$$

where $x_1^* = (((a+b\mu)+\sqrt{(a+b\mu)^2+4ab\mu/(e^{a\tau}-1)})/2b)(>a/b)$. It follows from the first equation of system (2.5) that $dx_1(t)/dt \le x_1(t)(a-bx_1(t))$. So we consider the following comparison impulsive differential system:

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)(a-bx(t)), \quad t\neq n\tau,\\ \Delta x(t) &= \mu, \quad t=n\tau,\\ x(0^+) &= x_1(0^+). \end{aligned} \tag{4.3}$$

In view of Lemma 3.5, we obtain the periodic solution of system (4.3):

$$\widetilde{x(t)} = \frac{ax_1^* \exp\left(a(t-n\tau)\right)}{a-bx_1^* + bx_1^* \exp\left(a(t-n\tau)\right)}, \quad t \in (n\tau,(n+1)\tau],\ n \in \mathbb{Z}_+, \tag{4.4}$$

which is globally asymptotically stable, where $x_1^* = (((a+b\mu)+\sqrt{(a+b\mu)^2+4ab\mu/(e^{a\tau}-1)})/2b)(>a/b)$.

By Lemma 3.5 and comparison theorem of impulsive equation [21], we have $x_1(t) \le x(t)$ and $x(t) \to \widetilde{x_1(t)}$ as $t \to \infty$. Then there exists an integer $k_2 > k_1$, $n > k_2$ such that

$$x_1(t) \le x(t) \le \widetilde{x_1(t)} + \varepsilon_0, \quad n\tau < t \le (n+1)\tau,\ n > k_2. \tag{4.5}$$

That is

$$x_1(t) \le \widetilde{x_1(t)} + \varepsilon_0 \le \frac{ax_1^* e^{a\tau}}{a-bx_1^* + bx_1^* e^{a\tau}} + \varepsilon_0 \stackrel{\Delta}{=} \rho, \quad n\tau < t \le (n+1)\tau,\ n > k_2. \tag{4.6}$$

From (2.5) and (4.2), we get

$$\frac{dx_3(t)}{dt} \le re^{-w\tau_1}x_3(t-\tau_1) - [d_3 + E - k\beta(1-e^{-\theta\rho})]x_3(t), \quad t > n\tau + \tau_1,\ n > k_2. \tag{4.7}$$

Consider the following comparison differential system:

$$\frac{dy(t)}{dt} = re^{-w\tau_1}y(t-\tau_1) - [d_3 + E - k\beta(1-e^{-\theta\rho})]y(t), \quad t > n\tau + \tau_1,\ n > k_2. \tag{4.8}$$

From (4.2), we have $re^{-w\tau_1} < d_3 + E - k\beta(1-e^{-\theta\rho})$. According to Lemma 3.4, we have $\lim_{t\to\infty} y(t) = 0$.

Let $(x_1(t), x_3(t))$ be the solution of system (2.5) with initial conditions (2.6) and $x_3(\zeta) = \varphi_3(\zeta)\ (\zeta \in [-\tau_1, 0])$, $y(t)$ is the solution of system (4.8) with initial conditions $y(\zeta) = \varphi_3(\zeta)\ (\zeta \in [-\tau_1, 0])$. By the comparison theorem, we have $\lim_{t\to\infty} x_3(t) < \lim_{t\to\infty} y(t) = 0$. Incorporating with the positivity of $x_3(t)$, we know that

$$\lim_{t\to\infty} x_3(t) = 0. \tag{4.9}$$

Therefore, for any $\varepsilon_1 > 0$ (sufficiently small), there exists an integer $k_3 (k_3\tau > k_2\tau + \tau_1)$ such that $x_3(t) < \varepsilon_1$ for all $t > k_3\tau$.

For system (2.5), we have

$$x_1(t)(a - bx_1(t)) - \beta\varepsilon_1 \le \frac{dx_1(t)}{dt} \le (a - bx_1(t))x_1(t). \tag{4.10}$$

Then we have $z_1(t) \le z_1(t) \le z_2(t)$ and $z_1(t) \to \widetilde{x_1(t)}$, $z_2(t) \to \widetilde{x_1(t)}$ as $t \to \infty$. While $z_1(t)$ and $z_2(t)$ are the solutions of

$$\begin{aligned} \frac{dz_1(t)}{dt} &= z_1(t)(a - bz_1(t)) - \beta\varepsilon_1, \quad t \neq n\tau, \\ z_1(t^+) &= z_1(t) + \mu, \quad t = n\tau, \\ z_1(0^+) &= x_1(0^+), \\ \frac{dz_2(t)}{dt} &= z_2(t)[a - bz_2(t)], \quad t \neq n\tau, \\ z_2(t^+) &= z_2(t) + \mu, \quad t = n\tau, \\ z_2(0^+) &= x_1(0^+), \end{aligned} \tag{4.11}$$

respectively. From Lemma 3.6, for $n\tau < t \le (n+1)\tau$,

$$\widetilde{z_1(t)} = \frac{k_1[(k_1+b_1(u^*-a/2b))e^{2k_1b_1(t-n\tau)} - (k_1-b_1(u^*-a/2b))]}{b_1[k_1-b_1(u^*-a/2b)+(k_1+b_1(u^*-a/2b))e^{2k_1b_1(t-n\tau)}]} \left(> \frac{\sqrt{a^2/4b-\beta\varepsilon}}{b} + \frac{a}{2b} \right), \tag{4.12}$$

where $z_1^* = a/2b + (b_1\mu + \sqrt{(2k_1+b_1\mu)^2 + 4k_1b_1\mu/(e^{2k_1b_1\tau}-1)})/2b_1$, $k_1 = \sqrt{a^2/4b-\beta\varepsilon}$, $b_1 = \sqrt{b}$. Therefore, for any $\varepsilon_2 > 0$, there exists an integer k_4, $n > k_4$, such that

$$\widetilde{z_1(t)} - \varepsilon_2 < x_1(t) < \widetilde{x_1(t)} + \varepsilon_2. \tag{4.13}$$

Let $\varepsilon_1 \to 0$, from Remark 3.7, we have

$$\widetilde{x_1(t)} - \varepsilon_2 < x_1(t) < \widetilde{x_1(t)} + \varepsilon_2 \tag{4.14}$$

for t large enough, which implies $x_1(t) \to \widetilde{x_1(t)}$ as $t \to \infty$. This completes the proof. □

5. Permanence

The next work is to investigate the permanence of the system (2.4). Before starting our theorem, we give the definition of permanence.

Definition 5.1. System (2.4) is said to be permanent if there are constants m, $M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(x_1(t), x_2(t), x_3(t))$ with all initial values $x_1(0^+) > 0$, $x_2(0^+) > 0$, $x_3(0^+) > 0$, $m \le x_1(t) < M/k$, $m \le x_2(t) \le M$, $m \le x_3(t) \le M$ hold for all $t \ge T_0$. Here T_0 may depend on the initial values $(x_1(0^+)$, $x_2(0^+), (x_3(0^+))$.

Theorem 5.2. *Suppose*

$$E < re^{-w\tau_1} - d_3 - d_4M + k\beta \times \left[1 - \exp\left\{-\theta\frac{((a-\beta x_3^*)+b\mu)+\sqrt{((a-\beta x_3^*)+b\mu)^2+4(a-\beta x_3^*)b\mu/(e^{(a-\beta x_3^*)\tau}-1)}}{2b}\right\}\right]. \tag{5.1}$$

Then there is a positive constant q such that each positive solution $(x_1(t), x_3(t))$ of (2.5) satisfies

$$x_3(t) \geq q \tag{5.2}$$

for t large enough. Where x_3^* is determined by the following equation:

$$\frac{1}{k\beta}(re^{-w\tau_1} - d_3 - E - d_4M) = 1 - \exp\left\{-\theta\left(\frac{\sqrt{b}\mu+\sqrt{\left(2\sqrt{a^2/4b-\beta x_3^*}+\sqrt{b}\mu\right)^2+4\mu\sqrt{ba^2-b\beta x_3^*}/\left(e^{2\sqrt{ba^2-b^2\beta x_3^*}\tau}-1\right)}}{2\sqrt{b}}\right)\right\}. \tag{5.3}$$

Proof. The second equation of (2.5) can be rewritten as

$$\frac{dx_3(t)}{dt} = [re^{-w\tau_1} + k\beta(1-e^{-\theta x_1(t)}) - d_3 - E - d_4x_3(t)]x_3(t) - re^{-w\tau_1}\frac{d}{dt}\int_{t-\tau_1}^{t} x_3(u)du. \tag{5.4}$$

Let us consider any positive solution $(x_1(t), x_3(t))$ of system (2.5). According to (5.4), $V(t)$ can be defined as

$$V(t) = x_3(t) + re^{-w\tau_1}\frac{d}{dt}\int_{t-\tau_1}^{t} x_3(u)du. \tag{5.5}$$

By calculating the derivative of $V(t)$ along the solution of (2.5), we have

$$\frac{dV(t)}{dt} = [re^{-w\tau_1} + k\beta(1-e^{-\theta x_1(t)}) - d_3 - E - d_4x_3(t)]x_3(t). \tag{5.6}$$

Due to Lemma 3.3, (5.6) can be written

$$\frac{dV(t)}{dt} > [re^{-w\tau_1} + k\beta(1-e^{-\theta x_1(t)}) - d_3 - E - d_4M]x_3(t). \tag{5.7}$$

Since

$$E < re^{-w\tau_1} - d_3 - d_4M + k\beta \times \left[1 - \exp\left\{-\theta \frac{((a-\beta x_3^*)+b\mu)+\sqrt{((a-\beta x_3^*)+b\mu)^2+4(a-\beta x_3^*)b\mu/(e^{(a-\beta x_3^*)\tau}-1)}}{2b}\right\}\right], \tag{5.8}$$

we can easily know that there exists a sufficiently small $\varepsilon > 0$ such that

$$re^{-w\tau_1} > d_3 + E + d_4M + k\beta \times \left[1 - \exp\left\{-\theta\left(\frac{((a-\beta x_3^*)+b\mu)}{2b} + \frac{\sqrt{((a-\beta x_3^*)+b\mu)^2+4(a-\beta x_3^*)b\mu/(e^{(a-\beta x_3^*)\tau}-1)}}{2b} + \varepsilon\right)\right\}\right]. \tag{5.9}$$

We claim that for any $t_0 > 0$, it is impossible that $x_3(t) < x_3^*$ for all $t > t_0$. Suppose that the claim is not valid. Then there is a $t_0 > 0$ such that $x_3(t) < x_3^*$ for all $t > t_0$. It follows from the first equation of (2.5) that for all $t > t_0$,

$$\frac{dx_1(t)}{dt} > x_1(t)(a - bx_1(t)) - \beta x_3^*. \tag{5.10}$$

Consider the following comparison impulsive system for all $t > t_0$:

$$\begin{aligned} \frac{dv(t)}{dt} &= x_1(t)(a - bx_1(t)) - \beta x_3^*, \quad t \neq n\tau, \\ \Delta v(t) &= \mu, \quad t = n\tau. \end{aligned} \tag{5.11}$$

By Lemma 3.6, for $t \in (n\tau, (n+1)\tau]$, we obtain

$$\widetilde{v(t)} = \frac{k_1[(k_1 + b_1(v^* - a/2b))e^{2k_1b_1(t-n\tau)} - (k_1 - b_1(v^* - a/2b))]}{b_1[k_1 - b_1(v^* - a/2b) + (k_1 + b_1(v^* - a/2b))e^{2k_1b_1(t-n\tau)}]} \tag{5.12}$$

is the unique positive periodic solution of (5.11) which is globally asymptotically stable, where $v^* = a/2b + (b_1\mu + \sqrt{(2k_1 + b_1\mu)^2 + 4k_1b_1\mu/(e^{2k_1b_1\tau} - 1)})/2b_1$, $k_1 = \sqrt{a^2/4b - \beta x_3^*}$, $b_1 = \sqrt{b}$.

By the comparison theorem for impulsive differential equation [21], we know that there exists $t_1(> t_0 + \tau_1)$ such that the following inequality holds for $t \geq t_1$:

$$x_1(t) \geq \widetilde{v(t)} - \varepsilon. \tag{5.13}$$

Thus

$$x_1(t) \geq v^* - \frac{b}{2a} - \varepsilon \tag{5.14}$$

for all $t \geq t_1$. We make notation as $\sigma \triangleq v^* - b/2a - \varepsilon$ for convenience. From (5.9), we have

$$re^{-w\tau_1} > k\beta\sigma + d_3 + E + d_4M. \tag{5.15}$$

By (5.6) and (5.14), we have

$$V'(t) > x_3(t)\left(re^{-w\tau_1} - k\beta\sigma - d_3 - E - d_4M\right) \tag{5.16}$$

for all $t > t_1$. Set

$$x_3^m = \min_{t\in[t_1,t_1+\tau_1]} x_3(t). \tag{5.17}$$

We will show that $x_3(t) \geq x_3^m$ for all $t \geq t_1$. Suppose the contrary. Then there is a $T_0 > 0$ such that $x_3(t) \geq x_3^m$ for $t_1 \leq t \leq t_1 + \tau_1 + T_0$, $x_3(t_1 + \tau_1 + T_0) = x_3^m$ and $x_3'(t_1 + \tau_1 + T_0) < 0$. Hence, the first equation of systems (2.5) and (5.14) imply that

$$\begin{aligned} x_3'(t_1+\tau_1+T_0) &= re^{-w\tau_1}x_3(t_1+T_0) + k\beta(1-\exp\{-\theta x_1(t_1+\tau_1+T_0)\})x_3(t_1+\tau_1+T_0) \\ &\quad - (d_3+E)x_3(t_1+\tau_1+T_0) - d_4x_3^2(t_1+\tau_1+T_0), \\ &\geq (re^{-w\tau_1} - \beta\sigma - d_3 - E - d_4M)x_3^m > 0. \end{aligned} \tag{5.18}$$

This is a contradiction. Thus, $x_3(t) \geq x_3^m$ for all $t > t_1$. As a consequence, (5.9) and (5.16) lead to

$$V'(t) > x_3^m\left(re^{-w\tau_1} - k\beta\sigma - d_3 - E - d_4M\right) > 0 \tag{5.19}$$

for all $t > t_1$. This implies that as $t \to \infty$, $V(t) \to \infty$. It is a contradiction to $V(t) \leq M(1 + r\tau_1e^{-w\tau_1} + k\beta(1 - e^{-\theta M}))$. Hence, the claim is true.

By the claim, we are left to consider two cases. First, $x_3(t) \geq x_3^*$ for all t large enough. Second, $x_3(t)$ oscillates about x_3^* for t large enough.

Define

$$q = \min\left\{\frac{x_3^*}{2}, q_1\right\}, \tag{5.20}$$

where $q_1 = x_3^*e^{-(d_3+E+d_4M)\tau_1}$. We will show that $x_3(t) \geq q$ for all t large enough. The conclusion is evident in first case. For the second case, let $t^* > 0$ and $\xi > 0$ satisfy $x_3(t^*) = x_3(t^* + \xi) = x_3^*$ and $x_3(t) < x_3^*$ for all $t^* < t < t^* + \xi$, where t^* is sufficiently large such that

$$x_3(t) > \sigma \quad \text{for } t^* < t < t^* + \xi. \tag{5.21}$$

$x_3(t)$ is uniformly continuous. The positive solutions of (2.5) are ultimately bounded and $x_3(t)$ is not affected by impulses. Hence, there is a $T(0 < t < \tau_1$ and T is independent of the choice of $t^*)$ such that $x_3(t) > x_3^*/3$ for $t^* < t < t^* + T$. If $\xi < T$, there is nothing to prove. Let us consider the case $T < \xi < \tau_1$. Since $x_3'(t) > -(d_3 + E + d_4M)x_3(t)$ and $x_3(t^*) = x_3^*$, it is clear that $x_3(t) \geq q_1$ for $t \in [t^*, t^* + \tau_1]$. Then, proceed exactly as the proof for the above claim. We see that $x_3(t) \geq q_1$ for $t \in [t^* + \tau_1, t^* + \xi]$ because the kind of interval $t \in [t^*, t^* + \xi]$ is chosen in an arbitrary way (we only need t^* to be large). We conclude that $x_3(t) \geq q$ for all large t. In view of our above discussion, the choice of q is independent of the positive solution, and we prove that any positive solution of (2.5) satisfies $x_3(t) \geq q$ for all sufficiently large t. This completes the proof of the theorem. □

Theorem 5.3. *Suppose*

$$E < re^{-w\tau_1} - d_3 - d_4 M + k\beta \times \left[1 - \exp\left\{-\theta \frac{((a-\beta x_3^*) + b\mu)}{2b} + \frac{\sqrt{((a-\beta x_3^*) + b\mu)^2 + 4(a-\beta x_3^*) b\mu/(e^{(a-\beta x_3^*)\tau} - 1)}}{2b}\right\}\right], \tag{5.22}$$

then the system (2.4) is permanent.

Proof. Denote $(x_1(t), x_2(t), x_3(t))$ is any solution of system (2.4). From the first equation of system (2.5) and Theorem 5.2, we have

$$\frac{dx_1(t)}{dt} \geq x_1(t)(a - bx_1(t)) - \beta(1 - e^{-\theta q}). \tag{5.23}$$

By the same argument as those in the proof of Theorem 4.1, we have that

$$\lim_{t\to\infty} x_1(t) \geq p, \tag{5.24}$$

where $p = (b_1\mu + \sqrt{(2k_1 + b_1\mu)^2 + 4k_1 b_1 \mu/(e^{2k_1 b_1 \tau} - 1)})/2b_1 - \varepsilon$, $k_1 = \sqrt{a^2/4b - \beta(1 - e^{-\theta q})}$, $b_1 = \sqrt{b}$.

In view of Theorem 4.1, the second equation of system (2.4) becomes

$$\frac{dx_2(t)}{dt} \geq r(p - e^{-w\tau_1} M) - wx_2(t). \tag{5.25}$$

It is easy to obtain

$$\lim_{t\to\infty} x_2(t) \geq \delta, \tag{5.26}$$

where $\delta = r(p - e^{-w\tau_1} M)/w - \varepsilon$. By Theorem 5.2 and the above discussion, system (2.4) is permanent. The proof of Theorem 5.3 is complete. □

6. Discussion

According to the fact of biological resource management, in this paper, a delayed stage-structured Ivlev's functional response predator-prey system with impulsive stocking on the prey and continuous harvesting on the predator is considered. We get the condition under which the predator-extinction periodic solution of system (2.4) is globally attractive, and obtained the condition for the permanent of system (2.4). From Theorems 4.1

and 5.3, we can easily guess there must exist a threshold μ^*. If $\mu < \mu^*$, the predator-extinction periodic solution $(\widetilde{x_1(t)}, 0, 0)$ of (2.4) is globally attractive. If $\mu > \mu^*$, system (2.4) is permanent. Or from Theorems 4.1 and 5.2, we can easily guess that there must exist a threshold E^*. If $E > E^*$, the predator-extinction periodic solution $(\widetilde{x_1(t)}, 0, 0)$ of (2.4) is globally attractive. If $E < E^*$, system (2.4) is permanent. The results show that the behavior of impulsive stocking on the prey plays an important role for the permanence of system (2.4), that is, it can prevent the predator from dying out. This can meet in biological balance protection. But there are some interesting problems: how does the impulsive stocking on prey affect the dynamical behavior of system (2.4)? What are the optimal harvesting policy of the system (2.4)? We will continue to study these problems in the future.

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References

[1] C. W. Clark, *Mathematical Bioeconomics*, Pure and Applied Mathematics, John Wiley & Sons, New York, NY, USA, 2nd edition, 1990.

[2] W. Wang and L. Chen, "A predator-prey system with stage-structure for predator," *Computers & Mathematics with Applications*, vol. 33, no. 8, pp. 83–91, 1997.

[3] H. I. Freedman and K. Gopalsamy, "Global stability in time-delayed single-species dynamics," *Bulletin of Mathematical Biology*, vol. 48, no. 5-6, pp. 485–492, 1986.

[4] A. d'Onofrio, "Stability properties of pulse vaccination strategy in SEIR epidemic model," *Mathematical Biosciences*, vol. 179, no. 1, pp. 57–72, 2002.

[5] M. G. Roberts and R. R. Kao, "The dynamics of an infectious disease in a population with birth pulse," *Mathematical Biology*, vol. 149, pp. 23–36, 2002.

[6] H. W. Hethcote, "The mathematics of infectious diseases," *SIAM Review*, vol. 42, no. 4, pp. 599–653, 2000.

[7] S. Gao, L. Chen, J. J. Nieto, and A. Torres, "Analysis of a delayed epidemic model with pulse vaccination and saturation incidence," *Vaccine*, vol. 24, no. 35-36, pp. 6037–6045, 2006.

[8] J. D. Murray, *Mathematical Biology*, vol. 19 of *Biomathematics*, Springer, Berlin, Germany, 1989.

[9] W. G. Aiello and H. I. Freedman, "A time-delay model of single-species growth with stage structure," *Mathematical Biosciences*, vol. 101, no. 2, pp. 139–153, 1990.

[10] P. Cull, "Global stability of population models," *Bulletin of Mathematical Biology*, vol. 43, no. 1, pp. 47–58, 1981.

[11] X. Liu and L. Chen, "Complex dynamics of Holling type II Lotka-Volterra predator-prey system with impulsive perturbations on the predator," *Chaos, Solitons & Fractals*, vol. 16, no. 2, pp. 311–320, 2003.

[12] R. E. Kooij and A. A. Zegeling, "A predator-prey model with Ivlev's functional response," *Journal of Mathematical Analysis and Applications*, vol. 198, no. 2, pp. 473–489, 1996.

[13] Y. N. Xiao and L. Chen, "A ratio-dependent predator-prey model with disease in the prey," *Applied Mathematics and Computation*, vol. 131, no. 2-3, pp. 397–414, 2002.

[14] Y. N. Xiao and L. Chen, "An SIS epidemic model with stage structure and a delay," *Acta Mathematicae Applicatae Sinica*, vol. 18, no. 4, pp. 607–618, 2002.

[15] Y. N. Xiao, L. Chen, and F. V. D. Bosch, "Dynamical behavior for a stage-structured SIR infectious disease model," *Nonlinear Analysis: Real World Applications*, vol. 3, no. 2, pp. 175–190, 2002.

[16] Y. N. Xiao and L. Chen, "On an SIS epidemic model with stage structure," *Journal of Systems Science and Complexity*, vol. 16, no. 2, pp. 275–288, 2003.
[17] Z. H. Lu, S. J. Gang, and L. Chen, "Analysis of an SI epidemic with nonlinear transmission and stage structure," *Acta Mathematica Scientia*, vol. 4, pp. 440–446, 2003.
[18] A. A. S. Zaghrout and S. H. Attalah, "Analysis of a model of stage-structured population dynamics growth with time state-dependent time delay," *Applied Mathematics and Computation*, vol. 77, no. 2-3, pp. 185–194, 1996.
[19] W. G. Aiello, H. I. Freedman, and J. Wu, "Analysis of a model representing stage-structured population growth with state-dependent time delay," *SIAM Journal on Applied Mathematics*, vol. 52, no. 3, pp. 855–869, 1992.
[20] V. Lakshmikantham, D. D. Baĭnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, vol. 6 of *Series in Modern Applied Mathematics*, World Scientific, Teaneck, NJ, USA, 1989.
[21] D. D. Baĭnov and P. S. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, vol. 66 of *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Longman Scientific & Technical, Harlow, UK, 1993.
[22] L. E. Caltagirone and R. L. Doutt, "Global behavior of an SEIRS epidemic model with delays, the history of the vedalia beetle importation to California and its impact on the development of biological control," *Annual Review of Entomology*, vol. 34, pp. 1–16, 1989.
[23] K. Yang, *Delay Differential Equations with Applications in Population Dynamics*, Academic Press, New York, NY, USA, 1987.
[24] L. Dong, L. Chen, and L. Sun, "Extinction and permanence of the predator-prey system with stocking of prey and harvesting of predator impulsively," *Mathematical Methods in the Applied Sciences*, vol. 29, no. 4, pp. 415–425, 2006.

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