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Research Article

Periodic Solution of Second-Order Hamiltonian Systems with a Change Sign Potential on Time Scales

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This paper is concerned with the second-order Hamiltonian system on time scales $\mathbb{T}$ of the form $u^{\Delta\Delta}(\rho(t))+\mu b(t)|u(t)|^{\mu-2}u(t)+\overline{\nabla}H(t,u(t))=0$, Δ-a.e. $t\in[0,T]_{\mathbb{T}}$, $u(0)-u(T)=u^{\Delta}(\rho(0))-u^{\Delta}(\rho(T))=0$, where $0,T\in\mathbb{T}$. By using the minimax methods in critical theory, an existence theorem of periodic solution for the above system is established. As an application, an example is given to illustrate the result. This is probably the first time the existence of periodic solutions for second-order Hamiltonian system on time scales has been studied by critical theory.

1. Introduction

The theory of calculus on time scales was introduced by Stefan Hilger in his Ph.D. thesis in 1988 [1]. It cannot only unify discrete and continuous calculus but also exhibit much more complicated dynamics on time scales [2–6]. In particular, dynamic equations on time scales have many important applications, such as, in the study of biological, heat transfer, stock market, and epidemic models [2, 5, 7–9]. Consequently, it has been attracted considerable amount of interest and is now a hot topic of still fairly theoretical exploration in mathematics.

Recently, for the existence problems of positive solutions for dynamic equations on time scales, some authors have obtained many results; for details, see [10–21] and the references therein. To the best of our knowledge, there is no work on the existence of periodic solutions for second-order Hamiltonian systems on time scales. In particular, there is very little work [22–24] on the existence of solutions of dynamic equations on time scales by using critical theory. Now, it is natural to use critical theory to consider the existence of periodic solutions for second-order Hamiltonian systems on time scales.

We make the blanket assumption that $0, T$ are points in $\mathbb{T}$, for an interval $(0, T)_{\mathbb{T}}$, we always mean $(0, T) \cap \mathbb{T}$. Other types of intervals are defined similarly. We say that a property holds for Δ-a.e. $t \in A \subset \mathbb{T}$ or Δ-a.e. on $A \subset \mathbb{T}$, whenever there exists a set $E \subset A$ with null Lebesgue Δ-measure such that this property holds for every $t \in A \setminus E$. We refer the reader to [3, 24, 25] for a broad introduction on Lebesgue Δ-measure.

In this paper, motivated by references [26, 27], we consider the following second-order Hamiltonian system on time scales $\mathbb{T}$ of the form

$$\begin{gathered} u^{\Delta\Delta}(\rho(t)) + \mu b(t)|u(t)|^{\mu-2}u(t) + \overline{\nabla} H(t, u(t)) = 0, \quad \Delta\text{-a.e. } t \in [0, T]_{\mathbb{T}}, \\ u(0) - u(T) = u^{\Delta}(\rho(0)) - u^{\Delta}(\rho(T)) = 0, \end{gathered} \tag{1.1}$$

where $T > 0, \mu > 2, b \in C([0, T]_{\mathbb{T}}, \mathbb{R})$, $H : [0, T]_{\mathbb{T}} \times \mathbb{R}^n \to \mathbb{R}, (t, x) \to H(t, x)$ is measurable in t for every $x \in \mathbb{R}^n$ and continuously differentiable in x for Δ-a.e. $t \in [0, T]_{\mathbb{T}}$ and $\overline{\nabla} H(t, u) = D_u H(t, u)$. By using the minimax methods in critical theory, we establish the existence of at least *one* nonzero solution for the problem (1.1). Our results are even new for the special cases of difference equation and include the results of Tang and Wu [27] for differential equation. Moreover, we prove some lemmas, which will be very important in proving the existence of periodic solutions in $H_T^1(\mathbb{T})$ spaces for many other second-order Hamiltonian systems on time scales. As an application, an example is given to illustrate the result.

There is a solution u of problems (1.1); we mean $u : \mathbb{T}_\kappa \to \mathbb{R}^n$ which is delta differential; u^{Δ} and $u^{\Delta\Delta}$ are both continuous Δ-a.e. on $\mathbb{T}^\kappa \cap \mathbb{T}_\kappa$, and u satisfies problems (1.1).

Now, we present some basic definitions which can be found in [3–5, 28]. Another excellent source on dynamical systems on measure chains is the book [6].

A time scale $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$. For $t \in \mathbb{T}$, the forward and back jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are well defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}. \tag{1.2}$$

In this definition, one puts $\inf \emptyset := \sup \mathbb{T}$ and $\sup \emptyset := \inf \mathbb{T}$, where $\emptyset$ denotes the empty set. A point $t \in \mathbb{T}$ is called left-dense if $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$.

If $\mathbb{T}$ has a right-scattered minimum m, define $\mathbb{T}_\kappa = \mathbb{T} - \{m\}$; otherwise, set $\mathbb{T}_\kappa = \mathbb{T}$. If $\mathbb{T}$ has a left-scattered maximum M, define $\mathbb{T}^\kappa = \mathbb{T} - \{M\}$; otherwise, set $\mathbb{T}^\kappa = \mathbb{T}$. The forward graininess is $\mu(t) := \sigma(t) - t$. Similarly, the backward graininess is $\nu(t) := t - \rho(t)$.

If $f : \mathbb{T} \to \mathbb{R}$ is a function and $t \in \mathbb{T}^\kappa$, then the delta derivative [4] of f at the point t is defined to be the number $f^{\Delta}(t)$ (provided it exists) with the property that, for any $\epsilon > 0$, there is a neighborhood $U \subset \mathbb{T}$ of t such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \le \epsilon |\sigma(t) - s|, \quad \forall\, s \in U. \tag{1.3}$$

If $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}_\kappa$, then the nabla derivative of f at the point t is defined by the number $f^\nabla(t)$ (provided it exists) with the property that, for any $\epsilon > 0$, there is a neighborhood $U \subset \mathbb{T}$ of t such that

$$\left| f(\rho(t)) - f(s) - f^\nabla(t)(\rho(t) - s) \right| \leq \epsilon \left| \rho(t) - s \right|, \quad \forall\, s \in U. \tag{1.4}$$

We refer the reader to [25] for measure on time scales; absolutely continuous on time scales can be found in [29]. We now provide the definition in [24, 30] and simply summarize the main points, which also be described in [31].

Let $a := \inf\{s : s \in \mathbb{T}\}$ and $b := \sup\{s : s \in \mathbb{T}\}$, defined a function $E : [a,b] \to \mathbb{R}$ by

$$E(t) := \sup\{s \in \mathbb{T} : s \leq t\}, \quad t \in [a,b]. \tag{1.5}$$

Now, suppose that $f : \mathbb{T}^\kappa \to \mathbb{R}$ is arbitrary function, if $f \circ E$ is measurable on the real interval $[a,b)$ in the usual Lebesgue senses, then we say f is measurable; if $f \circ E$ is integrable on the real interval $[a,b)$ in the usual Lebesgue senses, then we say f is integrable. Let $L^1(\mathbb{T})$ denote the set of such integrable functions on $\mathbb{T}$. Furthermore, for any $f \in L^1(\mathbb{T})$, we defined the integral of f by

$$\int_s^t f\Delta := \int_s^t f \circ E d\tau \quad \text{for } s,t \in \mathbb{T}, \tag{1.6}$$

with the norm defined by

$$\|f\|_{L^1(\mathbb{T})} = \int_a^b |f|\Delta \quad \text{for } f \in L^1(\mathbb{T}). \tag{1.7}$$

We use the notation $\int_s^t f\Delta$ to denote the Lebesgue integral of a function f between $s,t \in \mathbb{T}$ (when it is defined). That is, we use the same notation for the Lebesgue-type integral defined in [24, 30] as is commonly used in the time scale literature for a Riemann-type integral defined in terms of antiderivatives. A detailed discussion of the Lebesgue-type integral and it's relationship with the usual time scale integral is given in [24, 30]. With the Lebesgue integral defined, denote

$$\begin{gathered} L^2(\mathbb{T}) := \left\{ f \in L^1(\mathbb{T}) : |f|^2 \in L^1(\mathbb{T}) \right\}, \\ \|f\|_{L^2(\mathbb{T})} = \left(\int_a^b |f|^2 \Delta \right)^{1/2} \quad \text{for } f \in L^2(\mathbb{T}). \end{gathered} \tag{1.8}$$

It is shown in [31] that $L^2(\mathbb{T})$ is completed with respect to the norm $\|f\|_{L^2(\mathbb{T})}$.

Next, define the norm $\|\cdot\|$ on $C^1_{rd}(\mathbb{T})$ by

$$\|f\| = \left(\|f\|^2_{L^2(\mathbb{T})} + \left\| f^\Delta \right\|^2_{L^2(\mathbb{T}^\kappa)} \right)^{1/2} \quad \text{for } f \in C^1_{rd}(\mathbb{T}). \tag{1.9}$$

The space $H^1(\mathbb{T})$ is the completion of $C^1_{rd}(\mathbb{T})$ with respect to the norm $\|\cdot\|$ (see [31, Definition 4.1 and Remark 4.2]). The space $H^1(\mathbb{T})$ is a time scale analog to the usual Sobolev space $H^1(I)$ on a real interval I.

We refer the reader to [32] for an introduction on basic properties of Sobolev's spaces on bounded time scales.

Remark 1.1. If we replace $u : \mathbb{T} \to \mathbb{R}$ with $u : \mathbb{T} \to \mathbb{R}^n$, then the above discussion still holds.

The rest of the paper is organized as follows. In Section 2, we list some lemmas, which are important in proving the existence of periodic solutions. By applying these lemmas, we establish the existence of periodic solutions for problem (1.1). In the final section, an example is given to illustrate our main result.

2. Some Lemmas

In this section, to interpret Hamiltonian systems on time scales in a functional-analytic setting, we introduce some lemmas, which will be used in the rest of the paper and be very important in proving the existence of periodic solutions in $H^1_T(\mathbb{T})$ spaces for second-order Hamiltonian systems on time scales.

Let $H^1_T(\mathbb{T})$ be the Hilbert space given by

$$\begin{aligned} H^1_T(\mathbb{T}) = \{ u : [0,T]_{\mathbb{T}} \longrightarrow \mathbb{R}^n \mid u \text{ is absolutely continuous}, u(0) = u(T), \\ u^{\Delta}(t) \in L^2\left([0,T]_{\mathbb{T}_\kappa}, \mathbb{R}^n\right) \Big\}, \end{aligned} \tag{2.1}$$

with the norm defined by

$$\|u\| = \left(\int_0^T |u(t)|^2 \Delta + \int_0^T \left|u^{\Delta}(t)\right|^2 \Delta \right)^{1/2} \quad \text{for } u \in H^1_T(\mathbb{T}). \tag{2.2}$$

Moreover, we define

$$\|u\|_{L^1(\mathbb{T})} = \int_0^T |u(t)|\Delta, \|u\|_{L^2(\mathbb{T})} = \left(\int_0^T |u(t)|^2 \Delta \right)^{1/2}, \qquad \|u\|_{\infty} = \sup_{t \in [0,T]_{\mathbb{T}}} |u|. \tag{2.3}$$

We also define inner product on $H^1_T(\mathbb{T})$ by

$$(u, v) = \int_0^T \left[u(t) \cdot v(t) + u^{\Delta}(t) \cdot v^{\Delta}(t) \right] \Delta. \tag{2.4}$$

For $u \in H^1_T(\mathbb{T})$, let

$$\overline{u}(t) = \frac{1}{T} \int_0^T u(t) \Delta, \qquad \widetilde{u}(t) = u(t) - \overline{u}(t), \tag{2.5}$$

and let $\widetilde{H}_T^1(\mathbb{T})$ be the subspace of $H_T^1(\mathbb{T})$ given by $\widetilde{H}_T^1(\mathbb{T}) = \{u \in H_T^1(\mathbb{T}) \mid \overline{u}(t) = 0\}$.

In the following, we will prove several lemmas which are very important in proving the existence of periodic solutions for problem (1.1).

Lemma 2.1. *Let $p \in \overline{\mathbb{R}}$ be such that $p \geq 1$. Then, for every $q \in [1, +\infty)$, the immersion $H_T^1(\mathbb{T}) \hookrightarrow L^q(\mathbb{T})$ is compact.*

Proof. The proving is similar to the way as in proving of [32, Corollary 3.11], and we omit it here. □

The following two Lemmas are an immediate consequence of the [23, Proposition 3.6] (see also [23, Corollary 3.9]).

Lemma 2.2. *let $\{u_m\}_{m\in\mathbb{N}} \subset H_T^1(\mathbb{T})$ and $u \in H_T^1(\mathbb{T})$. If $\{u_m\}_{m\in\mathbb{N}}$ converges weakly in $H_T^1(\mathbb{T})$ to u, then $\{u_m\}_{m\in\mathbb{N}}$ converges uniformly to u on $[0,T]_{\mathbb{T}}$.*

Lemma 2.3. *If $u \in H_T^1(\mathbb{T})$, then*

$$\|u\|_\infty \leq c_1 \|u\|. \tag{2.6}$$

In particular, if $\int_0^T u(t)\Delta = 0$, then

$$\|u\|_\infty \leq T^{1/2} \left\| u^\Delta(t) \right\|_{L^2(\mathbb{T})}. \tag{2.7}$$

Lemma 2.4. *Let $L : [0,T]_{\mathbb{T}} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}, (t,x,y) \rightarrow L(t,x,y)$ be measurable in t for each $[x,y] \in \mathbb{R}^n \times \mathbb{R}^n$ and continuously differentiable in $[x,y]$ for Δ-a.e. $t \in [0,T]_{\mathbb{T}}$. If there exist $a \in C(\mathbb{R}^+, \mathbb{R}^+), b_1 \in L^1([0,T]_{\mathbb{T}}, \mathbb{R}^+)$, and $c \in L^2([0,T]_{\mathbb{T}}, \mathbb{R}^+)$, such that, for Δ-a.e. $t \in [0,T]_{\mathbb{T}}$ and each $[x,y] \in \mathbb{R}^n \times \mathbb{R}^n$, one has*

$$\begin{gathered}
|L(t,x,y)| \leq a(|x|)\left(b_1(t) + |y|^2\right), \\
|D_x L(t,x,y)| \leq a(|x|)\left(b_1(t) + |y|^2\right), \\
|D_y L(t,x,y)| \leq a(|x|)(c(t) + |y|),
\end{gathered} \tag{2.8}$$

then the functional φ defined by $\varphi(u) = \int_0^T L(t, u(t), u^\Delta(t))\Delta$ is continuously differential on $H_T^1(\mathbb{T})$ and

$$\langle \varphi'(u), v \rangle = \int_0^T \left[D_x L\left(t, u(t), u^\Delta(t)\right) \cdot v(t) + D_y L\left(t, u(t), u^\Delta(t)\right) \cdot v^\Delta(t) \right] \Delta. \tag{2.9}$$

Proof. In the following, we will prove that φ has a directional derivative $\varphi'(u) \in (H_T^1(\mathbb{T}))^*$ given by (2.9) and that the mapping

$$\varphi' : H_T^1(\mathbb{T}) \longrightarrow \left(H_T^1(\mathbb{T})\right)^* \qquad , u \longrightarrow \varphi'(u) \tag{2.10}$$

is continuous.

(i) It follows easily from (2.8) that φ is everywhere finite on $H_T^1(\mathbb{T})$. Fixing u and v in $H_T^1(\mathbb{T})$, we define

$$F(\lambda,t) = L\Big(t,u(t)+\lambda v(t),u^{\Delta}(t)+\lambda v^{\Delta}(t)\Big) \text{ for } \quad t\in[0,T]_{\mathbb{T}},\lambda\in[-1,1],$$
$$\psi(\lambda) = \varphi(u+\lambda v) = \int_0^T F(\lambda,t)\Delta = \int_0^T F(\lambda,t)\circ E dt = \int_0^T F(\lambda,E(t))dt. \tag{2.11}$$

We will apply Leibniz formula of differentiation under integral sign to ψ. According to assumption (2.8), one obtains

$$\begin{aligned}
&|D_\lambda F(\lambda,E(t))| \\
&\quad\le \left|\left(D_x L(t,u+\lambda v,u^{\Delta}+\lambda v^{\Delta})\circ E\right)\cdot(v\circ E)+\left(D_y L(t,u+\lambda v,u^{\Delta}+\lambda v^{\Delta})\circ E\right)\cdot(v^{\Delta}\circ E)\right| \\
&\quad\le a(|u(E(t))+\lambda v(E(t))|)\left[\left(b_1(t)+\left|u^{\Delta}+\lambda v^{\Delta}\right|^2\right)|v|+\left(c(t)+\left|u^{\Delta}+\lambda v^{\Delta}\right|\right)\left|v^{\Delta}\right|\right]\circ E \\
&\quad\le a_0\left[\left(b_1(t)+\left(\left|u^{\Delta}\right|+\left|u^{\Delta}\right|\right)^2\right)|v|+\left(c(t)+\left|u^{\Delta}\right|+\left|v^{\Delta}\right|\right)\left|v^{\Delta}\right|\right]\circ E,
\end{aligned} \tag{2.12}$$

where $a_0 = \max_{(\lambda,t)\in[-1,1]\times[0,T]} a(|u(E(t))+\lambda v(E(t))|)$.

It is obvious that $b_1\circ E\in L^1([0,T],\mathbb{R}^+)$, $(|u^{\Delta}\circ E|+|v^{\Delta}\circ E|)^2\in L^1([0,T],\mathbb{R}^+)$, $c\circ E\in L^2([0,T],\mathbb{R}^+)$. $v\in H_T^1(\mathbb{T})$ implies that $v^{\Delta}\circ E\in L^2([0,T],\mathbb{R}^+)$ and $v\circ E\in L^1([0,T],\mathbb{R}^+)$ hold, hence we have

$$|D_\lambda F(\lambda,E(t))| \le d\circ E\in L^1([0,T],\mathbb{R}^+). \tag{2.13}$$

In view of Leibniz formula and (1.6), we get

$$\begin{aligned}
\psi'(0) = \langle\varphi'(u),v\rangle &= \int_0^T D_\lambda F(0,E(t))dt \\
&= \int_0^T D_x L\Big(E(t),u(E(t)),u^{\Delta}(E(t))\Big)\cdot v(E(t))dt \\
&\quad+\int_0^T D_y L\Big(E(t),u(E(t)),u^{\Delta}(E(t))\Big)\cdot v^{\Delta}(E(t))dt \\
&= \int_0^T \Big[D_x L\Big(t,u(t),u^{\Delta}(t)\Big)\cdot v(t)+D_y L\Big(t,u(t),u^{\Delta}(t)\Big)\cdot v^{\Delta}(t)\Big]\Delta.
\end{aligned} \tag{2.14}$$

Moreover

$$\left|D_x L\left(t,u,u^{\Delta}\right)\right| \le a(|u|)\left(b_1(t)+\left|u^{\Delta}\right|^2\right) \in L^1([0,T]_{\mathbb{T}},\mathbb{R}^+),$$
$$\left|D_y L\left(t,u,u^{\Delta}\right)\right| \le a(|u|)\left(c(t)+\left|u^{\Delta}\right|\right) \in L^2([0,T]_{\mathbb{T}},\mathbb{R}^+). \tag{2.15}$$

Thus, from Lemma 2.3,

$$\int_0^T D_\lambda F(0,t)\Delta = \int_0^T \left[D_x L\left(t,u,u^{\Delta}\right)\cdot v + D_y L\left(t,u,u^{\Delta}\right)\cdot v^{\Delta}\right]\Delta$$
$$\le c_1\|v\|_\infty + c_2\left\|v^{\Delta}\right\|_{L^2(\mathbb{T})} \le c_3\|v\|, \tag{2.16}$$

and φ has a directional derivative $\varphi'(u)\in (H_T^1(\mathbb{T}))^*$, given by (2.9).

(ii) According to the theorem of Krasnosel'skii [33], assumption (2.8) implies that the mapping from $H_T^1(\mathbb{T})$ into $L^1\times L^2$ defined by $u \to (D_x L(t,u,u^{\Delta}), D_y L(t,u,u^{\Delta}))$ is continuous, thus, φ' is continuous from $H_T^1(\mathbb{T})$ into $(H_T^1(\mathbb{T}))^*$, and the proof is completed. □

We also need the following theorem, which was the generalized mountain pass theorem.

Lemma 2.5. *[34] Let E be a real Hilbert space with $E=E_1\oplus E_2$ and $E_2=E_1^{\perp}$. Suppose $I\in C^1(E,R)$, satisfies* (PS)*, and*

(I1) *$I(u)=1/2(Au,u)+m(u)$, where $Au=A_1P_1u+A_2P_2u$ and $A_i : E_i \to E_i$ is bounded and self-adjoint, $i=1,2$,*

(I2) *$m'(u)$ is compact, and*

(I3) *there exist a subspace $\widetilde{E}\subset E$ and sets $S\subset E, Q\subset \widetilde{E}$ and constants $\alpha>\omega$ such that*

(i) *$S\subset E_1$ and $I|_S \ge \alpha$;*

(ii) *Q is bounded and $I|_{\partial Q}\le \omega$;*

(iii) *S and ∂Q link.*

Then I possesses a critical value $c\ge\alpha$.

3. Existence Results

In this section, by using the minimax methods in critical theory, we establish the existence of at least *one* nonzero periodic solution for second-order Hamiltonian system (1.1) on time scales.

Throughout this section, the following is assumed.

(H1) Suppose that there exist $t_1,t_2\in[0,T]_{\mathbb{T}}$ and $b\in C([0,T]_{\mathbb{T}},\mathbb{R})$ satisfying $b(t)\ge (1/2)b(t_0)>0$ for all $t\in[t_1,t_2]_{\mathbb{T}}$, where $t_0\in[t_1,t_2]_{\mathbb{T}}$. In addition, $\int_0^T b(t)\Delta = 0$.

(H2) $\int_0^T H(t,x)\Delta \ge 0$ for all $x\in\mathbb{R}^n$ and Δ-a.e. $t\in[0,T]_{\mathbb{T}}$.

(H3) Assume that there exists $g \in L^1([0,T]_{\mathbb{T}}, \mathbb{R}^+)$ such that $|\overline{\nabla} H(t,x)| \leq g(t)$ for all $x \in \mathbb{R}^n$ and Δ-a.e. $t \in [0,T]_{\mathbb{T}}$.

(H4) Assume that there exist $\alpha_0 \in (0, 1/2T^2)$ and $r_0 > 0$ such that $|H(t,x)| \leq \alpha_0 |x|^2$ for all $\|x\| \leq r_0$ and Δ-a.e. $t \in [0,T]_{\mathbb{T}}$.

(H5) Assume that $\sigma(\rho(t)) = t$.

If

$$L(t,x,y) = L\left(t, u(t), u^{\Delta}(t)\right) = \frac{1}{2}|u^{\Delta}(t)|^2 - b(t)|u(t)|^{\mu} - H(t,u(t)), \tag{3.1}$$

then by Lemma 2.4, the functional φ is given by

$$\varphi(u) = \frac{1}{2}\int_0^T \left|u^{\Delta}(t)\right|^2 \Delta - \int_0^T b(t)|u(t)|^{\mu} \Delta - \int_0^T H(t,u(t))\Delta, \tag{3.2}$$

which is continuously differentiable on $H_T^1(\mathbb{T})$. Moreover

$$\begin{aligned} \langle \varphi'(u), v \rangle = \int_0^T u^{\Delta}(t) \cdot v^{\Delta}(t)\Delta - \mu \int_0^T b(t)|u(t)|^{\mu-2} u(t) \cdot v(t)\Delta \\ - \int_0^T \overline{\nabla} H(t,u(t)) \cdot v(t)\Delta \quad \forall u, v \in H_T^1(\mathbb{T}). \end{aligned} \tag{3.3}$$

That is, for all $u, v \in H_T^1(\mathbb{T})$, we get

$$\begin{aligned} \langle \varphi'(u), v \rangle &= -\int_0^T u^{\Delta\Delta}(\rho(t)) \cdot v(t)\Delta - \mu \int_0^T b(t)|u(t)|^{\mu-2} u(t) \cdot v(t)\Delta - \int_0^T \overline{\nabla} H(t,u(t)).v(t)\Delta \\ &= -\int_0^T \left(u^{\Delta\Delta}(\rho(t)) + \mu b(t)|u(t)|^{\mu-2}u(t) + \overline{\nabla} H(t,u(t))\right) \cdot v(t)\Delta. \end{aligned} \tag{3.4}$$

Hence , $u \in H_T^1(\mathbb{T})$ is a solution of problem (1.1) if and only if u is a critical point of φ.

Lemma 3.1. *Let a sequence $\{u_n(t)\} \subset H_T^1(\mathbb{T})$ be such that $\varphi'\ (u_n\ (t)) \ \rightarrow\ 0$ and let $\{u_n(t)\}$ be bounded in $H_T^1(\mathbb{T})$, then $\{u_n(t)\}$ has a convergent subsequence in $H_T^1(\mathbb{T})$.*

Proof. Since $\{u_n(t)\}$ is bounded in $H_T^1(\mathbb{T})$, it follows from [30, Theorem 4.12] that there exists a subsequence (still denoted by $\{u_n(t)\}$) which weakly converges to $u_0 \in H_T^1(\mathbb{T})$. By Lemma 2.2, we have

$$u_n \longrightarrow u_0 \quad \text{in } [0,T]_{\mathbb{T}}. \tag{3.5}$$

Hence, for $t \in [0,T]_{\mathbb{T}}$, there exists an $M > 0$ such that $|u_n(t)| \leq M, n = 1, 2, \ldots$.

Lemma 2.1 leads to

$$H_T^1(\mathbb{T}) \hookrightarrow L^2([0,T]_{\mathbb{T}}, \mathbb{R}^n) \text{ is compact.} \tag{3.6}$$

Hence

$$u_n \longrightarrow u_0 \quad \text{in } L^2([0,T]_{\mathbb{T}}, \mathbb{R}^n). \tag{3.7}$$

Since

$$\begin{aligned} &\langle \varphi'(u_n(t)) - \varphi'(u_m(t)), u_n(t) - u_m(t) \rangle \\ &\quad = \int_0^T \left(u_n^\Delta(t) - u_m^\Delta(t)\right) \cdot \left(u_n^\Delta(t) - u_m^\Delta(t)\right)\Delta \\ &\qquad - \mu \int_0^T b(t)\left(|u_n(t)|^{\mu-2}u_n(t) - |u_m(t)|^{\mu-2}u_m(t)\right) \cdot (u_n(t) - u_m(t))\Delta \\ &\qquad - \int_0^T \left(\overline{\nabla} H(t, u_n(t)) - \overline{\nabla} H(t, u_m(t))\right) \cdot (u_n(t) - u_m(t))\Delta, \end{aligned} \tag{3.8}$$

in view of (H1) and (H3), one has

$$\begin{aligned} \int_0^T \left|u_n^\Delta(t) - u_m^\Delta(t)\right|^2 \Delta \le{} & \|\varphi'(u_n(t)) - \varphi'(u_m(t))\| \|u_n(t) - u_m(t)\| \\ & + 2\mu M^{\mu-1} \|u_n(t) - u_m(t)\|_\infty \int_0^T |b(t)|\Delta \\ & + 2\|u_n(t) - u_m(t)\|_\infty \int_0^T g(t)\Delta \longrightarrow 0, \quad \text{as } n, m \longrightarrow \infty. \end{aligned} \tag{3.9}$$

Consequently

$$\|u_n(t) - u_m(t)\|^2 = \int_0^T |u_n^\Delta(t)\Delta - u_m^\Delta(t)|^2 \Delta + \int_0^T |u_n(t) - u_m(t)|^2 \Delta \longrightarrow 0 \quad \text{as } n, m \longrightarrow \infty, \tag{3.10}$$

which implies that $\{u_n\}$ is a Cauchy sequence in $H_T^1(\mathbb{T})$. By the completeness of $H_T^1(\mathbb{T})$, we obtain that $\{u_n\}$ is a convergent sequence in $H_T^1(\mathbb{T})$; the proof is completed. □

Now, we list our main result.

Theorem 3.2. *Suppose that $\mu > 2$, (H1), (H2), (H3), (H4), and (H5) hold, then the problem* (1.1) *has at least one nonzero solution.*

Proof. It suffices to show that all the conditions of Lemma 2.5 hold with respect to φ.

First, we show that φ satisfies the (PS) condition.

From Lemma 3.1, we only need to prove that $\{u_n\}$ is bounded. Otherwise, there exists a subsequence (still denoted by u_n) such that $\|u_n\| \to \infty$. Let $v_n = u_n/\|u_n\|$, then $\{v_n\}$ is bounded; it has a subsequence (we still denote $\{v_n\}$) which weakly converges to v_0. In view of Lemma 2.2, $\{v_n\}$ uniformly converges to v_0 in $[0,T]_{\mathbb{T}}$. Since $\|v_n\| = 1$ for all $n \in \mathbb{N}$, one has $\|v_n\| \neq 0$.

According to $\|u_n\| \to \infty$ as $n \to \infty$, $\mu > 2$ and (H3), for all $v_n, v \in H_T^1(\mathbb{T})$, we get

$$\begin{aligned}
\mu\left|\int_0^T b(t)|v_n(t)|^{\mu-2}v_n(t)\cdot v(t)\Delta\right| &\leq \|u_n\|^{1-\mu}\left|-\langle\varphi'(u_n),v\rangle\right| + \|u_n\|^{2-\mu}\left|\int_0^T v_n^{\Delta}(t)\cdot v^{\Delta}(t)\Delta\right| \\
&\quad + \|u_n\|^{1-\mu}\left|-\int_0^T \overline{\nabla}H(t,u_n(t))\cdot v(t)\Delta\right| \\
&\leq \|u_n\|^{1-\mu}\left\|\varphi'(u_n)\right\|\|v\| + C_1\|u_n\|^{2-\mu}\|v_n\|\|v\| \\
&\quad + \|u_n\|^{1-\mu}\|v\|_\infty\int_0^T g(t)\Delta \longrightarrow 0 \quad \text{as } n\longrightarrow\infty.
\end{aligned} \tag{3.11}$$

Thus, it follows from Lebesgue dominated convergence theorem on time scales [28] that

$$\left|\int_0^T b(t)|v_0(t)|^{\mu-2}v_0(t)\cdot v(t)\Delta\right| = 0 \quad \forall\, v\in H_T^1(\mathbb{T}). \tag{3.12}$$

By the arbitrariness of v, one has

$$b(t) = 0 \quad \text{for } \Delta\text{-a.e. } t\in[0,T]_{\mathbb{T}}, \tag{3.13}$$

which contradicts the condition (H1). Hence φ satisfies the (PS) condition.

Second, if

$$Au = u(t), \qquad m(u) = -\int_0^T b(t)|u(t)|^{\mu}\Delta - \int_0^T H(t,u(t))\Delta - \frac{1}{2}\int_0^T u^2(t)\Delta, \tag{3.14}$$

then it is easy to verify that (I1) and (I2) hold.

Third, we will prove that φ satisfies the condition (I3) in Lemma 2.5.

For arbitrary $u \in \widetilde{H}_T^1(\mathbb{T})$ with $\|u\| \le r_0$, (H4) and (2.7) imply

$$\begin{aligned}
\varphi(u) &= \frac{1}{2}\int_0^T \left|u^{\Delta}(t)\right|^2 \Delta - \int_0^T b(t)|u(t)|^{\mu}\Delta - \int_0^T H(t,u(t))\Delta \\
&\ge \frac{1}{2}\int_0^T \left|u^{\Delta}(t)\right|^2 \Delta - \|u(t)\|_{\infty}^{\mu}\int_0^T |b(t)|\Delta - \alpha_0\int_0^T |u(t)|^2\Delta \\
&\ge \frac{1}{2T}\|u\|_{\infty}^2 - \|u(t)\|_{\infty}^{\mu}\int_0^T |b(t)|\Delta - \alpha_0 T\|u(t)\|_{\infty}^2 \\
&\ge \left(\frac{1}{2T} - \alpha_0 T\right)\|u\|_{\infty}^2 - \|u(t)\|_{\infty}^{\mu}\int_0^T |b(t)|\Delta.
\end{aligned} \tag{3.15}$$

Choose $\rho > 0$ small enough such that

$$\alpha \triangleq \left(\frac{1}{2T} - \alpha_0 T\right)\rho^2 - \rho^{\mu}\int_0^T |b(t)|\Delta > 0, \tag{3.16}$$

thus

$$\varphi(u) \ge \alpha > \quad \forall\, u \in \widetilde{H}_T^1(\mathbb{T}) \quad \text{with } \|u\| = \rho. \tag{3.17}$$

If

$$S = \left\{u \in \widetilde{H}_T^1(\mathbb{T}), \|u\| = \rho\right\}, \tag{3.18}$$

then $\varphi|_s \ge \alpha > 0$; this implies that the condition (i) of Lemma 2.5 holds.

It is known that (H1) and (H2) lead to

$$\varphi(x) = -\int_0^T b(t)|x|^{\mu}\Delta - \int_0^T H(t,x)\Delta \le 0 \quad \forall\, x \in \mathbb{R}^n. \tag{3.19}$$

Choose $0 \ne e \in \widetilde{H}_T^1(\mathbb{T})$ such that $e(t) = 0$ for all $t \in [0,T]_{\mathbb{T}} \setminus [t_1,t_2]_{\mathbb{T}}$ and

$$\int_0^T b(t)e(t)\Delta = \int_{t_1}^{t_2} b(t)e(t)\Delta = 0. \tag{3.20}$$

For arbitrary $u \in H_T^1(\mathbb{T})$, let

$$\varphi_1(u) = \frac{1}{2}\int_0^T \left|u^{\Delta}(t)\right|^2\Delta, \quad \varphi_2(u) = -\int_0^T b(t)|u(t)|^{\mu}\Delta, \quad \varphi_3(u) = -\int_0^T H(t,u(t))\Delta, \tag{3.21}$$

then

$$\varphi(u) = \varphi_1(u) + \varphi_2(u) + \varphi_3(u). \tag{3.22}$$

For all $x \in \mathbb{R}^n$ and $r \geq 0$, in terms of (H3), one has

$$\begin{aligned}\varphi_3(x+re) - \varphi_3(x) &= \int_0^T \int_1^0 \overline{\nabla} H(t, x + sre(t)) \cdot re(t)\, ds \Delta \\ &\leq r \int_0^T g(t) |e(t)| \Delta \leq r \|e\|_\infty \|g\|_{L^1(\mathbb{T})}.\end{aligned} \tag{3.23}$$

Since $-\varphi_3(x) \geq 0$ for all $x \in \mathbb{R}^n$, we have

$$\varphi_3(x+re) \leq r \|e\|_\infty \|g\|_{L^1(\mathbb{T})} \quad \forall\, x \in \mathbb{R}^n, r \geq 0. \tag{3.24}$$

In terms of (3.20) and Hölder's inequality on time scales, one obtains

$$\begin{aligned}\int_{t_1}^{t_2} - b(t)\left(|x|^2 + r^2 |e(t)|^2\right) \Delta &= \int_{t_1}^{t_2} - b(t)(|x| + r|e(t)|)^2 \Delta \\ &\leq \left(\int_{t_1}^{t_2} - b(t)(|x| + r|e(t)|)^{\mu} \Delta\right)^{2/\mu} \\ &\quad \times \left(\int_{t_1}^{t_2} - b(t) \Delta\right)^{(\mu-2)/\mu} \quad \forall\, x \in \mathbb{R}^n, r \geq 0.\end{aligned} \tag{3.25}$$

Thus, by using $\int_0^T b(t)|x|^{\mu} \Delta = 0$, (3.25), and Hölder's inequality on time scales again, for all $x \in \mathbb{R}^n$ and $r \geq 0$, we obtain

$$\begin{aligned}\varphi_2(x+re) &= -\int_0^T b(t)\left(|x + re(t)|^{\mu} - |x|^{\mu}\right) \Delta \\ &= -\int_0^{t_1} b(t)|x+re(t)|^{\mu} \Delta - \int_{t_1}^{t_2} b(t)|x+re(t)|^{\mu} \Delta - \int_{t_2}^{T} b(t)|x+re(t)|^{\mu} \Delta + \int_0^T b(t)|x|^{\mu} \Delta \\ &= -\int_{t_1}^{t_2} b(t)|x+re(t)|^{\mu} \Delta + \int_{t_1}^{t_2} b(t)|x|^{\mu} \Delta \\ &\leq \left(\int_{t_1}^{t_2} - b(t)(|x|^2 + r^2|e(t)|^2) \Delta\right)^{\mu/2} \left(\int_{t_1}^{t_2} - b(t) \Delta\right)^{(2-\mu)/2} + \int_{t_1}^{t_2} b(t)|x|^{\mu} \Delta \\ &\leq -\int_{t_1}^{t_2} b(t)(|x| + r|e(t)|)^{\mu} \Delta + \int_{t_1}^{t_2} b(t)|x|^{\mu} \Delta \leq 0.\end{aligned} \tag{3.26}$$

(3.24) and (3.26) lead to

$$\begin{aligned}\varphi(x+re) &= \varphi_1(x+re)+\varphi_2(x+re)+\varphi_3(x+re)\\ &\leq \frac{1}{2}r^2\int_0^T \left|e^{\Delta}\right|^2\Delta + r\|e\|_\infty \|g\|_{L^1(\mathbb{T})} \quad \forall\ x\in\mathbb{R}^n, r\geq 0,\end{aligned} \tag{3.27}$$

which means that there exists $r_3 > 0$ such that

$$\varphi(x+re)\leq \frac{\alpha}{2} \quad \forall x\in\mathbb{R}^n, r\in[0,r_3]. \tag{3.28}$$

For all $R\geq 0$ and $r\geq 0$, let

$$\begin{aligned}h(R,r) &\triangleq \left(\int_{t_1}^{t_2} -b(t)\left(|R|^2+r^2|e(t)|^2\right)\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} -b(t)\Delta\right)^{(2-\mu)/2} + \int_{t_1}^{t_2} b(t)|R|^{\mu}\Delta\\ &= -\left(\int_{t_1}^{t_2} b(t)\left(|R|^2+r^2|e(t)|^2\right)\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2} + \int_{t_1}^{t_2} b(t)|R|^{\mu}\Delta.\end{aligned} \tag{3.29}$$

In view of $\int_{t_1}^{t_2} b(t) > 0$, for all $R\geq 0$ and $r\geq 0$, we get

$$\begin{aligned}\frac{\partial h}{\partial r}(R,r) &= -r\mu\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\left(\int_{t_1}^{t_2} b(t)\left(R^2+r^2|e(t)|^2\right)\Delta\right)^{\mu/2-1}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2}\\ &\leq -r\mu\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\left(\int_{t_1}^{t_2} R^2 b(t)\Delta\right)^{\mu/2-1}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2}\\ &\leq -r\mu R^{\mu-2}\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta.\end{aligned} \tag{3.30}$$

(3.30) and $h(R,0)=0$ imply that

$$h(R,r)\leq -\frac{1}{2}r^2\mu R^{\mu-2}\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta \quad \forall\ R\geq 0, \quad r\geq 0. \tag{3.31}$$

Note that there exists $R_1 > \rho > 0$ such that

$$\int_0^T \left|e^{\Delta}(t)\right|^2\Delta - \mu R^{\mu-2}\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta \leq -\frac{2}{r_3}\|e\|_\infty \|g\|_{L^1(\mathbb{T})} \quad \forall\ R\geq R_1. \tag{3.32}$$

Therefore, by using (3.24), (3.26), (3.31), and (3.32), for $x = R, r \in [r_3, R]$ and $R \geq R_1$, one has

$$\begin{aligned}
\varphi(x+re) &= \varphi_1(x+re) + \varphi_2(x+re) + \varphi_3(x+re) \\
&\leq \frac{1}{2}r^2\int_0^T |e^{\Delta}(t)|^2\Delta + h(R,r) + r\|e\|_\infty \|g\|_{L^1(\mathbb{T})} \\
&\leq \frac{1}{2}r^2\int_0^T |e^{\Delta}(t)|^2\Delta - \frac{1}{2}r^2\mu R^{\mu-2}\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta + r\|e\|_\infty \|g\|_{L(1)} \\
&\leq -\frac{r}{r_3}(r-r_3)\|e\|_\infty \|g\|_{L^1(\mathbb{T})} \leq 0.
\end{aligned} \tag{3.33}$$

Thus, (3.28) and (3.33) can lead to

$$\varphi(x+re) \leq \frac{\alpha}{2}, \quad \forall\ |x| = R, r \in [0,R], \quad R \geq R_1. \tag{3.34}$$

For all $x \in \mathbb{R}^n$ and $R \geq 0$, denote

$$f(x,R) \triangleq -\left(\int_{t_1}^{t_2} b(t)(|x|^2 + R^2|e(t)|^2)\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2} + \int_{t_1}^{t_2} b(t)|x|^{\mu}\Delta. \tag{3.35}$$

By using the similar way to inequality (3.26), one has

$$\varphi_2(x+Re) \leq f(x,R) \quad \forall\ x \in \mathbb{R}^n, \quad R \geq 0, \tag{3.36}$$

for all $x \in \mathbb{R}^n$ and $R \geq 0$, since

$$\begin{aligned}
\frac{\partial f}{\partial R}(x,R) &= -R\mu\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\left(\int_{t_1}^{t_2} b(t)(|x|^2 + R^2|e(t)|^2)\Delta\right)^{\mu/(2-1)}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2} \\
&\leq -\mu R^{\mu-1}\left(\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2}.
\end{aligned} \tag{3.37}$$

In view of (3.37) and $f(x,0) = 0$, for all $x \in \mathbb{R}^n$ and $R \geq 0$, we get

$$f(x,R) \leq -R^{\mu}\left(\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2}. \tag{3.38}$$

From (3.24), (3.36), and (3.38), for all $x \in \mathbb{R}^n$ and $R \geq 0$, we obtain

$$\begin{aligned}\varphi(x+Re) &= \varphi_1(x+Re)+\varphi_2(x+Re)+\varphi_3(x+Re)\\ &\leq \frac{1}{2}R^2\int_0^T \left|e^{\Delta}(t)\right|^2\Delta - R^{\mu}\left(\int_{t_1}^{t_2} b(t)|e(t)|^2\Delta\right)^{\mu/2}\left(\int_{t_1}^{t_2} b(t)\Delta\right)^{(2-\mu)/2} + R\|e\|_{\infty}\|g\|_{L^1(\mathbb{T})},\end{aligned} \tag{3.39}$$

which implies that there exists $R_2 > R_1$, such that

$$\varphi(x+Re) \leq 0 \quad \forall x \in \mathbb{R}^n, \quad R \geq R_2. \tag{3.40}$$

Now let

$$Q = \{x + re \mid x \in \mathbb{R}^n, \|x\| = R, r \in [0,R], R \geq R_2\}. \tag{3.41}$$

Thus, (3.19), (3.34), and (3.40) lead to $\varphi|_{\partial Q} \leq (1/2)\alpha$, which means that the condition (ii) of Lemma 2.5 is satisfied.

It is easy to see that S and ∂Q link. Hence, all the conditions of the generalized mountain pass theorem are satisfied. By Lemma 2.5, the problem (1.1) has at least *one* nonzero solution. □

4. An Example

In this section, we present a simple example to illustrate our result.

Let

$$\mathbb{T} = [0,0.3] \cup \{0.4, 0.45, 0.5, 0.55, 0.6\} \cup [0.7,1], \qquad T = 1. \tag{4.1}$$

Consider the following second-order Hamiltonian system on time scales $\mathbb{T}$ of the form

$$\begin{gathered}u^{\Delta\Delta}(\rho(t)) + \mu b(t)|u(t)|^{\mu-2}u(t) + \overline{\nabla}H(t,u(t)) = 0, \Delta\text{-a.e. } t \in [0,1]_{\mathbb{T}},\\ u(0)-u(1) = u^{\Delta}(0)-u^{\Delta}(1) = 0,\end{gathered} \tag{4.2}$$

where $\mu > 2$ is a constant; let $\varepsilon > 0$ is arbitrary small, $H(t,u) = \varepsilon u^2(t)$ for Δ-*a.e.* $t \in [0,1]_{\mathbb{T}}$ and

$$b(t) = \begin{cases} t, & t \in [0,0.3]_{\mathbb{T}},\\ -1.5t+0.75, & t \in [0.3,0.7]_{\mathbb{T}},\\ t-1, & t \in [0.7,1]_{\mathbb{T}}.\end{cases} \tag{4.3}$$

It is easy to verify that all the conditions of Theorem 3.2 are satisfied. By Theorem 3.2, we see that the problem (4.2) has at least *one* nonzero solution.

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