

## SEMIPRIME SF-RINGS WHOSE ESSENTIAL LEFT IDEALS ARE TWO-SIDED

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**ABSTRACT.** It is proved that if  $R$  is a semiprime ELT-ring and every simple right  $R$ -module is flat then  $R$  is regular. Is  $R$  regular if  $R$  is a semiprime ELT-ring and every simple right  $R$ -module is flat? In this note, we give a positive answer to the question.

**KEY WORDS AND PHRASES.** (Von Neumann) regular ring, SF-ring, ELT-ring.

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### 1. INTRODUCTION.

In [1] Yue Chi Ming proposed the following question: Is  $R$  regular if  $R$  is a semiprime ELT-ring and every simple right  $R$ -module is flat? In this note, we give a positive answer to the question.

All rings considered in this paper are associative with identity, and all modules are unital. A ring  $R$  is (Von Neumann) regular provided that for every  $a \in R$  there exists  $b \in R$  such that  $a = aba$  (see [2]).  $R$  is called a strongly regular ring if for each  $a \in R, a \in a^2R$ . Following [1], call  $R$  and ELT-ring if every essential left ideal is an ideal of  $R$ . We call  $R$  a right SF-ring if every simple right  $R$ -module is flat (see [3]).

### 2. MAIN RESULTS.

We begin by stating following lemmas which will be used in proof of our main result.

**LEMMA 1.** ([4], p.30, Exercise 19) If  $R$  is a semiprime ring, then  $Soc({}_R R) = Soc(R_R)$ .

**LEMMA 2.** ([5], Corollary 8.5) If  $R$  is a semiprime ring, then every minimal left (right) ideal is generated by an idempotent.

**LEMMA 3.** ([3], Proposition 3.2) Let  $R$  be a left (right) SF-ring. If  $I$  is an ideal of  $R$ , then  $R/I$  also is a left (right) SF-ring.

**LEMMA 4.** ([3], Theorem 4.10) Let  $R$  be a left (right) SF-ring. If every maximal right (left) ideal of  $R$  is an ideal, then  $R$  strongly regular.

**LEMMA 5.** If  $R$  is a semiprime ELT and right SF-ring, then  $R$  is fully left (right) idempotent.

**PROOF.** From Lemma 1,  $Soc({}_R R) = Soc(R_R)$ . Now we write  $S$  instead of  $Soc({}_R R)$ . By Lemma 2,  $S$  is fully left (right) idempotent. Since  $R$  is an ELT-ring, and every maximal left ideal of  $R/S$  is an image of a maximal essential left ideal of  $R$  under the natural map  $v: R \rightarrow R/S$ , hence every maximal left ideal of  $R/S$  is an ideal. By Lemma 3,  $R/S$  is a right SF-ring. It follows from Lemma 4 that  $R/S$  is strongly regular, whence  $R/S$  is fully left (right) idempotent. Since  $S$  is fully left (right) idempotent, then  $R$  is fully left (right) idempotent.

Now we prove our main result which gives a positive answer to the question raised in [1].

THEOREM 2.1. If  $R$  is a semiprime ELT and right SF-ring, then  $R$  is regular.

PROOF. From Lemma 5,  $R$  is a fully left (right) idempotent ring. If  $P$  is a prime ideal of  $R$ , then it is easy to know that  $R/P$  is a fully right idempotent ring. Since  $R$  is ELT, this implies that  $R/P$  is an ELT-ring. By (see [6], Corollary 6),  $R/P$  is regular. Considering that  $R$  is fully idempotent, thus  $R$  is a regular ring (see [2], Corollary 1.18).

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