INTUITIONISTIC FUZZY ALPHA-CONTINUITY AND INTUITIONISTIC FUZZY PRECONTINUITY

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A characterization of intuitionistic fuzzy α-open set is given, and conditions for an IFS to be an intuitionistic fuzzy α-open set are provided. Characterizations of intuitionistic fuzzy precontinuous (resp., α-continuous) mappings are given.

1. Introduction

After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them. Using the notion of intuitionistic fuzzy sets, Çoker [5] introduced the notion of intuitionistic fuzzy topological spaces. In this paper, we define the notion of intuitionistic fuzzy semiopen (resp., preopen and α-open) mappings and investigate relation among them. We give a characterization of intuitionistic fuzzy α-open set, and provide conditions for an IFS to be an intuitionistic fuzzy α-open set. We discuss characterizations of intuitionistic fuzzy precontinuous (resp., α-continuous) mappings. We give a condition for a mapping of IFTSs to be an intuitionistic fuzzy α-continuous mapping.

2. Preliminaries

Definition 2.1 (Atanassov [1]). An *intuitionistic fuzzy set* (IFS) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}, \tag{2.1}$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 (Atanassov [1]). Let A and B be IFSs of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle \mid x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

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(c) $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle \mid x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle \mid x \in X\}$,
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, we will use the notation $A = \langle x, \mu_A, \gamma_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \mid x \in X\}$. A constant fuzzy set taking value $\alpha \in [0,1]$ will be denoted by $\underline{\alpha}$. The IFSs $0_\sim$ and $1_\sim$ are defined to be $0_\sim = \langle x, \underline{0}, \underline{1}\rangle$ and $1_\sim = \langle x, \underline{1}, \underline{0}\rangle$, respectively. Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point* (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) := \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases} \tag{2.2}$$

Let f be a mapping from a set X to a set Y. If

$$B = \Big\{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\Big\} \tag{2.3}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \Big\{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\Big\} \tag{2.4}$$

and the *image* of A under f, denoted by $f(A)$, is an IFS of Y defined by

$$f(A) = \langle y, f(\mu_A), f(\gamma_A)\rangle, \tag{2.5}$$

where

$$f(\mu_A)(y) := \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \varnothing, \\ 0 & \text{otherwise,} \end{cases} \tag{2.6}$$

$$f(\gamma_A)(y) := \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \varnothing, \\ 1 & \text{otherwise,} \end{cases} \tag{2.7}$$

for each $y \in Y$. Çoker [5] generalized the concept of fuzzy topological space, first initiated by Chang [4], to the case of intuitionistic fuzzy sets as follows.

Definition 2.3 (Çoker [5, Definition 3.1]). An *intuitionistic fuzzy topology* (IFT) on X is a family τ of IFSs in X satisfying the following axioms:
(T1) $0_\sim, 1_\sim \in \tau$,
(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(T3) $\bigcup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case, the pair (X,τ) is called an *intuitionistic fuzzy topological space* (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS A in IFTS (X,τ) is called an *intuitionistic fuzzy closed set* (IFCS) in X.

Definition 2.4 (Çoker [5, Definition 3.13]). Let (X,τ) be an IFTS and let $A = \langle x, \mu_A, \gamma_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* of A are defined by

$$\begin{aligned} \operatorname{int}(A) &= \bigcup\{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\}, \\ \operatorname{cl}(A) &= \bigcap\{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}. \end{aligned} \tag{2.8}$$

Note that for any IFS A in (X,τ), we have

$$\operatorname{cl}(\bar{A}) = \overline{\operatorname{int}(A)}, \qquad \operatorname{int}(\bar{A}) = \overline{\operatorname{cl}(A)}. \tag{2.9}$$

3. Intuitionistic fuzzy openness

Definition 3.1 [7]. An IFS A in an IFTS (X,τ) is called
(i) an *intuitionistic fuzzy semiopen set* (IFSOS) if

$$A \subseteq \operatorname{cl}(\operatorname{int}(A)), \tag{3.1}$$

(ii) an *intuitionistic fuzzy α-open set* (IFαOS) [3] if

$$A \subseteq \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))), \tag{3.2}$$

(iii) an *intuitionistic fuzzy preopen set* (IFPOS) if

$$A \subseteq \operatorname{int}(\operatorname{cl}(A)), \tag{3.3}$$

(iv) an *intuitionistic fuzzy regular open set* (IFROS) if

$$\operatorname{int}(\operatorname{cl}(A)) = A. \tag{3.4}$$

An IFS A is called an *intuitionistic fuzzy semiclosed set*, *intuitionistic fuzzy α-closed set*, *intuitionistic fuzzy preclosed set*, and *intuitionistic fuzzy regular closed set*, respectively (IFSCS, IFαCS, IFPCS, and IFRCS, resp.), if the complement of A is an IFSOS, IFαOS, IFPOS, and IFROS, respectively.

In the following diagram, we provide relations between various types of intuitionistic fuzzy openness (intuitionistic fuzzy closedness):

$$\begin{tikzcd}
\boxed{\text{IFROS (IFRCS)}} \arrow[d, Rightarrow] & \\
\boxed{\text{IFOS (IFCS)}} \arrow[d, Rightarrow] & \\
\boxed{\text{IF}\alpha\text{OS (IF}\alpha\text{CS)}} \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\
\boxed{\text{IFPOS (IFPCS)}} \arrow[d, Rightarrow] & \boxed{\text{IFSOS (IFSCS)}} \arrow[dl, Rightarrow] \\
\boxed{\text{IF}\beta\text{OS (IF}\beta\text{CS)}} &
\end{tikzcd} \tag{3.5}$$

The reverse implications are not true in the above diagram (see [7]). The following is a characterization of an IFαOS.

Theorem 3.2. *An IFS A in an IFTS (X,τ) is an IFαOS if and only if it is both an IFSOS and an IFPOS.*

Proof. Necessity follows from the diagram given above. Suppose that A is both an IFSOS and an IFPOS. Then $A \subseteq \text{cl}(\text{int}(A))$, and so

$$\text{cl}(A) \subseteq \text{cl}\big(\text{cl}\big(\text{int}(A)\big)\big) = \text{cl}\big(\text{int}(A)\big). \tag{3.6}$$

It follows that $A \subseteq \text{int}(\text{cl}(A)) \subseteq \text{int}(\text{cl}(\text{int}(A)))$, so that A is an IFαOS. $\square$

We give condition(s) for an IFS to be an IFαOS.

Theorem 3.3. *Let A be an IFS in an IFTS (X,τ). If B is an IFSOS such that $B \subseteq A \subseteq \text{int}(\text{cl}(B))$, then A is an IFαOS.*

Proof. Since B is an IFSOS, we have $B \subseteq \mathrm{cl}(\mathrm{int}(B))$. Thus,

$$A \subset \mathrm{int}(\mathrm{cl}(B)) \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{cl}(\mathrm{int}(B)))) = \mathrm{int}(\mathrm{cl}(\mathrm{int}(B))) \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A))), \tag{3.7}$$

and so A is an IFαOS. □

LEMMA 3.4. *Any union of IFαOSs (resp., IFPOSs) is an IFαOS (resp., IFPOS).*

The proof is straightforward.

THEOREM 3.5. *An IFS A in an IFTS X is intuitionistic fuzzy α-open (resp., intuitionistic fuzzy preopen) if and only if for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFαOS (resp., IFPOS) $B_{p_{(\alpha,\beta)}}$ such that $p_{(\alpha,\beta)} \in B_{p_{(\alpha,\beta)}} \subseteq A$.*

Proof. If A is an IFαOS (resp., IFPOS), then we may take $B_{p_{(\alpha,\beta)}} = A$ for every $p_{(\alpha,\beta)} \in A$. Conversely assume that for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFαOS (resp., IFPOS) $B_{p_{(\alpha,\beta)}}$ such that $p_{(\alpha,\beta)} \in B_{p_{(\alpha,\beta)}} \subseteq A$. Then,

$$A = \bigcup \{p_{(\alpha,\beta)} \mid p_{(\alpha,\beta)} \in A\} \subseteq \bigcup \{B_{p_{(\alpha,\beta)}} \mid p_{(\alpha,\beta)} \in A\} \subseteq A, \tag{3.8}$$

and so $A = \bigcup\{B_{p_{(\alpha,\beta)}} \mid p_{(\alpha,\beta)} \in A\}$, which is an IF$\alpha$OS (resp., IFPOS) by Lemma 3.4. □

Definition 3.6. Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then, f is called
(i) an *intuitionistic fuzzy open mapping* if $f(A)$ is an IFOS in Y for every IFOS A in X,
(ii) an *intuitionistic fuzzy α-open mapping* if $f(A)$ is an IFαOS in Y for every IFOS A in X,
(iii) an *intuitionistic fuzzy preopen mapping* if $f(A)$ is an IFPOS in Y for every IFOS A in X,
(iv) an *intuitionistic fuzzy semiopen mapping* if $f(A)$ is an IFSOS in Y for every IFOS A in X.

We have the following implications in which reverse implications are not valid, where "IF" means "intuitionistic fuzzy":

$$\begin{tikzcd} \boxed{\text{IF open mapping}} \arrow[d, Rightarrow] & \\ \boxed{\text{IF } \alpha\text{-open mapping}} \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\ \boxed{\text{IF semiopen mapping}} & \boxed{\text{IF preopen mapping}} \end{tikzcd} \tag{3.9}$$

Let $A = \langle x, \mu_A, \gamma_A \rangle$, $B = \langle x, \mu_B, \gamma_B \rangle$, and $C = \langle x, \mu_C, \gamma_C \rangle$ be IFSs in $I = [0,1]$ defined by

$$\mu_A(x) = \begin{cases} 0, & 0 \le x \le \dfrac{1}{2}, \\ 2x - 1, & \dfrac{1}{2} \le x \le 1, \end{cases} \qquad \gamma_A(x) = \begin{cases} 1, & 0 \le x \le \dfrac{1}{2}, \\ 2(1 - x), & \dfrac{1}{2} \le x \le 1, \end{cases}$$

$$\mu_B(x) = \begin{cases} 1, & 0 \le x \le \dfrac{1}{4}, \\ 2 - 4x, & \dfrac{1}{4} \le x \le \dfrac{1}{2}, \\ 0, & \dfrac{1}{2} \le x \le 1, \end{cases} \qquad \gamma_B(x) = \begin{cases} 0, & 0 \le x \le \dfrac{1}{4}, \\ 4x - 1, & \dfrac{1}{4} \le x \le \dfrac{1}{2}, \\ 1, & \dfrac{1}{2} \le x \le 1, \end{cases} \tag{3.10}$$

$$\mu_C(x) = \begin{cases} 0, & 0 \le x \le \dfrac{1}{4}, \\ \dfrac{1}{3}(4x - 1), & \dfrac{1}{4} \le x \le 1, \end{cases} \qquad \gamma_C(x) = \begin{cases} 1, & 0 \le x \le \dfrac{1}{4}, \\ \dfrac{4}{3}(1 - x), & \dfrac{1}{4} \le x \le 1. \end{cases}$$

Then $\tau_1 = \{0_\sim, 1_\sim, B, A \cup B\}$, $\tau_2 = \{0_\sim, 1_\sim, \overline{C}\}$, and $\tau_3 = \{0_\sim, 1_\sim, C\}$ are IFTSs on I. Define a mapping $f : I \to I$ by $f(x) = \min\{2x, 1\}$ for each $x \in I$. Then $f(0_\sim) = 0_\sim$, $f(1_\sim) = 1_\sim$, $f(A) = 0_\sim$, and $f(B) = \overline{A} = f(A \cup B)$. It is easy to verify that $\overline{A}$ is an IFαOS in (I, τ_2). Since $\overline{A} \notin \tau_2$, we know that the mapping $f : (I, \tau_1) \to (I, \tau_2)$ is intuitionistic fuzzy α-open which is not intuitionistic fuzzy open. We also note that $\overline{A}$ is an IFSOS but not an IFPOS in (I, τ_1). Hence, $f : (I, \tau_1) \to (I, \tau_1)$ is an intuitionistic fuzzy semiopen mapping which is not intuitionistic fuzzy preopen, and so, also not intuitionistic fuzzy α-open. Further, $\overline{A}$ is an IFPOS which is not an IFSOS in (I, τ_3). Therefore, $f : (I, \tau_1) \to (I, \tau_3)$ is an intuitionistic fuzzy preopen mapping which is not intuitionistic fuzzy semiopen, and thus, also not intuitionistic fuzzy α-open.

Theorem 3.7. *Let $f : (X, \tau) \to (Y, \kappa)$ and $g : (Y, \kappa) \to (Z, \delta)$ be mappings of IFTSs. If f is intuitionistic fuzzy open and g is intuitionistic fuzzy α-open (resp., intuitionistic fuzzy preopen), then $g \circ f$ is intuitionistic fuzzy α-open (resp., intuitionistic fuzzy preopen).*

The proof is straightforward.

Theorem 3.8. *A mapping $f : (X, \tau) \to (Y, \kappa)$ is intuitionistic fuzzy α-open if and only if it is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen.*

Proof. Necessity follows from the above second diagram (3.9). Assume that f is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen and let A be an IFOS in X. Then, $f(A)$ is an IFPOS as well as an IFSOS in Y. It follows from Theorem 3.2 that $f(A)$ is an IFαOS so that f is an intuitionistic fuzzy α-open mapping. $\square$

4. Intuitionistic fuzzy continuity

Definition 4.1 [7]. Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then f is called an *intuitionistic fuzzy precontinuous mapping* if $f^{-1}(B)$ is an IFPOS in X for every IFOS B in Y.

Theorem 4.2. *For a mapping f from an IFTS (X,τ) to an IFTS (Y,κ), the following are equivalent.*

(i) *f is intuitionistic fuzzy precontinuous.*
(ii) *$f^{-1}(B)$ is an IFPCS in X for every IFCS B in Y.*
(iii) $\operatorname{cl}(\operatorname{int}(f^{-1}(A))) \subseteq f^{-1}(\operatorname{cl}(A))$ *for every IFS A in Y.*

Proof. (i)⇒(ii). The proof is straightforward.

(ii)⇒(iii). Let A be an IFS in Y. Then $\operatorname{cl}(A)$ is intuitionistic fuzzy closed. It follows from (ii) that $f^{-1}(\operatorname{cl}(A))$ is an IFPCS in X so that

$$\operatorname{cl}\left(\operatorname{int}\left(f^{-1}(A)\right)\right) \subseteq \operatorname{cl}\left(\operatorname{int}\left(f^{-1}\left(\operatorname{cl}(A)\right)\right)\right) \subseteq f^{-1}\left(\operatorname{cl}(A)\right). \tag{4.1}$$

(iii)⇒(i). Let A be an IFOS in Y. Then $\overline{A}$ is an IFCS in Y, and so

$$\operatorname{cl}\left(\operatorname{int}\left(f^{-1}(\overline{A})\right)\right) \subseteq f^{-1}\left(\operatorname{cl}(\overline{A})\right) = f^{-1}(\overline{A}). \tag{4.2}$$

This implies that

$$\begin{aligned} \overline{\operatorname{int}\left(\operatorname{cl}\left(f^{-1}(A)\right)\right)} &= \operatorname{cl}\left(\overline{\operatorname{cl}\left(f^{-1}(A)\right)}\right) = \operatorname{cl}\left(\operatorname{int}\left(\overline{f^{-1}(A)}\right)\right) \\ &= \operatorname{cl}\left(\operatorname{int}\left(f^{-1}(\overline{A})\right)\right) \subseteq f^{-1}(\overline{A}) = \overline{f^{-1}(A)}, \end{aligned} \tag{4.3}$$

and thus $f^{-1}(A) \subseteq \operatorname{int}(\operatorname{cl}(f^{-1}(A)))$. Hence $f^{-1}(A)$ is an IFPOS in X, and f is intuitionistic fuzzy precontinuous. □

Definition 4.3 [9]. Let $p_{(\alpha,\beta)}$ be an IFP of an IFTS (X,τ). An IFS A of X is called an *intuitionistic fuzzy neighborhood* (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Theorem 4.4. *Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then the following assertions are equivalent.*

(i) *f is intuitionistic fuzzy precontinuous.*
(ii) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFPOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.*
(iii) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFPOS B in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.*

Proof. (i)⇒(ii). Let $p_{(\alpha,\beta)}$ be an IFP in X and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS B in Y such that $f(p_{(\alpha,\beta)}) \in B \subseteq A$. Since f is intuitionistic fuzzy precontinuous, we know that $f^{-1}(B)$ is an IFPOS in X and

$$p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(B) \subseteq f^{-1}(A). \tag{4.4}$$

Thus (ii) is valid.

(ii)⇒(iii). Let $p_{(\alpha,\beta)}$ be an IFP in X and let A be an IFN of $f(p_{(\alpha,\beta)})$. The condition (ii) implies that there exists an IFPOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$ so that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (iii) is true.

(iii)⇒(i). Let B be an IFOS in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in B$, and so B is an IFN of $f(p_{(\alpha,\beta)})$ since B is an IFOS. It follows from (iii) that there exists an IFPOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$ so that

$$p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B). \tag{4.5}$$

Applying Theorem 3.5 induces that $f^{-1}(B)$ is an IFPOS in X. Therefore, f is intuitionistic fuzzy precontinuous. □

Definition 4.5 [7]. Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then f is called an *intuitionistic fuzzy α-continuous mapping* if $f^{-1}(B)$ is an IFαOS in X for every IFOS B in Y.

Theorem 4.6. *Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ) that satisfies*

$$\operatorname{cl}(\operatorname{int}(\operatorname{cl}(f^{-1}(B)))) \subseteq f^{-1}(\operatorname{cl}(B)) \tag{4.6}$$

for every IFS B in Y. Then f is intuitionistic fuzzy α-continuous.

Proof. Let B be an IFOS in Y. Then $\overline{B}$ is an IFCS in Y, which implies from hypothesis that

$$\operatorname{cl}(\operatorname{int}(\operatorname{cl}(f^{-1}(\overline{B})))) \subseteq f^{-1}(\operatorname{cl}(\overline{B})) = f^{-1}(\overline{B}). \tag{4.7}$$

It follows that

$$\begin{aligned}\overline{\operatorname{int}(\operatorname{cl}(\operatorname{int}(f^{-1}(B))))} &= \operatorname{cl}(\overline{\operatorname{cl}(\operatorname{int}(f^{-1}(B)))}) \\ &= \operatorname{cl}(\operatorname{int}(\overline{\operatorname{int}(f^{-1}(B))})) \\ &= \operatorname{cl}(\operatorname{int}(\operatorname{cl}(\overline{f^{-1}(B)}))) \\ &= \operatorname{cl}(\operatorname{int}(\operatorname{cl}(f^{-1}(\overline{B})))) \subseteq f^{-1}(\overline{B}) \\ &= \overline{f^{-1}(B)}\end{aligned} \tag{4.8}$$

so that $f^{-1}(B) \subseteq \operatorname{int}(\operatorname{cl}(\operatorname{int}(f^{-1}(B))))$. This shows that $f^{-1}(B)$ is an IFαOS in X. Hence, f is intuitionistic fuzzy α-continuous. □

Theorem 4.7. *Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then the following assertions are equivalent.*

(i) *f is intuitionistic fuzzy α-continuous.*

(ii) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFαOS B such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.*

(iii) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFαOS B such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.*

Proof. (i)⇒(ii). Let $p_{(\alpha,\beta)}$ be an IFP in X and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since f is intuitionistic fuzzy α-continuous, $B := f^{-1}(C)$ is an IFαOS and

$$p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) = B \subseteq f^{-1}(A). \tag{4.9}$$

Thus (ii) is valid.

(ii)⇒(iii). Let $p_{(\alpha,\beta)}$ be an IFP in X and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFαOS B such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$ by (ii). Thus, we have $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (iii) is valid.

(iii)⇒(i). Let B be an IFOS in Y and take $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in f(f^{-1}(B)) \subseteq B$. Since B is an IFOS, it follows that B is an IFN of $f(p_{(\alpha,\beta)})$ so from (iii), there exists an IFαOS A such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This shows that

$$p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B). \tag{4.10}$$

Using Theorem 3.5, we know that $f^{-1}(B)$ is an IFαOS in X, and hence f is intuitionistic fuzzy α-continuous. □

Combining Theorems 4.6, 4.7, and [8, Theorems 3.12 and 3.13], we have the following characterization of an intuitionistic fuzzy α-continuous mapping.

Theorem 4.8. *Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). Then the following assertions are equivalent.*

(i) *f is intuitionistic fuzzy α-continuous.*

(ii) *If C is an IFCS in Y, then $f^{-1}(C)$ is an IFαCS in X.*

(iii) $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B)))) \subseteq f^{-1}(\mathrm{cl}(B))$ *for every IFS B in Y.*

(iv) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFαOS B such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.*

(v) *For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFαOS B such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.*

Some aspects of intuitionistic fuzzy continuity, intuitionistic fuzzy almost continuity, intuitionistic fuzzy weak continuity, intuitionistic fuzzy α-continuity, intuitionistic fuzzy precontinuity, intuitionistic fuzzy semicontinuity, and intuitionistic fuzzy β-continuity are studied in [7] as well as in several papers. The relation among these types of intuitionistic fuzzy continuity is given in [7] as follows, where "IF" means "intuitionistic fuzzy":

$$
\begin{tikzcd}
\boxed{\text{IF weak continuity}} & \\
\boxed{\text{IF almost continuity}} \arrow[u, Rightarrow] & \\
\boxed{\text{IF continuity}} \arrow[u, Rightarrow] \arrow[d, Rightarrow] & \\
\boxed{\text{IF } \alpha\text{-continuity}} \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\
\boxed{\text{IF semicontinuity}} \arrow[d, Rightarrow] & \boxed{\text{IF precontinuity}} \\
\boxed{\text{IF } \beta\text{-continuity}} &
\end{tikzcd}
\tag{4.11}
$$

The reverse implications are not true in the above diagram in general (see [7]).

THEOREM 4.9. *Let f be a mapping from an IFTS (X,τ) to an IFTS (Y,κ). If f is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, then it is intuitionistic fuzzy α-continuous.*

Proof. Let B be an IFOS in Y. Since f is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, $f^{-1}(B)$ is both an IFPOS and an IFSOS in X. It follows from Theorem 3.2 that $f^{-1}(B)$ is an IFαOS in X so that f is intuitionistic fuzzy α-continuous. $\square$

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