Internat. J. Math. & Math. Sci.
Vol. 10 No. 1 (1987) 1-8

HOLOMORPHIC EXTENSION OF GENERALIZATIONS OF H^p FUNCTIONS. II

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(Received July 28, 1986)

ABSTRACT. In a previous article we have obtained a holomorphic extension theorem (edge of the wedge theorem) concerning holomorphic functions in tubes in $\mathbb{C}^n$ which generalize the Hardy H^p functions for the cases $1 < p \leq 2$. In this paper we obtain a similar holomorphic extension theorem for the cases $2 < p < \infty$.

KEY WORDS AND PHRASES. Generalization of H^p Functions in Tube Domains, Holomorphic Extension, Fourier-Laplace Transform, Edge of the Wave Theorem.
1980 AMS SUBJECT CLASSIFICATION CODE. 32A07, 32A10, 32A25, 32A35, 32A40, 46F20.

1. INTRODUCTION.

This paper is a continuation of Carmichael [1]. The definitions of cone C in $\mathbb{R}^n$ with vertex at the origin $\bar{0} = (0, 0, \ldots, 0)$ in $\mathbb{R}^n$, regular cone, and projection of a cone are all contained in Carmichael [1, p. 417] as are the definitions of the indicatrix function $u_C(t)$ of the cone C and the number ρ_C which characterizes the nonconvexity of the cone C. $C^* = \{t \in \mathbb{R}^n: \langle y,t\rangle \geq 0$ for all $y \in C\}$ is the dual cone of the cone C; C^* is always closed and convex (Vladimirov [2, p. 218]). O(C) will denote the convex hull (convex envelope) of a cone C. Following Vladimirov [3, p. 930], we say that a cone $C \subset \mathbb{R}^n$ with interior points has an admissible set of vectors if there are vectors $e_k \in C$, $|e_k| = 1$, $k = 1,2, \ldots, n$, which form a basis for $\mathbb{R}^n$; equivalently we say that such a set of n vectors in C is admissible for the cone C. Let $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ be any of the 2^n n-tuples whose entries are 0 or 1. $C_\mu = \{y \in \mathbb{R}^n: (-1)^{\mu_j} y_j > 0, j = 1, \ldots, n\}$ is a quadrant in $\mathbb{R}^n$ and there are 2^n such quadrants. We note that any quadrant C_μ is a regular cone in $\mathbb{R}^n$.

Let $\mathcal{S}'$ denote the space of tempered distributions. The subspace $\mathcal{S}'_p$ of $\mathcal{S}'$, $1 \leq p < \infty$, is defined to be the set of all measurable functions $g(t)$, $t \in \mathbb{R}^n$, such that there exists a real number $b \geq 0$ for which $((1 + |t|^p)^{-b} g(t)) \in L^p$ (Carmichael [4, p. 83]).

We now define the norm growth and the space of functions which are of interest in this paper. Let B denote a proper open subset in $\mathbb{R}^n$ and let $T^B = \mathbb{R}^n + iB$ be the associated tube in $\mathbb{C}^n$. Let $0 < p < \infty$ and $A \geq 0$. Let $d(y)$ denote the distance from $y \in B$ to the complement of B in $\mathbb{R}^n$. The space $S_A^p(T^B)$ (Carmichael [4, pp. 80-81]) is the set of all functions $f(z)$, $z = x+iy \in T^B$, which are holomorphic in T^B and which satisfy

$$||f(x+iy)||_{L^p} = \left(\int_{\mathbb{R}^n} |f(x+iy)|^p \, dx \right)^{1/p} \leq$$
$$\leq M \quad (1+(d(y))^{-r})^s \quad \exp(2\pi A|y|), \; y \in B, \tag{1.1}$$

for some constants $r \geq 0$ and $s \geq 0$ which can depend on f, p, and A but not on $y \in B$ and for some constant $M = M(f,p,A,r,s)$ which can depend on f, p, A, r, and s but not on $y \in B$. If $B = C$, a cone, then $d(y)$ in (1.1) is the distance from $y \in C$ to the boundary of C. We have defined and studied the functions $S_A^p(T^B)$ in Carmichael [1] and Carmichael [4-7] and have stated our motivation in studying these functions in Carmichael [4, p. 81].

In Carmichael [1, Theorem] we proved a holomorphic extension theorem (edge of the wedge theorem) for holomorphic functions in T^C which satisfy (1.1) for $y \in C$ where C is a finite union of open convex cones in $\mathbb{R}^n$ and for the cases $1 < p \leq 2$. We did not consider the cases $2 < p < \infty$ in Carmichael [1] because at that time we did not know whether elements of $S_A^p(T^C)$, $2 < p < \infty$, had distributional boundary values for any base C of the tube T^C; in fact we did not know anything about the basic structure of $S_A^p(T^C)$, $2 < p < \infty$.

In Carmichael [6] we have recently proved that, indeed, elements of $S_A^p(T^C)$, $2 < p < \infty$, do have distributional boundary values in the strong topology of $\mathcal{D}'$ where C is a polygonal cone or a regular cone. A polygonal cone is a more general cone than a regular cone (Carmichael [6, section 2]); a polygonal cone is a finite union of open convex cones which satisfy a certain intersection property and each of which is the image of the first quadrant $C_{\bar{0}}$ under a nonsingular linear transformation. For our purposes here the main importance of the results in Carmichael [6] is that we now know that elements of $S_A^p(T^C)$, $2 < p < \infty$, for regular cones C do have distributional boundary values in the strong topology of $\mathcal{D}'$; and we also use some technical details from Carmichael [6] here.

The purpose of this paper is to prove a holomorphic extension theorem like that in Carmichael [1, Theorem] for holomorphic functions in a tube T^C, where C is a finite union of open convex cones, which satisfy (1.1) for $y \in C$ and for the values $2 < p < \infty$. In so doing we complete this holomorphic extension problem for functions that generalize the H^p functions in tubes to the values $2 < p < \infty$; we have already obtained this type of result for $1 < p \leq 2$ in Carmichael [1] as we have noted above. For the values $2 < p < \infty$, the analysis to obtain our basic result is somewhat different than for the cases $1 < p \leq 2$; although there is some overlap in the technical details. We obtain our general holomorphic extension theorem here by first proving a special case corresponding to special cones and then use this special case to prove the general case.

2. HOLOMORPHIC EXTENSION THEOREM.

Let $C = \bigcup_{j=1}^{m} C_j$ where each C_j is a regular cone in $\mathbb{R}^n$. Let $f(z)$ be holomorphic in T^C and satisfy (1.1) for $y \in C$ and $2 < p < \infty$. For any $y \in C_j$, $j = 1, \ldots, m$, the distance from y to the boundary of C is larger than or equal to the distance from y to the boundary of C_j from which we have $f(z) \in S_A^p(T^{C_j})$, $j = 1, \ldots, m$, $2 < p < \infty$. By Carmichael [6, Corollary 1] for each $j = 1, \ldots, m$ there is an element $V_j \in \mathcal{S}'$ such that

$$\lim_{\substack{y \to \bar{0} \\ y \in C_j}} f(x+iy) = \mathcal{F}[V_j] \in \mathcal{S}', \; j = 1, \ldots, m, \tag{2.1}$$

in the strong topology of $\mathcal{S}'$ with this boundary value being unique and being obtained independently of how $y \to \bar{0}$, $y \in C_j$, $j = 1, \ldots, m$. Here $\mathcal{F}[V_j]$ means the $\mathcal{S}'$ Fourier transform which maps $\mathcal{S}'$ one-one and onto $\mathcal{S}'$ (Schwartz [8, Chapter 7]). (As usual in the papers Carmichael [1] and Carmichael [4 - 7], by $y \to \bar{0}$, $y \in C$, for a cone C we mean $y \to \bar{0}$, $y \in C'$, for every compact subcone C' of C.)

Before stating our theorem we first need to make a technical discussion which is needed for the theorem. Let the open cone C be the union of a finite number of regular cones, $C = \bigcup_{j=1}^{m} C_j$, as in the preceding paragraph. For each of the regular cones C_j, $j = 1, \ldots, m$, consider $C_j \cap C_\mu$ for the 2^n quadrants C_μ in $\mathbb{R}^n$. Let $C_{j,k}$, $k = 1, \ldots, r_j$, be an enumeration of the intersections $C_j \cap C_\mu$ which are nonempty; and for each $j = 1, \ldots, m$, r_j is a positive integer with 2^n as an upper bound. Each $C_{j,k}$, $k = 1, \ldots, r_j$, $j = 1 \ldots, m$, is an open convex cone that is contained in or is a quadrant C_μ in $\mathbb{R}^n$. Now put $\Gamma = \bigcup_{j=1}^{m} \bigcup_{k=1}^{r_j} C_{j,k}$. We have $\Gamma \subseteq C$ and Γ is the finite union of open convex cones each of which is contained in or is a quadrant in $\mathbb{R}^n$. We have that $O(\Gamma) = O(C)$ and $1 \le \rho_C \le \rho_\Gamma < \infty$ (Vladimirov [2, p. 220]).

We now state our holomorphic extension theorem (edge of the wedge theorem); in this theorem Γ is the cone constructed from the cone C as in the preceding paragraph.

THEOREM. Let C be an open cone in $\mathbb{R}^n$ which is the union of a finite number of regular cones, $C = \bigcup_{j=1}^{m} C_j$, such that $(O(C))^*$ contains interior points and has an admissible set of vectors. Let $f(z)$, $z = x+iy$, be holomorphic in T^C and satisfy (1.1) for $y \in C$ and $2 < p < \infty$. Let the boundary values of $f(x+iy)$ in the strong topology of $\mathcal{S}'$ corresponding to each connected component C_j, $j = 1, \ldots, m$, of C given in (2.1) be equal in $\mathcal{S}'$. There is a function $F(z)$ which is holomorphic in $T^{O(C)}$ and which satisfies $F(z) = f(z)$, $z \in T^C$, where $F(z)$ is of the form

$$F(z) = P(z)\, H(z), \; z \in T^{O(C)}, \tag{2.2}$$

with $P(z)$ being a polynomial in z and $H(z) \in S^2_{A\rho_\Gamma}(T^{O(C)}) \cap S^q_{A\rho_\Gamma}(T^{O(C)})$,

$(1/u) + (1/q) = 1$, for all u, $1 < u \le 2$.

For our purposes here we have assumed that the C_j, $j = 1, \ldots, m$, in the Theorem are regular cones instead of the more general polygonal cones (Carmichael [6, section 2]) because the hypothesis that $(O(C))^*$ contain interior points and an admissible set of vectors cannot be true for polygonal cones that are not regular cones. By comparing the statement of the Theorem with the statement of Carmichael [1, Theorem] we see that these two results have the same type of conclusion; this is what we desired. We thus have been able to extend the result Carmichael [1, Theorem] to the cases $2 < p < \infty$.

We will prove the Theorem by first proving a special case of it corresponding to the cones C_j, $j = 1, \ldots, m$, being contained in quadrants. After proving this special case we will then use it to give a proof of the Theorem. We now present the desired special case of the Theorem.

LEMMA. Let C be an open cone in $\mathbb{R}^n$ of the form $C = \bigcup_{j=1}^{m} C_j$ where each C_j, $j = 1, \ldots, m$, is an open convex cone that is contained in or is any of the 2^n quadrants C_μ in $\mathbb{R}^n$, and let $(O(C))^*$ contain interior points and have an admissible set of vectors. Let $f(z)$, $z = x+iy$, be holomorphic in T^C and satisfy (1.1) for $y \in C$ and $2 < p < \infty$. Let the boundary values of $f(x+iy)$ in the strong topology of $\mathcal{S}'$ corresponding to each connected component C_j, $j = 1, \ldots, m$, of C given in (2.1) be equal in $\mathcal{S}'$. There is a function $F(z)$ which is holomorphic in $T^{O(C)}$ and which satisfies $F(z) = f(z)$, $z \in T^C$, where $F(z)$ has the form given in (2.2) with $P(z)$ being a polynomial in z and $H(z) \in S^2_{A\rho_C}(T^{O(C)}) \cap S^q_{A\rho_C}(T^{O(C)})$, $(1/u) + (1/q) = 1$, for all u, $1 < u \leq 2$.

PROOF. An open convex cone that is contained in or is any of the 2^n quadrants C_μ in $\mathbb{R}^n$ is a regular cone. Recall the discussion in the first paragraph of section 2. By this discussion we have $f(z) \in S^p_A(T^{C_j})$, $j = 1, \ldots, m$, $2 < p < \infty$, and we have the existence of elements $V_j \in \mathcal{S}'$, $j = 1, \ldots, m$, such that (2.1) holds. In addition, by the proof of Carmichael [6, Lemma 1] these elements $V_j \in \mathcal{S}'$ have $\operatorname{supp}(V_j) \subseteq \{t: u_{C_j}(t) \leq A\}$ and

$$f(z) = \langle V_j, \exp(2\pi i\langle z,t\rangle)\rangle, \quad z \in T^{C_j}, \ j = 1, \ldots, m. \tag{2.3}$$

By hypothesis the boundary values in (2.1) satisfy

$$\mathcal{F}[V_1] = \mathcal{F}[V_2] = \ldots = \mathcal{F}[V_m]$$

in $\mathcal{S}'$. Since the Fourier transform is a topological isomorphism of $\mathcal{S}'$ onto $\mathcal{S}'$ we then have

$$V_1 = V_2 = \ldots = V_m \tag{2.4}$$

in $\mathcal{S}'$, we denote the common value in (2.4) by V, and $V \in \mathcal{S}'$. Since the support of each V_j is contained in $\{t: u_{C_j}(t) \leq A\}$, $j = 1, \ldots, m$, then by exactly the same proof as in Carmichael [1, equation (2.4) on p. 419 through equation (2.6) on p. 420] we obtain that $\operatorname{supp}(V) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$; and $\{t: u_{O(C)}(t) \leq A\rho_C\} = (O(C))^* + \overline{N(\bar{0}; A\rho_C)}$ (Vladimirov [3, Lemma 1, p. 936]) with $\overline{N(\bar{0}; A\rho_C)}$ being the closure of the open ball in $\mathbb{R}^n$ centered at the origin $\bar{0}$ in $\mathbb{R}^n$ with radius $A\rho_C$. The dual cone $(O(C))^*$ is closed and convex; and by hypothesis in this theorem, $(O(C))^*$ contains interior points and has an admissible set of vectors. Any element of $\mathcal{S}'$ has finite order; we denote the order of $V \in \mathcal{S}'$ by m_0. By Vladimirov [3, Theorem 1, p. 930] we have

$$V = \prod_{k=1}^{n} \langle e_k, \text{gradient}\rangle^{m_0+2} \quad G(t) \tag{2.5}$$

where $\{e_k\}_{k=1}^n$ is an admissible set of vectors for the cone $(O(C))^*$, $G(t)$ is a continuous function of $t \in \mathbb{R}^n$ which is unique corresponding to $\{e_k\}_{k=1}^n$ and the order m_0 of $V \in \mathcal{S}'$, $\text{supp}(G) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\} = (O(C))^* + \overline{N(\bar{0};A\rho_C)}$, and

$$|G(t)| \leq K\,(1 + |t|)^{3m_0 + 1},\ t \in \mathbb{R}^n, \tag{2.6}$$

where the constant K is independent of $t \in \mathbb{R}^n$. (In Vladimirov [3, Theorem 1, p. 930] the term "acute" in our present situation means that $((O(C))^*)^* = \overline{O(C)}$ (Vladimirov [2, p. 218]) should have non-empty interior (Vladimirov [3, p. 930]) which is certainly the case in this Theorem.) Since $G(t)$ is continuous on $\mathbb{R}^n$ then $\text{supp}(G) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$ as a function (Schwartz [8, Chapter 1, sections 1 and 3]). (This fact is also obtained in the proof of Vladimirov [3, Theorem 1], and the containment $\text{supp}(G) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$ gives the support of $G(t)$ as a function here as well as a distribution.) Choose a function $\lambda(t) \in C^\infty$, $t \in \mathbb{R}^n$, such that for any n-tuple α of nonnegative integers $|D^\alpha\lambda(t)| \leq M_\alpha$, $t \in \mathbb{R}^n$, where M_α is a constant which depends only on α; and for $\epsilon > 0$, $\lambda(t) = 1$ for t on an ϵ neighborhood of $\{t: u_{O(C)}(t) \leq A\rho_C\}$ and $\lambda(t) = 0$ for $t \in \mathbb{R}^n$ but not on a 2ϵ neighborhood of $\{t: u_{O(C)}(t) \leq A\rho_C\}$ (Carmichael [1, p. 420] and [4, p. 94]). For $z \in T^{O(C)}$ we have $(\lambda(t)\exp(2\pi i\langle z,t\rangle)) \in \mathcal{S}$ as a function of $t \in \mathbb{R}^n$. Recalling that $\text{supp}(V) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$ we put

$$F(z) = \langle V, \exp(2\pi i\langle z,t\rangle)\rangle = \langle V,\lambda(t)\exp(2\pi i\langle z,t\rangle)\rangle,\ z \in T^{O(C)}. \tag{2.7}$$

From (2.5) and $\text{supp}(G) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$ as a function we have (Vladimirov [3, (3.1), p. 931])

$$F(z) = \left(\prod_{k=1}^{n} \langle e_k, -2\pi i z\rangle^{m_0+2}\right) H(z),\ z \in T^{O(C)}, \tag{2.8}$$

where

$$H(z) = \int_{\{t:\ u_{O(C)}(t) \leq A\rho_C\}} G(t)\exp(2\pi i\langle z,t\rangle)\,dt,\ z \in T^{O(C)}. \tag{2.9}$$

Since $G(t)$ is continuous on $\mathbb{R}^n$ and satisfies (2.6) for all $t \in \mathbb{R}^n$ we have $G(t) \in \mathcal{S}'_u$ for all u, $1 \leq u < \infty$, as can easily be seen by choosing $b = 3m_0 + 3$ in the definition of $\mathcal{S}'_u$ (section 1). Combining this fact with the support of $G(t)$ as a function, which is $\text{supp}(G) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$, and Carmichael [4, Theorem 6.1, p. 98] yield

$$(\exp(-2\pi\langle y,t\rangle)\,G(t)) \in L^u,\ y \in O(C),\ 1 \leq u < \infty, \tag{2.10}$$

and

$$||\exp(-2\pi\langle y,t\rangle)\,G(t)||_{L^u} \leq M\,(1 + (d(y))^{-r})^s \exp(2\pi A\rho_C|y|),\ y \in O(C),\ 1 \leq u < \infty, \tag{2.11}$$

for constants $r = r(G,u,A) \geq 0$, $s = s(G,u,A) \geq 0$, and $M = M(G,u,A,r,s) > 0$ which are independent of $y \in O(C)$; and we emphasize that (2.10) and (2.11) hold for all u, $1 \leq u < \infty$. Now (2.10), (2.11), and Carmichael [4, Theorem 5.1, p. 97] combine to prove that $H(z)$ in (2.9) satisfies $H(z) \in S^q_{A\rho_C}(T^{O(C)})$, $(1/u) + (1/q) = 1$, for all

u, $1 < u \leq 2$; and in particular $H(z) \in S^2_{A\rho_C}(T^{O(C)})$. Since $H(z)$ is holomorphic in $T^{O(C)}$ then so is $F(z)$, which is defined in (2.7), because of (2.8); and (2.8) is the desired representation (2.2) of $F(z)$ in the statement of the Lemma where the polynomial $P(z)$ is

$$P(z) = \prod_{k=1}^{n} \langle e_k, -2\pi i z\rangle^{m_0+2}$$

and $H(z) \in S^2_{A\rho_C}(T^{O(C)}) \cap S^q_{A\rho_C}(T^{O(C)})$, $(1/u) + (1/q) = 1$, $1 < u \leq 2$, is given in (2.9). From (2.4), the fact that $\operatorname{supp}(V) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$ in $\mathcal{S}'$, and the definition of $\lambda(t)$ preceding (2.7), we can write (2.3) as

$$\begin{aligned} f(z) &= \langle V, \lambda(t) \exp(2\pi i\langle z,t\rangle)\rangle \\ &= \langle V, \exp(2\pi i\langle z,t\rangle)\rangle, \ z \in T^{C_j}, \ j = 1, \ldots ,m. \end{aligned}$$

These identities and (2.7) prove that $F(z)$ is the desired holomorphic extension of $f(z)$ to $T^{O(C)}$ and $F(z) = f(z)$, $z \in T^C$. The proof of the Lemma is complete.

We can obtain a holomorphic extension result like that in the Lemma without the assumption that $(O(C))^*$ contains interior points and has an admissible set of vectors. But we loose the detailed information concerning the holomorphic extension function as we see in the following corollary to the Lemma.

COROLLARY 1. Let $C = \bigcup_{j=1}^{m} C_j$ where each C_j, $j = 1, \ldots ,m$, is an open convex cone that is contained in or is any of the 2^n quadrants C_μ in $\mathbb{R}^n$; and let $f(z)$, $z = x+iy$, be holomorphic in T^C and satisfy (1.1) for $y \in C$ and $2 < p < \infty$. Let the boundary values of $f(x+iy)$ in the strong topology of $\mathcal{S}'$ corresponding to each C_j, $j = 1, \ldots ,m$, given in (2.1) be equal in $\mathcal{S}'$. There is a function $F(z)$ which is holomorphic in $T^{O(C)}$ and which satisfies $F(z) = f(z)$, $z \in T^C$.

PROOF. Proceeding as in the proof of the Lemma, obtain (2.4) and call V the common value. By the proof of the Lemma, $\operatorname{supp}(V) \subseteq \{t: u_{O(C)}(t) \leq A\rho_C\}$. Define $F(z)$, $z \in T^{O(C)}$, as in (2.7). By the necessity of Vladimirov [2, Theorem 2, p. 239], $F(z)$ is holomorphic in $T^{O(C)}$; and $F(z) = f(z)$, $z \in T^C$, because of (2.3), (2.4), and the definition (2.7) of $F(z)$ as in the Lemma.

Using the Lemma we can now give a proof of the Theorem.

PROOF OF THE THEOREM. From the cone C construct the cone $\Gamma = \bigcup_{j=1}^{m} \bigcup_{k=1}^{r_j} C_{j,k}$ as in the second paragraph of this section. We have that each $C_{j,k}$, $k = 1, \ldots ,r_j$, $j = 1, \ldots ,m$, is an open convex cone that is contained in or is a quadrant C_μ in $\mathbb{R}^n$. Further we have $\Gamma \subseteq C$, $O(\Gamma) = O(C)$ and $1 \leq \rho_C \leq \rho_\Gamma < \infty$. Since the distance from $y \in C_{j,k}$ to the boundary of $C_{j,k}$ is less than or equal to the distance from y to the boundary of C, we have $f(z) \in S^p_A(T^{C_{j,k}})$, $k = 1, \ldots , r_j$, $j = 1, \ldots ,m$. By hypothesis the m boundary values given in (2.1) of $f(x+iy)$ corresponding to each connected component C_j, $j = 1, \ldots ,m$, of C are equal in $\mathcal{S}'$; we denote the common value of the boundary values as $\mathfrak{F}[V] \in \mathcal{S}'$ for some $V \in \mathcal{S}'$. Since each boundary value $\mathfrak{F}[V_j] = \mathfrak{F}[V]$, $j = 1, \ldots ,m$, is obtained uniquely and independently of how $y \to \bar{0}$, $y \in C_j$, then we have

$$\lim_{\substack{y\to\bar{0} \\ y\in C_{j,k}}} f(x+iy) = \mathfrak{F}[V_j] = \mathfrak{F}[V], \ k = 1, \ldots ,r_j, \ j = 1, \ldots , m, \quad (2.12)$$

in $\mathcal{S}'$. Since we have $O(\Gamma) = O(C)$, than $(O(\Gamma))^* = (O(C))^*$ contains interior points and has an admissible set of vectors by the hypothesis on $(O(C))^*$. In addition $f(z)$ is holomorphic in T^Γ and satisfies (1.1) for $y \in \Gamma$ and $2 < p < \infty$ from the hypothesis on $f(z)$ in this Theorem corresponding to the cone C and the facts that $\Gamma \subseteq C$ and the distance from $y \in \Gamma$ to the boundary of Γ is less than or equal to the distance from y to the boundary of C. Thus by these facts, (2.12), and the Lemma, we have the existence of a function $F(z)$ which is holomorphic in $T^{O(\Gamma)} = T^{O(C)}$ and which satisfies $F(z) = f(z)$, $z \in T^\Gamma$, where $F(z)$ is of the form

$$F(z) = P(z)\,H(z),\ z \in T^{O(\Gamma)} = T^{O(C)},$$

with $P(z)$ being a polynomial in z and $H(z) \in S^2_{A\rho_\Gamma}(T^{O(C)}) \cap S^q_{A\rho_\Gamma}(T^{O(C)})$, $(1/u) + (1/q) = 1$, for all u, $1 < u \leq 2$. Now consider each C_j, $j = 1, \ldots, m$; we have that both $f(z)$ and $F(z)$ are holomorphic in T^{C_j}, $j = 1, \ldots, m$. For each $j = 1, \ldots, m$, we further have that $f(z) = F(z)$, $z \in \bigcup_{k=1}^{r_j} T^{C_{j,k}}$, with $\bigcup_{k=1}^{r_j} T^{C_{j,k}} \subseteq T^{C_j}$. It thus follows (Vladimirov [2, p. 39]) that $f(z) = F(z)$, $z \in T^{C_j}$, $j = 1, \ldots, m$; hence $f(z) = F(z)$, $z \in T^C$. The proof of the Theorem is complete.

We have the following corollary to the Theorem which is similar to the Corollary 1 of the Lemma. The proof of the following corollary is obtained by the construction of the proof of the Theorem and the use of Corollary 1 in place of the use of the Lemma. We leave the obvious details to the reader.

COROLLARY 2. Let $C = \bigcup_{j=1}^{m} C_j$ where each C_j is a regular cone in $\mathbb{R}^n$ and let $f(z)$, $z = x+iy$, be holomorphic in T^C and satisfy (1.1) for $y \in C$ and $2 < p < \infty$. Let the boundary values of $f(x+iy)$ in the strong topology of $\mathcal{S}'$ corresponding to each C_j, $j = 1, \ldots, m$, given in (2.1) be equal in $\mathcal{S}'$. There is a function $F(z)$ which is holomorphic in $T^{O(C)}$ and which satisfies $F(z) = f(z)$, $z \in T^C$.

An additional fact concerning the holomorphic extension function $F(z)$ in the Theorem can be observed as we now note. From the proof of the Theorem and the construction of the Lemma we have that the analytic extension function $F(z)$ in the Theorem has the form

$$F(z) = \langle V, \exp(2\pi i\langle z,t\rangle)\rangle = \langle V, \lambda(t)\exp(2\pi i\langle z,t\rangle)\rangle,\ z \in T^{O(\Gamma)} = T^{O(C)},$$

with $V \in \mathcal{S}'$ and $\operatorname{supp}(V) \subseteq \{t: u_{O(\Gamma)}(t) \leq A\rho_\Gamma\} = \{t: u_{O(C)}(t) \leq A\rho_\Gamma\}$. We thus have

$$\lim_{\substack{y \to \bar{0} \\ y \in O(C)}} F(x+iy) = \mathcal{F}[V] \in \mathcal{S}'$$

in the strong topology of $\mathcal{S}'$ by the boundary value proof in Carmichael [4, Corollary 4.1, p. 93]. Thus $F(x+iy)$ has the same $\mathcal{S}'$ boundary value on the distinguished boundary $\mathbb{R}^n + i\bar{0}$ of $T^{O(C)}$ as the original function $f(x+iy)$ does from each connected component T^{C_j}, $j = 1, \ldots, m$, of T^C.

3. ACKNOWLEDGEMENT.

This material is based upon work supported by the National Science Foundation under Grant No. DMS-8418435.

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