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Research Article

Dynamic Inequalities on Time Scales with Applications in Permanence of Predator-Prey System

Meng Hu and Lili Wang

Department of Mathematics, Anyang Normal University, Anyang, Henan 455000, China

Correspondence should be addressed to Meng Hu, humeng2001@126.com

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By using the theory of calculus on time scales and some mathematical methods, several dynamic inequalities on time scales are established. Based on these results, we derive some sufficient conditions for permanence of predator-prey system incorporating a prey refuge on time scales. Finally, examples and numerical simulations are presented to illustrate the feasibility and effectiveness of the results.

1. Introduction

An important and ubiquitous problem in predator-prey theory and related topics in mathematical ecology concerns the long-term coexistence of species. In the past few years, permanence of different classes of continuous or discrete ecosystem has been studied wildly both in theories and applications; we refer the readers to [1–6] and the references therein.

However, in the natural world, there are many species whose developing processes are both continuous and discrete. Hence, using the only differential equation or difference equation cannot accurately describe the law of their developments. Therefore, there is a need to establish correspondent dynamic models on new time scales.

The theory of calculus on time scales (see [7] and references cited therein) was initiated by Stefan Hilger in his Ph.D. thesis in 1988 [8] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work; one may see [9–15]. Therefore, it is practicable to study that on time scales which can unify the continuous and discrete situations. However, to the best of the authors' knowledge, there are few papers considered permanence of predator-prey system on time scales.

Motivated by the previous, in this paper, we first establish some dynamic inequalities on time scales by using the theory of calculus on time scales and some mathematical methods, then, based on these results, as an application, we will study the permanence of the following delayed predator-prey system incorporating a prey refuge with Michaelis-Menten and Beddington-DeAngelis functional response on time scales:

$$\begin{aligned}
x^{\Delta}(t) &= x(t)\left[b(t) - c(t)x(\delta_-(\tau_1,t)) - \frac{d(t)y(t)(1-m)}{a(t)y(t)+x(t)(1-m)}\right.\\
&\qquad \left. - \frac{h(t)z(t)(1-m)}{k(t)+p(t)x(t)(1-m)+n(t)z(t)}\right],\\
y^{\Delta}(t) &= y(t)\left[-d_1(t) + \frac{f(t)x(\delta_-(\tau_2,t))(1-m)}{a(t)y(\delta_-(\tau_2,t))+x(\delta_-(\tau_2,t))(1-m)}\right],\\
z^{\Delta}(t) &= z(t)\left[-d_2(t) + \frac{g(t)x(\delta_-(\tau_3,t))(1-m)}{k(t)+p(t)x(\delta_-(\tau_3,t))(1-m)+n(t)z(\delta_-(\tau_3,t))}\right],
\end{aligned} \tag{1.1}$$

where $t \in \mathbb{T}$, $\mathbb{T}$ is a time scale. $x(t)$ denotes the density of prey specie and $y(t)$ and $z(t)$ denote the density of two predators species. $a(t)$, $b(t)$, $c(t)$, $d(t)$, $f(t)$, $g(t)$, $h(t)$, $k(t)$, $p(t)$, $n(t)$, $d_1(t)$, $d_2(t)$ are continuous, positive, and bounded functions, $m \in [0,1]$ is a constant, and m denotes the prey refuge parameter. $\delta_-(\tau_i,t)$, $i = 1,2,3$, are delay functions with $t \in \mathbb{T}$ and $\tau_i \in [0,\infty)_{\mathbb{T}}$, $i = 1,2,3$, where δ_- be a backward shift operator on the set $\mathbb{T}^*$ and $\mathbb{T}^*$ is a nonempty subset of the time scale $\mathbb{T}$. For the ecological justification of (1.1), one can refer to [2, 12].

The initial conditions of (1.1) are of the form

$$x(t) = \varphi_1(t), \qquad y(t) = \varphi_2(t), \qquad z(t) = \varphi_3(t), \quad t \in [\delta_-(\tau,0),0]_{\mathbb{T}}, \quad \varphi_i(0) > 0, \quad i = 1,2,3, \tag{1.2}$$

where $\tau = \max\{\tau_i\}, \tau_i \geq 0, i = 1,2,3$.

For convenience, we introduce the notation

$$f^u = \sup_{t\in\mathbb{T}}\{f(t)\}, \qquad f^l = \inf_{t\in\mathbb{T}}\{f(t)\}, \tag{1.3}$$

where f is a positive and bounded function.

2. Dynamic Inequalities on Time Scales

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$, and the graininess μ: $\mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \qquad \mu(t) = \sigma(t) - t. \tag{2.1}$$

A point $t \in \mathbb{T}$ is called left dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left scattered if $\rho(t) < t$, right dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

The basic theories of calculus on time scales, one can see [7].

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $p : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R})$. We define the set $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0, \text{ for all } t \in \mathbb{T}\}$.

If r is a regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau\right\} \tag{2.2}$$

for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \dfrac{\operatorname{Log}(1+hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases} \tag{2.3}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions and define

$$p \oplus q = p + q + \mu pq, \qquad \ominus p = -\frac{p}{1+\mu p}, \qquad p \ominus q = p \oplus (\ominus q). \tag{2.4}$$

Lemma 2.1 (See [7]). *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;

(iii) $e_p(t,s) = 1/e_p(s,t) = e_{\ominus p}(s,t)$;

(iv) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(v) $(e_{\ominus p}(t,s))^\Delta = (\ominus p)(t)e_{\ominus p}(t,s)$.

Lemma 2.2. *Assume that $a > 0$, $b > 0$ and $-a \in \mathcal{R}^+$. Then*

$$y^\Delta(t) \geq (\leq) b - ay(t), \quad y(t) > 0, \ t \in [t_0, \infty)_\mathbb{T} \tag{2.5}$$

implies

$$y(t) \geq (\leq) \frac{b}{a}\left[1 + \left(\frac{ay(t_0)}{b} - 1\right)e_{(-a)}(t, t_0)\right], \quad t \in [t_0, \infty)_\mathbb{T}. \tag{2.6}$$

Proof. We only prove the "≥" case; the proof of the "≤" case is similar. For

$$
\begin{aligned}
\left[y e_{\ominus(-a)}(\cdot, t_0)\right]^{\Delta}(t) &= y^{\Delta}(t) e_{\ominus(-a)}(\sigma(t), t_0) + y(t)(\ominus(-a)) e_{\ominus(-a)}(t, t_0) \\
&= y^{\Delta}(t) e_{\ominus(-a)}(\sigma(t), t_0) + y(t) \frac{(\ominus(-a))}{1+\mu(t)(\ominus(-a))} e_{\ominus(-a)}(\sigma(t), t_0) \\
&= \left[y^{\Delta}(t) - (\ominus(\ominus(-a))) y(t)\right] e_{\ominus(-a)}(\sigma(t), t_0) \\
&= \left[y^{\Delta}(t) - (-a) y(t)\right] e_{\ominus(-a)}(\sigma(t), t_0),
\end{aligned}
\tag{2.7}
$$

then, integrate both side from t_0 to t to conclude

$$
\begin{aligned}
y(t) e_{\ominus(-a)}(t, t_0) - y(t_0) &= \int_{t_0}^{t} \left[y^{\Delta}(s) - (-a) y(s)\right] e_{\ominus(-a)}(\sigma(s), t_0) \Delta s \\
&\geq \int_{t_0}^{t} b e_{\ominus(-a)}(\sigma(s), t_0) \Delta s \\
&= b \int_{t_0}^{t} e_{(-a)}(t_0, \sigma(s)) \Delta s,
\end{aligned}
\tag{2.8}
$$

that is, $y(t) \geq y(t_0) e_{(-a)}(t, t_0) + b \int_{t_0}^{t} e_{(-a)}(t, \sigma(s)) \Delta s$. So

$$
\begin{aligned}
y(t) &\geq y(t_0) e_{(-a)}(t, t_0) + b \int_{t_0}^{t} e_{(-a)}(t, \sigma(s)) \Delta s \\
&= y(t_0) e_{(-a)}(t, t_0) - \frac{b}{a} \int_{t_0}^{t} e_{(-a)}(t, \sigma(s))(-a) \Delta s \\
&= y(t_0) e_{(-a)}(t, t_0) - \frac{b}{a}\left[e_{(-a)}(t, t_0) - 1\right] \\
&= e_{(-a)}(t, t_0)\left[y(t_0) - \frac{b}{a}\right] + \frac{b}{a},
\end{aligned}
\tag{2.9}
$$

that is, $y(t) \geq (b/a)[1 + (a y(t_0)/b - 1) e_{(-a)}(t, t_0)]$. This completes the proof. □

Lemma 2.3. *Assume that* $a > 0$, $b > 0$. *Then*

$$
y^{\Delta}(t) \geq (\leq) b - a y(\sigma(t)), \qquad y(t) > 0, \ t \in [t_0, \infty)_{\mathbb{T}}
\tag{2.10}
$$

implies

$$
y(t) \geq (\leq) \frac{b}{a}\left[1 + \left(\frac{a y(t_0)}{b} - 1\right) e_{\ominus a}(t, t_0)\right], \qquad t \in [t_0, \infty)_{\mathbb{T}}.
\tag{2.11}
$$

Proof. We only prove the "≥" case; the proof of the "≤" case is similar. For

$$[ye_a(\cdot,t_0)]^{\Delta}(t) = y^{\Delta}(t)e_a(t,t_0) + ae_a(t,t_0)y(\sigma(t)) = e_a(t,t_0)\left[y^{\Delta}(t) + ay(\sigma(t))\right], \tag{2.12}$$

then, integrate both side from t_0 to t to conclude

$$\begin{aligned} y(t)e_a(t,t_0) - y(t_0) &= \int_{t_0}^{t} e_a(s,t_0)\left[y^{\Delta}(s) + ay(\sigma(s))\right]\Delta s \\ &\geq b\int_{t_0}^{t} e_a(s,t_0)\Delta s, \end{aligned} \tag{2.13}$$

then

$$\begin{aligned} y(t) &\geq e_{\ominus a}(t,t_0)y(t_0) + b\int_{t_0}^{t} e_{\ominus a}(t,s)\Delta s \\ &= e_{\ominus a}(t,t_0)y(t_0) + b\int_{t_0}^{t} (1+\mu(s)a)e_a(s,t)\frac{1}{1+\mu(s)a}\Delta s \\ &= e_{\ominus a}(t,t_0)y(t_0) + b\int_{t_0}^{t} e_a(\sigma(s),t)\frac{1}{1+\mu(s)a}\Delta s \\ &= e_{\ominus a}(t,t_0)y(t_0) + b\int_{t_0}^{t} e_{\ominus a}(t,\sigma(s))\frac{1}{1+\mu(s)a}\Delta s \\ &= e_{\ominus a}(t,t_0)y(t_0) - \frac{b}{a}\int_{t_0}^{t} e_{\ominus a}(t,\sigma(s))(\ominus a)\Delta s \\ &= e_{\ominus a}(t,t_0)y(t_0) - \frac{b}{a}(e_{\ominus a}(t,t_0) - 1) \\ &= e_{\ominus a}(t,t_0)\left[y(t_0) - \frac{b}{a}\right] + \frac{b}{a}, \end{aligned} \tag{2.14}$$

that is, $y(t) \geq (b/a)[1 + (ay(t_0)/b - 1)e_{\ominus a}(t,t_0)]$. This completes the proof. □

Lemma 2.4. *Assume that* $a > 0$, $b > 0$ *and* $-b \in \mathcal{R}^+$. *Then*

$$y^{\Delta}(t) \leq (\geq)y(\sigma(t))(b - ay(t)), \quad y(t) > 0, \ t \in [t_0,\infty)_{\mathbb{T}} \tag{2.15}$$

implies

$$y(t) \leq (\geq)\frac{b}{a}\left[1 + \left(\frac{b}{ay(t_0)} - 1\right)e_{(-b)}(t,t_0)\right]^{-1}, \quad t \in [t_0,\infty)_{\mathbb{T}}. \tag{2.16}$$

Proof. We only prove the "≤" case; the proof of the "≥" case is similar.

Let $y(t) = 1/x(t)$, then $(1/x(t))^{\Delta} \leq (1/x(\sigma(t)))(b - a/x(t))$, that is, $-x^{\Delta}(t)/(x(t)x(\sigma(t))) \leq (1/x(\sigma(t)))(b - a/x(t))$, so, $x^{\Delta}(t) \geq a - bx(t)$. By Lemma 2.2, we have $x(t) \geq (a/b)[1 + (bx(t_0)/a - 1)e_{(-b)}(t,t_0)]$. Therefore, $y(t) \leq (b/a)[1 + (b/ay(t_0) - 1)e_{(-b)}(t,t_0)]^{-1}$. This completes the proof. □

Lemma 2.5. *Assume that $a > 0$, $b > 0$. Then*

$$y^{\Delta}(t) \leq (\geq) y(t)\big(b - ay(\sigma(t))\big), \quad y(t) > 0,\ t \in [t_0, \infty)_{\mathbb{T}} \tag{2.17}$$

implies

$$y(t) \leq (\geq) \frac{b}{a}\left[1 + \left(\frac{b}{ay(t_0)} - 1\right)e_{\ominus b}(t,t_0)\right], \quad t \in [t_0,\infty)_{\mathbb{T}}. \tag{2.18}$$

Proof. We only prove the "≤" case; the proof of the "≥" case is similar.

Let $y(t) = 1/x(t)$, then $(1/x(t))^{\Delta} \leq (1/x(t))(b - a/x(\sigma(t)))$, that is, $-x^{\Delta}(t)/(x(t)x(\sigma(t))) \leq (1/x(t))(b - a/x(\sigma(t)))$, so, $x^{\Delta}(t) \geq a - bx(\sigma(t))$. By Lemma 2.3, we have $x(t) \geq (a/b)[1 + (bx(t_0)/a - 1)e_{\ominus b}(t,t_0)]$. Therefore, $y(t) \leq (b/a)[1 + (b/ay(t_0)-1)e_{\ominus b}(t,t_0)]^{-1}$. This completes the proof. □

Definition 2.6 (see [16]). Let $\mathbb{T}^*$ be a nonempty subset of the time scale $\mathbb{T}$ and $t_0 \in \mathbb{T}^*$ a fixed number. The operator δ_- associated with $t_0 \in \mathbb{T}^*$ (called the initial point) is said to be backward shift operator on the set $\mathbb{T}^*$. The variable $s \in [t_0,\infty)_{\mathbb{T}}$ *in* $\delta_-(s,t)$ is called the shift size. The value $\delta_-(s,t)$ in $\mathbb{T}^*$ indicate s units translation of the term $t \in \mathbb{T}^*$ to the left.

Now, we state some different time scales with their corresponding backward shift operators: let $\mathbb{T} = \mathbb{R}$ and $t_0 = 0$; then $\delta_-(s,t) = t - s$; let $\mathbb{T} = \mathbb{Z}$ and $t_0 = 0$; then $\delta_-(s,t) = t - s$; let $\mathbb{T} = h\mathbb{Z}$ and $t_0 = 0$; then $\delta_-(s,t) = t - s$; let $\mathbb{T} = \mathbb{N}^{1/2}$ and $t_0 = 0$; then $\delta_-(s,t) = \sqrt{t^2 - s^2}$; let $\mathbb{T} = 2^{\mathbb{N}}$ and $t_0 = 1$; then $\delta_-(s,t) = t/s$; and so on.

Lemma 2.7. *If $a \in \mathcal{R}^+$ and $y^{\Delta}(t) \leq (\geq) a(t)y(t), t \in [t_0,\infty)_{\mathbb{T}}$, then*

$$y(\delta_-(s,t)) \geq (\leq) e_{\ominus a}(\sigma(t), \delta_-(s,t)) y(\sigma(t)), \tag{2.19}$$

where $\delta_-(s,t)$ be defined in Definition 2.6 and $(s,t) \in [t_0,\infty)_{\mathbb{T}} \times [s,\infty)_{\mathbb{T}}$.

Proof. We only prove the "≤" case; the proof of the "≥" case is similar. For

$$\begin{aligned}
\left[ye_{\ominus a}(\cdot,t_0)\right]^{\Delta}(t) &= y^{\Delta}(t)e_{\ominus a}(\sigma(t),t_0) + y(t)(\ominus a)(t)e_{\ominus a}(t,t_0) \\
&= y^{\Delta}(t)e_{\ominus a}(\sigma(t),t_0) + y(t)\frac{(\ominus a)(t)}{1+\mu(t)(\ominus a)(t)}e_{\ominus a}(\sigma(t),t_0) \\
&= \left[y^{\Delta}(t) - (\ominus(\ominus a))(t)y(t)\right]e_{\ominus a}(\sigma(t),t_0) \\
&= \left[y^{\Delta}(t) - a(t)y(t)\right]e_{\ominus a}(\sigma(t),t_0) \\
&\leq 0,
\end{aligned} \tag{2.20}$$

then, integrate both side from $\delta_-(s,t)$ to $\sigma(t)$ to conclude

$$y(\sigma(t))e_{\ominus a}(\sigma(t),t_0) \le y(\delta_-(s,t))e_{\ominus a}(\delta_-(s,t),t_0), \tag{2.21}$$

then

$$y(\delta_-(s,t)) \ge e_{\ominus a}(\sigma(t),\delta_-(s,t))y(\sigma(t)). \tag{2.22}$$

This completes the proof. □

3. Permanence

As an application, based on the results obtained in Section 2, we will establish a permanent result for system (1.1).

Definition 3.1. System (1.1) is said to be permanent if there exists a compact region $D \subseteq \operatorname{Int}\mathbb{R}^3_+$, such that for any positive solution $(x(t),y(t),z(t))$ of system (1.1) with initial condition (1.2) eventually enters and remains in region D.

In this section, we consider the time scale $\mathbb{T}$ that satisfies $e_a(\sigma(t),\delta_-(\tau,t))$ to be a constant on $[0,\infty)_{\mathbb{T}}$, where $a \in \mathcal{R}^+$, $\tau \in [0,\infty)_{\mathbb{T}}$ are constants. For example, let $t_0 = 0$ (the initial point), when $\mathbb{T} = \mathbb{R}$, then $e_a(\sigma(t),\delta_-(\tau,t)) = e^{a\tau}$; when $\mathbb{T} = \mathbb{Z}$, then $e_a(\sigma(t),\delta_-(\tau,t)) = (1+a)^{1+\tau}$; when $\mathbb{T} = h\mathbb{Z}$, then $e_a(\sigma(t),\delta_-(\tau,t)) = (1+ah)^{1+\tau/h}$, and so on.

For convenience, we introduce the following notations:

$$\begin{aligned}
M_1 &= \frac{b^u e_{b^u}(\sigma(t),\delta_-(\tau_1,t))}{c^l} + \varepsilon,\\
M_2 &= \frac{\left(f^u - d_1^l\right)(1-m)M_1 e_{(f^u-d_1^l)}(\sigma(t),\delta_-(\tau_2,t))}{a^l d_1^l} + \varepsilon,\\
M_3 &= \frac{\left(\left(g^u - p^l d_2^l\right)/p^l - k^l d_2^l/(p^u(1-m)M_1)\right)p^u M_1(1-m)e_{(g^u/p^l-d_2^l)}(\sigma(t),\delta_-(\tau_3,t))}{d_2^l n^l} + \varepsilon,\\
m_1 &= \frac{\left[b^l - d^u(1-m)/a^l - h^u(1-m)/n^l\right]e_{\beta^l}(\sigma(t),\delta_-(\tau_1,t))}{c^u} - \varepsilon,\\
m_2 &= \frac{(m_1/2)\left(f^l - d_1^u\right)(1-m)e_{(-d_1^u)}(\sigma(t),\delta_-(\tau_2,t))}{a^u d_1^u} - \varepsilon,\\
m_3 &= \frac{\left[(m_1/2)p^l(1-m)\left(g^l - d_2^u\right) - k^u d_2^u\right]e_{(-d_2^u)}(\sigma(t),\delta_-(\tau_3,t))}{n^u d_2^u} - \varepsilon,
\end{aligned} \tag{3.1}$$

where $\beta^l = b^l - d^u(1-m)/a^l - h^u(1-m)/n^l - c^u M_1$.

Hereafter, we assume that

(H_1) $f(t) - d_1(t) \in \mathcal{R}^+$;

(H_2) $f^u - d_1^l > 0$;

(H_3) $g(t)/p(t) - d_2(t) \in \mathcal{R}^+$;

(H_4) $(g^u - p^l d_2^l)/p^l - k^l d_2^l/(p^u(1-m)M_1) > 0$;

(H_5) $b(t) - d(t)(1-m)/a(t) - h(t)(1-m)/n(t) - c(t)M_1 \in \mathcal{R}^+$;

(H_6) $b^l - d^u(1-m)/a^l - h^u(1-m)/n^l > 0$;

(H_7) $-d_1(t) \in \mathcal{R}^+$;

(H_8) $f^l - d_1^u > 0$;

(H_9) $-d_2(t) \in \mathcal{R}^+$;

(H_{10}) $(m_1/2)(1-m)(g^l - d_2^u p^u) - k^u d_2^u > 0$.

Proposition 3.2. *Assume that* $(x(t), y(t), z(t))$ *is any positive solution of system* (1.1) *with initial condition* (1.2). *If (H1)–(H_4) hold, then*

$$x(t) \le M_1, \qquad y(t) \le M_2, \qquad z(t) \le M_3. \tag{3.2}$$

Proof. Assume that $(x(t), y(t), z(t))$ is any positive solution of system (1.1) with initial condition (1.2). From the first equation of system (1.1), for $t \in [\tau_1, \infty)_{\mathbb{T}}$, we have

$$x^{\Delta}(t) \le x(t)(b(t) - c(t)x(\delta_-(\tau_1, t))). \tag{3.3}$$

From (3.3), we can see $x^{\Delta}(t) \le b(t)x(t)$; then by Lemma 2.7, we can get

$$x(\delta_-(\tau_1, t)) \ge e_{\ominus b}(\sigma(t), \delta_-(\tau_1, t))x(\sigma(t)). \tag{3.4}$$

Together with (3.3) and (3.4), we have

$$\begin{aligned} x^{\Delta}(t) &\le x(t)(b(t) - c(t)e_{\ominus b}(\sigma(t), \delta_-(\tau_1, t))x(\sigma(t))) \\ &\le x(t)\Big(b^u - c^l e_{\ominus b^u}(\sigma(t), \delta_-(\tau_1, t))x(\sigma(t))\Big). \end{aligned} \tag{3.5}$$

By Lemma 2.5, for arbitrary small positive constant ε, there exists $T_1 > 0$ such that

$$\begin{aligned} x(t) &\le \frac{b^u}{c^l e_{\ominus b^u}(\sigma(t), \delta_-(\tau_1, t))} + \varepsilon \\ &= \frac{b^u e_{b^u}(\sigma(t), \delta_-(\tau_1, t))}{c^l} + \varepsilon := M_1, \quad t \in [T_1, \infty)_{\mathbb{T}}. \end{aligned} \tag{3.6}$$

Again, from the second equation of system (1.1) and (3.6), for $t \in [T_1 + \tau_2, \infty)_{\mathbb{T}}$, we have

$$y^{\Delta}(t) \le y(t)\left(-d_1(t) + \frac{f(t)(1-m)M_1}{a(t)y(\delta_-(\tau_2,t)) + (1-m)M_1}\right). \tag{3.7}$$

From (3.7), we can see $y^{\Delta}(t) \le (-d_1(t) + f(t))y(t)$; then by (H_1) and Lemma 2.7, we can get

$$y(\delta_-(\tau_2,t)) \ge e_{(\ominus\alpha)}(\sigma(t), \delta_-(\tau_2,t))y(\sigma(t)), \tag{3.8}$$

where $\alpha(t) = f(t) - d_1(t)$.

Together with (3.7) and (3.8), we have

$$\begin{aligned} y^{\Delta}(t) &\le y(t)\left(-d_1(t) + \frac{f(t)(1-m)M_1}{a(t)e_{(\ominus\alpha)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t)) + (1-m)M_1}\right) \\ &\le y(t)\left(\frac{(1-m)M_1\big(f(t)-d_1(t)\big) - a(t)d_1(t)e_{(\ominus\alpha)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t))}{a(t)e_{(\ominus\alpha)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t)) + (1-m)M_1}\right) \\ &\le y(t)\left(f^u - d_1^l - \frac{a^l d_1^l e_{\ominus(f^u - d_1^l)}(\sigma(t),\delta_-(\tau_2,t))}{(1-m)M_1}y(\sigma(t))\right). \end{aligned} \tag{3.9}$$

By (H_2) and Lemma 2.5, for arbitrary small positive constant ε, there exists $T_2 > T_1 + \tau_2$ such that

$$\begin{aligned} y(t) &\le \frac{\left(f^u - d_1^l\right)(1-m)M_1}{a^l d_1^l e_{\ominus(f^u - d_1^l)}(\sigma(t),\delta_-(\tau_2,t))} + \varepsilon \\ &= \frac{\left(f^u - d_1^l\right)(1-m)M_1 e_{(f^u - d_1^l)}(\sigma(t),\delta_-(\tau_2,t))}{a^l d_1^l} + \varepsilon := M_2, \quad t \in [T_2,\infty)_{\mathbb{T}}. \end{aligned} \tag{3.10}$$

Similarly, under conditions (H_3)-(H_4), by Lemmas 2.5 and 2.7, we can get that, for arbitrary small positive constant ε, there exists $T_3 > T_2 + \tau_3$ such that

$$\begin{aligned} z(t) \le{} & \frac{\left(\left(g^u - p^l d_2^l\right)/p^l - k^l d_2^l/\left(p^u(1-m)M_1\right)\right)p^u M_1(1-m)e_{(g^u/p^l - d_2^l)}(\sigma(t),\delta_-(\tau_3,t))}{d_2^l n^l} \\ & + \varepsilon := M_3, \quad t \in [T_3,\infty)_{\mathbb{T}}. \end{aligned} \tag{3.11}$$

The conclusion of Proposition 3.2 follows. This completes the proof. □

Proposition 3.3. *Assume that* $(x(t), y(t), z(t))$ *is any positive solution of system* (1.1) *with initial condition* (1.2). *If* (H_1)*–*(H_{10}) *hold, then*

$$x(t) \geq m_1, \qquad y(t) \geq m_2, \qquad z(t) \geq m_3. \tag{3.12}$$

Proof. Assume that $(x(t), y(t), z(t))$ is any positive solution of system (1.1) with initial condition (1.2). From Theorem 3.4, there exists a $T_1 > 0$, such that $x(t) \leq M_1,\ t \in [T_1, \infty)_{\mathbb{T}}$. By the first equation of system (1.1), for $t \in [T_1 + \tau_1, \infty)_{\mathbb{T}}$, we have

$$x^{\Delta}(t) \geq x(t)\left(b(t) - \frac{d(t)(1-m)}{a(t)} - \frac{h(t)(1-m)}{n(t)} - c(t)x(\delta_{-}(\tau_1, t))\right), \tag{3.13}$$

then

$$x^{\Delta}(t) \geq x(t)\left(b(t) - \frac{d(t)(1-m)}{a(t)} - \frac{h(t)(1-m)}{n(t)} - c(t)M_1\right). \tag{3.14}$$

By (3.14), (H_5), and Lemma 2.7, we can get

$$x(\delta_{-}(\tau_1, t)) \leq e_{\ominus\beta}(\sigma(t), \delta_{-}(\tau_1, t))x(\sigma(t)), \tag{3.15}$$

where $\beta = b(t) - d(t)(1-m)/a(t) - h(t)(1-m)/n(t) - c(t)M_1$.

Together with (3.13) and (3.15), we have

$$\begin{aligned} x^{\Delta}(t) &\geq x(t)\left(b(t) - \frac{d(t)(1-m)}{a(t)} - \frac{h(t)(1-m)}{n(t)} - c(t)e_{\ominus\beta}(\sigma(t), \delta_{-}(\tau_1, t))x(\sigma(t))\right) \\ &\geq x(t)\left(b^l - \frac{d^u(1-m)}{a^l} - \frac{h^u(1-m)}{n^l} - c^u e_{\ominus\beta^l}(\sigma(t), \delta_{-}(\tau_1, t))x(\sigma(t))\right), \end{aligned} \tag{3.16}$$

where $\beta^l = b^l - d^u(1-m)/a^l - h^u(1-m)/n^l - c^u M_1$.

By (H_6) and Lemma 2.5, for arbitrary small positive constant ε, there exists $T_4 > T_1 + \tau_1$ such that

$$\begin{aligned} x(t) &\geq \frac{b^l - d^u(1-m)/a^l - h^u(1-m)/n^l}{c^u e_{\ominus\beta^l}(\sigma(t), \delta_{-}(\tau_1, t))} - \varepsilon \\ &= \frac{\left[b^l - d^u(1-m)/a^l - h^u(1-m)/n^l\right]e_{\beta^l}(\sigma(t), \delta_{-}(\tau_1, t))}{c^u} - \varepsilon := m_1, \quad t \in [T_4, \infty)_{\mathbb{T}}. \end{aligned} \tag{3.17}$$

Again, from the second equation of system (1.1) and (3.17), for $t \in [T_4 + \tau_2, \infty)_{\mathbb{T}}$, we have

$$y^{\Delta}(t) \geq y(t)\left(-d_1(t) + \frac{(m_1/2)f(t)(1-m)}{a(t)y(\delta_{-}(\tau_2, t)) + (m_1/2)(1-m)}\right). \tag{3.18}$$

From (3.18), we can see $y^{\Delta}(t) \geq -d_1(t)y(t)$; then by ($H_7$) and Lemma 2.7, we can get

$$y(\delta_-(\tau_2,t)) \leq e_{\ominus(-d_1)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t)). \tag{3.19}$$

Together with (3.18) and (3.19), we have

$$\begin{aligned} y^{\Delta}(t) &\geq y(t)\left(-d_1(t) + \frac{(m_1/2)f(t)(1-m)}{a(t)e_{\ominus(-d_1)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t)) + (m_1/2)(1-m)}\right) \\ &\geq y(t)\left(\frac{(m_1/2)\left(f^l - d_1^u\right)(1-m) - a^u d_1^u e_{\ominus(-d_1^u)}(\sigma(t),\delta_-(\tau_2,t))y(\sigma(t))}{a^u e_{\ominus(-d_1^u)}(\sigma(t),\delta_-(\tau_2,t))M_2 + (m_1/2)(1-m)}\right). \end{aligned} \tag{3.20}$$

By (H_8) and Lemma 2.5, for arbitrary small positive constant ε, there exists $T_5 > T_4 + \tau_2$ such that

$$\begin{aligned} y(t) &\geq \frac{(m_1/2)\left(f^l - d_1^u\right)(1-m)}{a^u d_1^u e_{\ominus(-d_1^u)}(\sigma(t),\delta_-(\tau_2,t))} - \varepsilon \\ &= \frac{(m_1/2)\left(f^l - d_1^u\right)(1-m)e_{(-d_1^u)}(\sigma(t),\delta_-(\tau_2,t))}{a^u d_1^u} - \varepsilon := m_2, \quad t \in [T_5,\infty)_{\mathbb{T}}. \end{aligned} \tag{3.21}$$

Similarly, under conditions (H_9) and (H_{10}), by Lemmas 2.5 and 2.7, we can get that, for arbitrary small positive constant ε, there exists $T_6 > T_5 + \tau_3$ such that

$$\begin{aligned} z(t) \geq{}& \frac{\left[(m_1/2)p^l(1-m)\left(g^l - d_2^u\right) - k^u d_2^u\right]e_{(-d_2^u)}(\sigma(t),\delta_-(\tau_3,t))}{n^u d_2^u} \\ &- \varepsilon := m_3, \quad t \in [T_6,\infty)_{\mathbb{T}}. \end{aligned} \tag{3.22}$$

The conclusion of Proposition 3.3 follows. This completes the proof. □

Together with Propositions 3.2 and 3.3, we can obtain the following theorem.

Theorem 3.4. *Assume that (H_1)–(H_{10}) hold; then system* (1.1) *is permanent.*

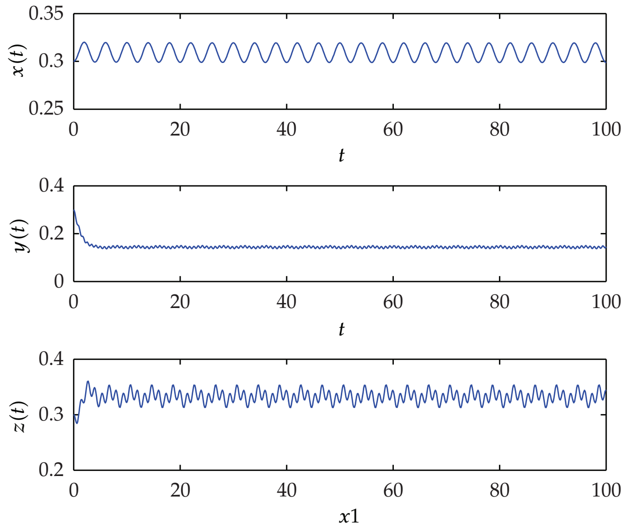

Figure 1: $\mathbb{T} = \mathbb{R}$, $m = 0.6$. Dynamics behavior of system (4.1) with initial condition $x(0) = 0.3$, $y(0) = 0.3$, $z(0) = 0.3$.

4. Examples and Simulations

Consider the following system on time scales with $m = 0.6$:

$$\begin{aligned}
x^{\Delta}(t) &= x(t)\left[\left(0.08 + 0.05\sin\frac{\pi}{2}t\right) - 0.2x(\delta_{-}(\tau_1, t)) - \frac{0.05y(t)(1-m)}{2y(t) + x(t)(1-m)}\right.\\
&\qquad\left. - \frac{0.05z(t)(1-m)}{0.005 + x(t)(1-m) + 2z(t)}\right],\\
y^{\Delta}(t) &= y(t)\left[-\left(0.7 - 0.25\cos\frac{5\pi}{2}t\right) + \frac{2x(\delta_{-}(\tau_2, t))(1-m)}{2y(\delta_{-}(\tau_2, t)) + x(\delta_{-}(\tau_2, t))(1-m)}\right],\\
z^{\Delta}(t) &= z(t)\left[-\left(0.75 + 0.2\sin\frac{3\pi}{2}t\right) + \frac{4x(\delta_{-}(\tau_3, t))(1-m)}{0.005 + x(\delta_{-}(\tau_3, t))(1-m) + 2z(\delta_{-}(\tau_3, t))}\right].
\end{aligned} \tag{4.1}$$

Let $\mathbb{T} = \mathbb{R}$; then $\mu(t) = 0$. Obviously, (H_1), (H_3), (H_5), (H_7), and (H_9) hold. Taking $\tau_i = 1$, $i = 1, 2, 3$, by a direct calculation, we can get

(H_2) $f^u - d_1^l = 1.5500 > 0$;

(H_4) $(g^u - p^l d_2^l)/p^l - k^l d_2^l/(p^u(1-m)M_1) = 3.4407 > 0$;

(H_6) $b^l - d^u(1-m)/a^l - h^u(1-m)/n^l = 0.0100 > 0$;

(H_8) $f^l - d_1^u = 1.0500 > 0$;

(H_{10}) $(m_1/2)(1-m)(g^l - d_2^u p^u) - k^u d_2^u = 0.0218 > 0$.

From the above results, we can see that all conditions of Theorem 3.4 hold. So, system (4.1) is permanent, see Figure 1.

Let $\mathbb{T} = \mathbb{Z}$; then $\mu(t) = 1$. It is easy to check (H_1), (H_3), (H_5), (H_7), and (H_9) hold. Taking $\tau_i = 1$, $i = 1, 2, 3$, by a direct calculation, we can get

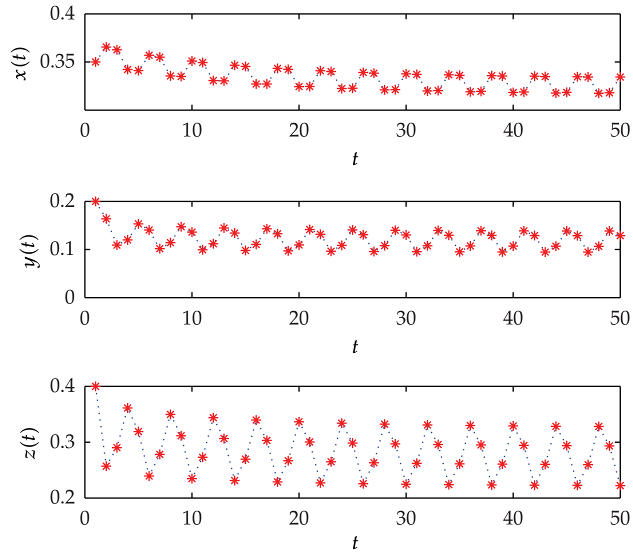

Figure 2: $\mathbb{T} = \mathbb{Z}$, $m = 0.6$. Dynamics behavior of system (4.1) with initial condition $x(0) = 0.35$, $y(0) = 0.2$, $z(0) = 0.4$.

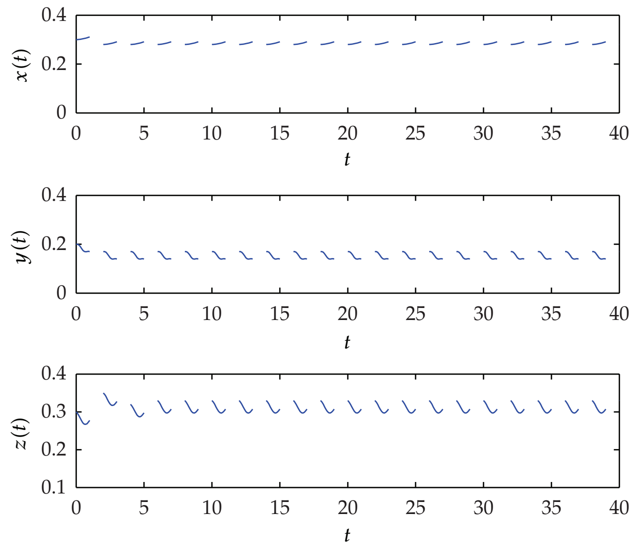

Figure 3: $\mathbb{T} = \bigcup_{k=0}^{\infty}[2k, 2k+1]$, $m = 0.6$. Dynamics behavior of system (4.1) with initial condition $x(0) = 0.3$, $y(0) = 0.2$, $z(0) = 0.3$.

(H_2) $f^u - d_1^l = 1.5500 > 0$;

(H_4) $(g^u - p^l d_2^l)/p^l - k^l d_2^l/(p^u(1-m)M_1) = 3.4086 > 0$;

(H_6) $b^l - d^u(1-m)/a^l - h^u(1-m)/n^l = 0.0100 > 0$;

(H_8) $f^l - d_1^u = 1.0500 > 0$;

(H_{10}) $(m_1/2)(1-m)(g^l - d_2^u p^u) - k^u d_2^u = 0.0244 > 0$.

From the above results, we can see that all conditions of Theorem 3.4 hold. So, system (4.1) is permanent, see Figure 2.

Let $\mathbb{T} = \bigcup_{k=0}^{\infty}[2k, 2k+1]$; then $\mu(t) = \begin{cases} 0, & \text{for } t \in \bigcup_{k=0}^{\infty}[2k,2k+1), \\ 1, & \text{for } t \in \bigcup_{k=0}^{\infty}\{2k+1\}. \end{cases}$ It is easy to check (H_1), (H_3), (H_5), (H_7), and (H_9) hold. Taking $\tau_i = 2, i = 1, 2, 3$, by a direct calculation, we can get

(H_2) $f^u - d_1^l = 1.5500 > 0$;

(H_6) $b^l - d^u(1-m)/a^l - h^u(1-m)/n^l = 0.0100 > 0$;

(H_8) $f^l - d_1^u = 1.0500 > 0$.

Furthermore, if $t \in \bigcup_{k=0}^{\infty}[2k, 2k+1)$, then

(H_4) $(g^u - p^l d_2^l)/p^l - k^l d_2^l/(p^u(1-m)M_1) = 3.4418 > 0$;

(H_{10}) $(m_1/2)(1-m)(g^l - d_2^u p^u) - k^u d_2^u = 0.0175 > 0$;

if $t \in \bigcup_{k=0}^{\infty}\{2k+1\}$, then

(H_4) $(g^u - p^l d_2^l)/p^l - k^l d_2^l/(p^u(1-m)M_1) = 3.4133 > 0$;

(H_{10}) $(m_1/2)(1-m)(g^l - d_2^u p^u) - k^u d_2^u = 0.0233 > 0$.

From the above results, we can see that all conditions of Theorem 3.4 hold. So, system (4.1) is permanent, see Figure 3.

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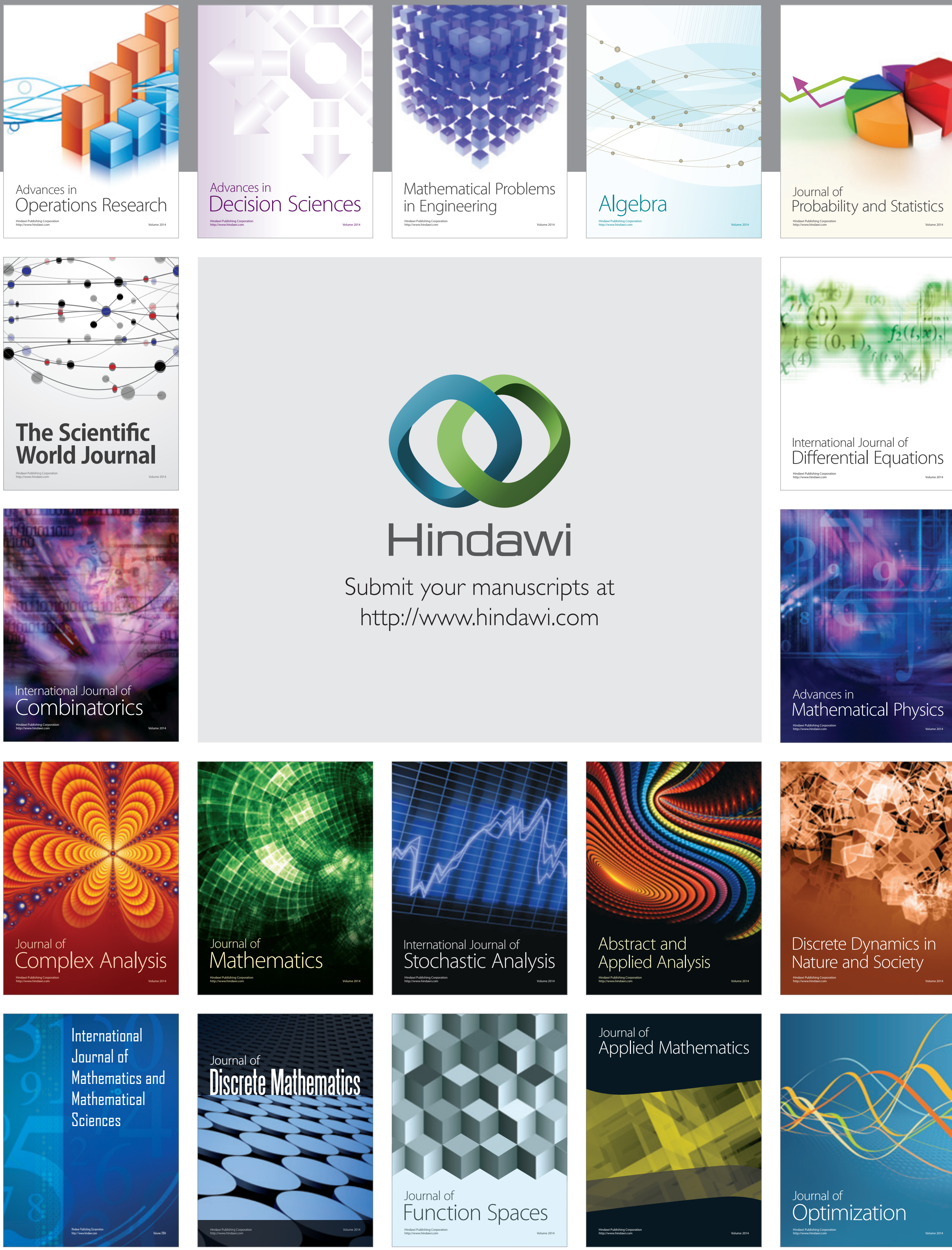
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