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ON COEFFICIENT BOUNDS OF A CERTAIN CLASS p-VALENT λ-SPIRAL FUNCTIONS OF ORDER α

M.K. AOUF

Department of Mathematics, Faculty of Science
University of Mansoura
Mansoura, Egypt

and

Department of Mathematics, Faculty of Science
University of Qatar
P.O. Box 2713
Doha - Qatar

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ABSTRACT. Let $S^{\lambda}(A,B,p,\alpha)(|\lambda| < \frac{\pi}{2}, -1 \leq A < B \leq 1$ and $o \leq \alpha < p)$, denote the class of functions $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$ analytic in $U = \{z: |z| < 1\}$, which satisfy for $z = re^{i\theta} \in U$

$$e^{i\lambda} \sec\lambda \frac{zf'(z)}{f(z)} - ip \tan \lambda = \frac{p + [pB+(A-B)(p-\alpha)]w(z)}{1 + Bw(z)},$$

$w(z)$ is analytic in U with $w(o) = o$ and $|w(z)| \leq |z|$ for $z \in U$. In this paper we obtain the bounds of a_n and we maximize $|a_{p+2} - \mu a^2_{p+1}|$ over the class $S^{\lambda}(A,B,p,\alpha)$ for complex values of μ.

KEY WORDS AND PHRASES. p-Valent, analytic, bounds, λ-spirallike functions of order α.
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1. INTRODUCTION.

Let A_p (p a fixed integer greater than zero) denote the class of functions $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$ which are analytic in $U = \{z: |z| < 1\}$. We use Ω to denote the class of bounded analytic functions $w(z)$ in U satisfies the conditions $w(o) = o$ and $|w(z)| \leq |z|$ for $z \in U$. Also let $P(p,\alpha)$(with p a postive integer) denote the class of functions with positive real part of order α that have the form

$$P(z) = p + \sum_{k=1}^{\infty} c_k z^k \tag{1.1}$$

which are analytic in U and satisfy the conditions $P(o) = p$ and $Re\{P(z)\} > \alpha$ $(o \leq \alpha < p)$ in U. The class $P(p,\alpha)$ was introduced by Patil and Thakare [1]. It was shown in [1] that the function $P \in P(p,\alpha)$ if and only if

$$P(z) = \frac{p-(p-2\alpha)w(z)}{1+w(z)}, \quad w \in \Omega . \tag{1.2}$$

For $|\lambda| < \frac{\pi}{2}$ and p a fixed integer greater than zero, let $S^\lambda(p,\alpha)$ denote the class of functions $f(z) \in A_p$ which satisfy

$$\operatorname{Re}\left\{ e^{i\lambda} \frac{zf'(z)}{f(z)} \right\} > \alpha \cos\lambda \tag{1.3}$$

for $z \in U$ and $o \leq \alpha < p$. We say the functions in $S^\lambda(p,\alpha)$ are p-valent λ-spiral-like of order α. The class $S^\lambda(p,\alpha)$ was introduced by Patil and Thakare [1]. It was shown in [1] that $f \in S^\lambda(p,\alpha)$ if and only if there exists a function $P \in P(p,\alpha)$ such that

$$e^{i\lambda} \frac{zf'(z)}{f(z)} = \cos\lambda \, . \, P(z) + ip \sin\lambda . \tag{1.4}$$

Let $P(A,B)(-1 \leq A < B \leq 1)$ denote the class of functions $P_1(z) = 1 + \sum_{n=1}^{\infty} b_n z^n$ analytic in U and such that $P_1(z) \in P(A,B)$ if and only if

$$P_1(z) = \frac{1+Aw(z)}{1+Bw(z)}, \quad w \in \Omega, \quad z \in U . \tag{1.5}$$

The class $P(A,B)$ was introduced by Janowski [2].

For $-1 \leq A < B \leq 1$ and $o \leq \alpha < p$, denote by $P(A,B,p,\alpha)$ the class of functions $P_2(z)$ of form (1.1) which satisfy that $P_2(z) \in P(A,B,p,\alpha)$ if and only if

$$P_2(z) = (p-\alpha)P_1(z) + \alpha, \quad P_1(z) \in P(A,B) \tag{1.6}$$

Using (1.5) in (1.6), one can show that $P_2(z) \in P(A,B,p,\alpha)$ if and only if

$$P_2(z) = \frac{p + [pB+(A-B)(p-\alpha)]w(z)}{1+Bw(z)}, \quad w \in \Omega . \tag{1.7}$$

Also let $S^\lambda(A,B,p,\alpha)(|\lambda| < \frac{\pi}{2}, -1 \leq A < B \leq 1$ and $o \leq \alpha < p)$ denote the class of functions $f(z) \in A_p$ which satify

$$e^{i\lambda} \frac{zf'(z)}{f(z)} = \cos\lambda \, P_2(z) + ip \sin\lambda, \; P_2(z) \in P(A,B,p,\alpha) . \tag{1.8}$$

Using (1.7) in (1.8) one can easily show that:

$f(z) \in S^\lambda(A,B,p,\alpha)$ if and only if

$$\text{(i)} \quad e^{i\lambda} \sec\lambda \frac{zf'(z)}{f(z)} - ip \tan\lambda = \frac{p + [pB+(A-B)(p-\alpha)]w(z)}{1+Bw(z)}, \quad w \in \Omega. \tag{1.9}$$

$$\text{(ii)} \quad \frac{zf'(z)}{f(z)} = \frac{p + [pB+(A-B)(p-\alpha)\cos\lambda \, e^{-i\lambda}]w(z)}{1+Bw(z)}, \quad w \in \Omega. \tag{1.10}$$

We shall need the following lemma in our investigation:

LEMMA 1[3]. Let $w(z) = \sum_{k=1}^{\infty} b_k z^k \in \Omega$, if ν is any complex number, then

$$|b_2 - \nu b_1^2| \leq \max\{1, |\nu|\}. \tag{1.11}$$

Equality is attained for $w(z) = z^2$ and $w(z) = z$.

2. COEFFICIENT ESTIMATES FOR THE CLASS $S^{\lambda}(A,B,p,\alpha)$.

LEMMA 2. If integers p and m are greater than zero; $0 \leqq \alpha < p$, $|\lambda| < \frac{\pi}{2}$ and $-1 \leqq A < B \leqq 1$, then

$$\prod_{j=0}^{m-1} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2} =$$

$$\frac{\cos^2\lambda}{m^2} \{(B-A)^2(p-\alpha)^2 + \sum_{k=1}^{m-1} [k^2(B^2-1)\sec^2\lambda + (B-A)^2(p-\alpha)^2 + 2kB(B-A)(p-\alpha)] \prod_{j=0}^{k-1} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2}\}. \quad (2.1)$$

PROOF. We prove the lemma by induction on m. For $m=1$, the lemma is obvious. Next suppose that the result is true for $m=q-1$. We have

$$\frac{\cos^2\lambda}{q^2} \{(B-A)^2(p-\alpha)^2 + \sum_{k=1}^{q-1} [k^2(B^2-1)\sec^2\lambda + (B-A)^2(p-\alpha)^2 + 2kB(B-A)(p-\alpha)] \times \prod_{j=0}^{k-1} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2}\} =$$

$$\frac{\cos^2\lambda}{q^2} \{(B-A)^2(p-\alpha)^2 + \sum_{k=1}^{q-2} [k^2(B^2-1)\sec^2\lambda + (B-A)^2(p-\alpha)^2 + 2kB(B-A)(p-\alpha)] \times \prod_{j=0}^{k-1} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2} +$$

$$[(q-1)^2(B^2-1)\sec^2\lambda + (B-A)^2(p-\alpha)^2 + 2(q-1)B(B-A)(p-\alpha)] x \prod_{j=0}^{q-2} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2}\}$$

$$= \prod_{j=0}^{q-2} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2} \times$$

$$\{\frac{(q-1)^2B^2 + (B-A)^2(p-\alpha)^2\cos^2\lambda + 2(q-1)\cdot B(B-A)(p-\alpha)\cos^2\lambda}{q^2}\}$$

$$= \prod_{j=0}^{q-1} \left|\frac{(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj}{(j+1)}\right|^2 .$$

Showing that the result is valid for $m=q$. This proves the lemma.

THEOREM 1. If $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \in S^{\lambda}(A,B,p,\alpha)$, then

$$|a_n| \leqq \prod_{k=0}^{n-(p+1)} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bk|}{k+1} \quad (2.2)$$

for $n \geq p+1$ and these bounds are sharp for all admissible A,B,λ and α and for each n.

PROOF. As $f \in S^{\lambda}(A,B,p,\alpha)$, from (1.9), we have

$$e^{i\lambda}\sec\lambda \cdot \frac{zf'(z)}{f(z)} - ip\tan\lambda = \frac{p+[pB+(A-B)(p-\alpha)]w(z)}{1+Bw(z)}, \quad w \in \Omega .$$

This may be written as

$$\{Be^{i\lambda}\sec\lambda\, sf'(z) + [-pB+(B-A)(p-\alpha)-ipB\tan\lambda]f(z)\}\ w(z)$$

$$= (p+ip\tan\lambda)f(z) - e^{i\lambda}\sec\lambda\, zf'(z) .$$

Hence

$$\Big[Be^{i\lambda}\sec\lambda\{pz^p + \sum_{k=1}^{\infty}(p+k)a_{p+k}z^{p+k}\} +$$

$$[-pB+(B-A)(p-\alpha)-ipB\tan\lambda]\ \{z^p + \sum_{k=1}^{\infty} a_{p+k}z^{p+k}\}\Big]\ w(z)$$

$$= (p+ip\tan\lambda)\ \{z^p + \sum_{k=1}^{\infty}(p+k)a_{p+k}z^{p+k}\} -$$

$$e^{i\lambda}\sec\lambda\ \{pz^p + \sum_{k=1}^{\infty}(p+k)a_{p+k}z^{p+k}\}$$

or

$$\Big[pBe^{i\lambda}\sec\lambda + [-pB+(B-A)(p-\alpha) - ipB\tan\lambda] +$$

$$\sum_{k=1}^{\infty}\{(p+k)Be^{i\lambda}\sec\lambda + [-pB+(B-A)(p-\alpha) - ipB\tan\lambda]\}a_{p+k}z^k\Big]\ w(z)$$

$$= (p+ip\tan\lambda - p\,e^{i\lambda}\sec\lambda) + \sum_{k=1}^{\infty}\{p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda\}a_{p+k}z^k$$

which may be written as

$$\sum_{k=o}^{\infty}\Big[\{(p+k)Be^{i\lambda}\sec\lambda + [-pB+(B-A)(p-\alpha) - ipB\tan\lambda]\}\ a_{p+k}z^k\Big]\ w(z)$$

$$= \sum_{k=o}^{\infty}[p+ip\tan\lambda - (p+k)e^{i\lambda}\sec\lambda]a_{p+k}z^k \qquad (2.3)$$

where $a_p=1$ and $w(z) = \sum_{k=o}^{\infty} b_{k+1}\, z^{k+1}$.

Equating coefficients of z^m on both sides of (2.3), we obtain

$$\sum_{k=o}^{m-1}\{(p+k)Be^{i\lambda}\sec\lambda + [-pB + (B-A)(p-\alpha) - ipB\tan\lambda]\}\ a_{p+k}\, b_{m-k}$$

$$= \{p+ip\tan\lambda - (p+m)e^{i\lambda}\sec\lambda\}\, a_{p+m} ;$$

which shows that a_{p+m} on right-hand side depends only on

$$a_p,\ a_{p+1},\ \ldots,\ a_{p+(m-1)}$$

of left-hand side. Hence we can write

$$\sum_{k=o}^{m-1}\Big[\{(p+k)B\,e^{i\lambda}\sec\lambda+[-pB+(B-A)(p-\alpha)-ip\,B\tan\lambda\,]\}\,a^{p+k}\,z^{k}\Big]\,w(z)=$$

$$=\sum_{k=o}^{m}[p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda]a_{p+k}\,z^{k}+\sum_{k=m+1}^{\infty}A_k z^k$$

for $m=1,2,3,\ldots$ and a proper choice of $A_k (k \geqq 0)$.

Let $z = re^{i\theta}$, $0 < r < 1$, $0 \leqq \theta \leqq 2\pi$, then

$$\sum_{k=o}^{m-1}|(p+k)Be^{i\lambda}\sec\lambda+[-pB+(B-A)(p-\alpha)-ip\,B\tan\lambda]|^2\,|a_{p+k}|^2\,r^{2k}$$

$$=\frac{1}{2\pi}\int_0^{2\pi}\Big|\sum_{k=o}^{m-1}\{(p+k)Be^{i\lambda}\sec\lambda+[-pB+(B-A)(p-\alpha)-ipB\cdot\tan\lambda]\}\,a_{p+k}\,r^k\,e^{i\theta k}\Big|^2\,d\theta$$

$$\geqq\frac{1}{2\pi}\cdot\int_0^{2\pi}\Big|\sum_{k=o}^{m-1}\{(p+k)Be^{i\lambda}\sec\lambda+[-pB+(B-A)(p-\alpha)-ip\cdot B\cdot\tan\lambda]\}\,a_{p+k}r^k e^{i\theta k}\Big|^2\,|w(re^{i\theta})|^2\cdot d\theta$$

$$\geqq\frac{1}{2\pi}\int_0^{2\pi}\Big|\sum_{k=o}^{m}\{p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda\}a_{p+k}r^k e^{i\theta k}+\sum_{k=m+1}^{\infty}A_k r^k e^{i\theta k}\Big|^2\cdot d\theta$$

$$\geqq\sum_{k=o}^{m}|p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda|^2\,|a_{p+k}|^2\,r^{2k}+\sum_{k=m+1}^{\infty}|A_k|^2\,r^{2k}$$

$$\geqq\sum_{k=o}^{m}|p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda|^2\,|a_{p+k}|^2\,r^{2k}\,. \tag{2.4}$$

Setting $r \to 1$ in (2.4), the inequality (2.4) may be written as

$$\sum_{k=o}^{m-1}\{\,|(p+k)Be^{i\lambda}\sec\lambda+[-pB+(B-A)(p-\alpha)-ip\,B\tan\lambda]|^2-$$

$$-|p+ip\tan\lambda-(p+k)e^{i\lambda}\sec\lambda|^2\,\}\,|a_{p+k}|^2$$

$$\geqq|p+ip\tan\lambda-(p+m)e^{i\lambda}\sec\lambda|^2\,|a_{p+m}|^2\,. \tag{2.5}$$

Simplification of (2.5) leads to

$$|a_{p+m}|^2\leqq\frac{\cos^2\lambda}{m^2}\cdot\sum_{k=o}^{m-1}\{k^2(B^2-1)\sec^2\lambda+(B-A)(p-\alpha)[(B-A)(p-\alpha)+2kB]\}|a_{p+k}|^2 \tag{2.6}$$

Replacing $p+m$ by n in (2.6), we are led to

$$|a_n|^2\leqq\frac{\cos^2\lambda}{(n-p)^2}\cdot\sum_{k=o}^{n-(p+1)}\{k^2(B^2-1)\sec^2\lambda+(B-A)(p-\alpha)[(B-A)(p-\alpha)+2kB]\}\,|a_{p+k}|^2, \tag{2.7}$$

where $n \geqq p+1$.

For $n=p+1$, (2.7) reduces to

$$|a_{p+1}|^2 \leqq (B-A)^2(p-\alpha)^2\cos^2\lambda$$

or
$$|a_{p+1}|^2 \leqq (B-A)(p-\alpha)\cos\lambda \tag{2.8}$$

which is equivalent to (2.2).

To establish (2.2) for $n > p+1$, we will apply induction argument.

Fix n, $n \geqq p+2$, and suppose (2.2) holds for $k = 1,2,\ldots, n-(p+1)$. Then

$$|a_n|^2 \leqq \frac{\cos^2\lambda}{(n-p)^2} \{(B-A)^2(p-\alpha)^2 +$$

$$\sum_{k=1}^{n-(p+1)} \{k^2(B^2-1)\sec^2\lambda + (B-A)(p-\alpha)[(B-A)(p-\alpha) + 2kB]\}$$

$$\times \prod_{j=o}^{k-1} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2}\} . \tag{2.9}$$

Thus from (2.7), (2.9) and <u>lemma</u> <u>2</u> with $m=n-p$, we obtain

$$|a_n|^2 \leqq \prod_{j=o}^{n-(p+1)} \frac{|(B-A)(p-\alpha)\cos\lambda\, e^{-i\lambda} + Bj|^2}{(j+1)^2} .$$

This completes the proof of (2.2). This proof is based on a technique found in Clunie [4].

For sharpness of (2.2) consider

$$f(z) = \frac{z^p}{(1-B\delta z)^{(\frac{B-A}{B})(p-\alpha)\cos\lambda e^{-i\lambda}}}, \quad |\delta| = 1,\ B \neq 0$$

Remarks on Theorem 1:

(1) Setting $B=1$ and $A=-1$ in Theorem 1, we get the result of Patil and Thakare [1].

(2) Setting $B=1$, $A=-1$ and $p=1$ in Theorem 1, we get the result of Libera [5].

(3) Setting $B=1$, $A=-1$, $p=1$ and $\alpha=0$ in Theorem 1, we get the result of Zamorski [6].

(4) Setting $B=1$, $A=-1$, $p=1$ and $\lambda=0$ in Theorem 1, we get the result of Robertson [7] and Schild [8].

THEOREM 2. If $f(z)=z^p + \sum_{k=p+1}^{\infty} a_k z^k \in S^\lambda(A,B,p,\alpha)$ and μ is any complex number, then

$$|a_{p+2}-\mu\, a^2_{p+1}| \leqq \frac{(B-A)(p-\alpha)}{2}\cos\lambda \max\{1, |(B-A)(p-\alpha)(2\mu-1)\cos\lambda - e^{i\lambda}|\} \tag{2.10}$$

This inequality is sharp for each μ.

PROOF. As $f \in S^\lambda(A,B,p,\alpha)$, from (1.9) we have

$$e^{i\lambda}\sec\lambda \frac{zf'(z)}{f(z)} - ip\tan\lambda = \frac{p+[pB+(A-B)(p-\alpha)]w(z)}{1+Bw(z)} \tag{2.11}$$

where $w(z) = \sum_{k=1}^{\infty} b_k z^k \in \Omega$.

Rewriting the form (2.11) as

$$w(z) = \frac{p-e^{i\lambda}\sec\lambda\,\frac{zf'(z)}{f(z)} + ip\tan\lambda}{Be^{i\lambda}\sec\lambda\,\frac{zf'(z)}{f(z)} + [-pB+(B-A)(p-\alpha)-ip\tan\lambda\, B]}$$

$$= \frac{e^{i\lambda}\sec\lambda[pf(z)-zf'(z)]}{Be^{i\lambda}\sec\lambda\cdot(zf'(z)) + [-Bp\, e^{i\lambda}\sec\lambda + (B-A)(p-\alpha)]f(z)}$$

$$= \frac{-e^{i\lambda}\sec\lambda \sum_{k=1}^{\infty} k a_{p+k} z^k}{(B-A)(p-\alpha)[1+\sum_{k=1}^{\infty} a_{p+k} z^k] + Be^{i\lambda}\sec\lambda \sum_{k=1}^{\infty} k\, a_{p+k} z^k}$$

$$= \frac{-e^{i\lambda}\sec\lambda \sum_{k=1}^{\infty} k\, a_{p+k} z^k}{(B-A)(p-\alpha) + \sum_{k=1}^{\infty} [\{(B-A)(p-\alpha)+\kappa Be^{i\lambda}\sec\lambda\}\, a_{p+k}]z^k}$$

$$-e^{i\lambda}\sec\lambda\, [\frac{a_{p+1}}{(B-A)(p-\alpha)}\, z + \frac{1}{(B-A)(p-\alpha)}\, x$$

$$x\ \{2\, a_{p+2} - (\frac{(B-A)(p-\alpha)+Be^{i\lambda}\sec\lambda}{(B-A)(p-\alpha)})a^2_{p+1}\}\ z^2 + \ldots]$$

and then comparing coefficients of z and z^2 on both sides, we have

$$b_1 = -\frac{e^{i\lambda}\sec\lambda}{(B-A)(p-\alpha)}\, a_{p+1}$$

$$b_2 = -\frac{e^{i\lambda}\sec\lambda}{(B-A)^2(p-\alpha)^2}\, [2(B-A)(p-\alpha)a_{p+2} - \{(B-A)(p-\alpha)+e^{i\lambda}\sec\lambda\}\, a^2_{p+1}].$$

Thus

$$a_{p+1} = -\frac{(B-A)(p-\alpha)}{e^{i\lambda}\sec\lambda}\, b_1$$

and

$$a_{p+2} = -\frac{(B-A)(p-\alpha)}{2\, e^{i\lambda}\sec\lambda}\, b_2 + \frac{(B-A)(p-\alpha)+e^{i\lambda}\sec\lambda}{2(B-A)(p-\alpha)}\, a^2_{p+1}$$

Hence

$$a_{p+2} - \mu\, a^2_{p+1} =$$

$$-\frac{(B-A)(p-\alpha)}{2e^{i\lambda}\sec\lambda}\, b_2 + [\frac{(B-A)(p-\alpha)+e^{i\lambda}\sec\lambda}{2(B-A)(p-\alpha)} - \mu]a^2_{p+1}$$

$$= -\frac{(B-A)(p-\alpha)}{2e^{i\lambda}\sec\lambda}\, b_2 + [\frac{(B-A)(p-\alpha)+e^{i\lambda}\sec\lambda}{2(B-A)(p-\alpha)} - \mu]\frac{(B-A)^2(p-\alpha)^2}{e^{2i\lambda}\sec^2\lambda}\, b_1^2\ . \tag{2.12}$$

Thus taking modulus of both sides of (2.12), we are led to

$$|a_{p+2} - \mu a^2_{p+1}| =$$

$$\frac{(B-A)(p-\alpha)}{2} \cos\lambda \left| b_2 - \left\{ \frac{(B-A)(p-\alpha)+e^{i\lambda}\sec\lambda}{2(B-A)(p-\alpha)} - \mu \right\} \frac{2(B-A)(p-\alpha)}{e^{i\lambda}\sec\lambda} b_1^2 \right|$$

$$= \frac{(B-A)(p-\alpha)}{2} \cos\lambda \left| b_2 - \left\{ \frac{e^{i\lambda}\sec\lambda - (B-A)(p-\alpha)(2\mu-1)}{e^{i\lambda}\sec\lambda} \right\} b_1^2 \right|. \qquad (2.13)$$

Using lemma 1 in (2.13), we get

$$|a_{p+2} - \mu a^2_{p+1}| \leq \frac{(B-A)(p-\alpha)}{2} \cos\lambda \max\{1, |(B-A)(p-\alpha)(2\mu-1)\cos\lambda - e^{i\lambda}|\}$$

and since (1.11) is sharp, then (2.10) is also sharp.

Remark on Theorem 2. Setting (i) B=1 and A=-1, (ii) B=1, A=-1 and p=1, (iii) B=1, A=-1, p=1 and α=0, (iv) B=1, A=-1 and λ=0, in Theorem 2, we get the results of Patil and Thakare [1].

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