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A NOTE ON WEAKLY QUASI CONTINUOUS FUNCTIONS

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ABSTRACT. In [1], it was shown that a function $f : X \to Y$ is weakly quasi continuous if and only if $f^{-1}(Cl(V)) \subset Cl(Int(f^{-1}(Cl(V))))$ for every open set V of Y. By utilizing this result, the present author [2] showed that a function $f : X \to Y$ is weakly quasi continuous if and only if for every regular closed set F of Y, $f^{-1}(F)$ is semi-open in X. In this note, the author shows that these results are false and corrects the proofs of Theorem 6.1.7 and Lemma 6.4.4 of [2].

KEY WORDS AND PHRASES. *weakly quasi continuous.*
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The purpose of this note is to point out that Theorem 2 of [1] and Theorem 4.2 of [2] are false, and to correct the proofs of Theorem 6.1.7 and Lemma 6.4.4 of [2]. Let S be a subset of a topological space. The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively. A subset S is said to be *semi-open* [3] (resp. *regular-closed*) if $S \subset Cl(Int(S))$ (resp. $S = Cl(Int(S))$). A function $f : X \to Y$ is said to be *semi-continuous* [3] (resp. *semi-open* [4]) if for every open set V of Y (resp. X), $f^{-1}(V)$ (resp. $f(V)$) is semi-open in X (resp. Y). A function $f : X \to Y$ is said to be *almost-continuous* [5] if for each $x \in X$ and each open neighborhood V of $f(x)$, $Cl(f^{-1}(V))$ is a neighborhood of x.

DEFINITION. A function $f : X \to Y$ is said to be *weakly quasi continuous* [1] (briefly *w.q.c.*) if for each $x \in X$, each open set U containing x and each open set V containing $f(x)$, there exists an open set G of X such that $\emptyset \neq G \subset U$ and $f(G) \subset Cl(V)$.

Popa and Stan [1] obtained the following characterization of w.q.c. functions.

THEOREM A ([1, Theorem 2]). A function $f : X \to Y$ is w.q.c. if and only if $f^{-1}(Cl(V)) \subset Cl(Int(f^{-1}(Cl(V))))$ for every open set V of Y.

In [2], among others, the author established the following three statements.

THEOREM B ([2, Theorem 4.2]). A function $f : X \to Y$ is w.q.c. if and only if for every regular closed set F of Y, $f^{-1}(F)$ is semi-open in X.

THEOREM C ([2, Theorem 6.1.7]). The composition $g \circ f : X \to Z$ of a continuous function $f : X \to Y$ and a semi-continuous function $g : Y \to Z$ is not necessarily w.q.c.

LEMMA D ([2, Lemma 6.4.4]). Let $f : X \to Y$ be an open continuous surjection and $g : Y \to Z$ a function. If $g \circ f : X \to Z$ is w.q.c., then g is w.q.c.

The author utilized Theorem A in order to prove Theorem B. Moreover, Theorem B was utilized in the proofs of Theorem C and Lemma D. However, it follows from Example 2 (below) that the necessity of Theorem A is false and hence so is Theorem B. Thus, it is necessary to revise the proofs of Theorem C and Lemma D. For this purpose, we have the following modification of Theorem A.

THEOREM 1. A function $f : X \to Y$ is w.q.c. if and only if for every open set V of Y, $f^{-1}(V) \subset Cl(Int(f^{-1}(Cl(V))))$.

PROOF. *Necessity*. Suppose that f is w.q.c. Let V be any open set of Y and $x \in f^{-1}(V)$. For each open set G of X containing x, there exists an open set U of X such that $\emptyset \neq U \subset G$ and $f(U) \subset Cl(V)$. Therefore, it follows that $U \subset f^{-1}(Cl(V))$ and $U \subset Int(f^{-1}(Cl(V)))$. Since $\emptyset \neq U \subset G \cap Int(f^{-1}(Cl(V)))$, $x \in Cl(Int(f^{-1}(Cl(V))))$ and hence $f^{-1}(V) \subset Cl(Int(f^{-1}(Cl(V))))$.

Sufficiency. Let x be any point of X, G any open set of X containing x and V any open set of Y containing $f(x)$. By hypothesis, we have $x \in f^{-1}(V) \subset Cl(Int(f^{-1}(Cl(V))))$ and hence $G \cap Int(f^{-1}(Cl(V))) \neq \emptyset$. Put $G \cap Int(f^{-1}(Cl(V))) = U$, then we obtain $\emptyset \neq U \subset G$ and $f(U) \subset Cl(V)$. This shows that f is w.q.c.

The following example shows that the necessities of Theorems A and B are both false.

EXAMPLE 2. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is w.q.c. by Theorem 1. Let $V = \{a\} \in \sigma$, then $Cl(V) = \{a, c\}$ is a regular closed set of (X, σ). However, $f^{-1}(Cl(V)) = \{a, c\}$ and $Cl(Int(f^{-1}(Cl(V)))) = \{a\}$. Thus, $f^{-1}(Cl(V))$ is not semi-open in (X, τ) and $f^{-1}(Cl(V)) \not\subset Cl(Int(f^{-1}(Cl(V))))$.

REMARK 3. (1) It follows immediately from Theorem 1 that the sufficiencies of Theorems A and B are both true.

(2) In the proof of [2, Theorem 6.1.7], the set $V = \{b, c, d\}$ is clopen in (Z, θ), $(g \circ f)^{-1}(V) = \{b, c, d\}$ and $Cl(Int((g \circ f)^{-1}(Cl(V)))) = \{b\}$. Therefore, by Theorem 1 $g \circ f$ is not w.q.c. and hence it is not semi-continuous.

Next, we give the correct proof of Lemma D in the improved form.

THEOREM 4. Let $f : X \to Y$ be a semi-open almost continuous surjection and $g : Y \to Z$ a function. If $g \circ f : X \to Z$ is w.q.c., then g is w.q.c.

PROOF. Let W be any open set of Z. Since $g \circ f$ is w.q.c., by Theorem 1 we have $(g \circ f)^{-1}(W) \subset Cl(Int((g \circ f)^{-1}(Cl(W))))$. Since f is almost continuous, for every subset A of X, $f(Cl(Int(A))) \subset Cl(f(Int(A)))$ [6, Theorem 6]. Moreover, since f is semi-open, $f(Int(A)) \subset Cl(Int(f(A)))$ [4, Theorem 9] and hence $f(Cl(Int(A))) \subset Cl(Int(f(A)))$ for every subset A of X. Therefore, we obtain $g^{-1}(W) \subset Cl(Int(g^{-1}(Cl(W))))$. It follows from Theorem 1 that g is w.q.c.

THEOREM 5. Let $f : X \to Y$ be an open continuous surjection. A function $g : Y \to Z$ is w.q.c. if and only if the composition $g \circ f : X \to Z$ is w.q.c.

PROOF. *Necessity*. Let W be any open set of Z. Since g is w.q.c., by Theorem 1 $g^{-1}(W) \subset Cl(Int(g^{-1}(Cl(W))))$. Since f is open continuous, for every subset B of Y $f^{-1}(Cl(Int(B))) \subset Cl(Int(f^{-1}(B)))$. Therefore, we obtain $(g \circ f)^{-1}(W) \subset Cl(Int((g \circ f)^{-1}(Cl(W))))$ and hence by Theorem 1 $g \circ f$ is w.q.c.

Sufficiency. Since an open continuous function is semi-open almost continuous, this is an immediate consequence of Theorem 4.

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