

## Research Article

# On Concircular $\phi$ -Recurrent $K$ -Contact Manifold Admitting Semisymmetric Metric Connection

**Venkatesha, K. T. Pradeep Kumar, C. S. Bagewadi,  
and Gurupadavva Ingalahalli**

*Department of Mathematics, Kuvempu University, Shankaraghatta, Shimoga 577 451, India*

Correspondence should be addressed to Venkatesha, vensmath@gmail.com

Received 29 March 2012; Accepted 29 May 2012

Academic Editor: J. Dydak

Copyright © 2012 Venkatesha et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In the present paper, we have studied  $\phi$ -recurrent and concircular  $\phi$ -recurrent  $K$ -contact manifold with respect to semisymmetric metric connection and obtained some interesting results.

## 1. Introduction

The idea of semisymmetric linear connection on a differentiable manifold was introduced by Friedmann and Schouten [1]. In [2], Hayden introduced idea of metric connection with torsion on a Riemannian manifold. Further, some properties of semisymmetric metric connection has been studied by Yano [3]. In [4], Golab defined and studied quarter-symmetric connection on a differentiable manifold with affine connection, which generalizes the idea of semisymmetric connection. Various properties of semisymmetric metric connection and quarter-symmetric metric connection have been studied by many geometers like Sharfuddin and Hussain [5], Amur and Pujar [6], Rastogi [7, 8], Mishra and Pandey [9], Bagewadi et al. [10–14], De et al. [15, 16], and many others.

The notion of local symmetry of a Riemannian manifold has been weakened by many authors in several ways to a different extent. As a weaker version of local symmetry, Takahashi [17] introduced the notion of local  $\phi$ -symmetry on a Sasakian manifold. Generalizing the notion of  $\phi$ -symmetry, De et al. [18] introduced the notion of  $\phi$ -recurrent Sasakian manifolds.

The paper is organized as follows. Section 2 is devoted to preliminaries. In Section 3, we study semisymmetric metric connection in a  $K$ -contact manifold. In Section 4, it is proved that a  $\phi$ -recurrent  $K$ -contact manifold with respect to semisymmetric metric connection is an

Einstein manifold. Finally, in Section 5 it is also shown that concircular  $\phi$ -recurrent  $K$ -contact manifold admitting semisymmetric metric connection is an Einstein manifold, and the characteristic vector field  $\xi$  and the vector field  $\rho$  associated to the 1-form  $A$  are codirectional.

## 2. Preliminaries

An  $n$ -dimensional differentiable manifold  $M$  is said to have an almost contact structure  $(\phi, \xi, \eta)$  if it carries a tensor field  $\phi$  of type  $(1, 1)$ , a vector field  $\xi$ , and a 1-form  $\eta$  on  $M$ , respectively, such that,

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0. \quad (2.1)$$

Thus a manifold  $M$  equipped with this structure  $(\phi, \xi, \eta)$  is called an almost contact manifold and is denoted by  $(M, \phi, \xi, \eta)$ . If  $g$  is a Riemannian metric on an almost contact manifold  $M$  such that,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad g(X, \xi) = \eta(X), \quad (2.2)$$

where  $X, Y$  are vector fields defined on  $M$ , then,  $M$  is said to have an almost contact metric structure  $(\phi, \xi, \eta, g)$ , and  $M$  with this structure is called an almost contact metric manifold and is denoted by  $(M, \phi, \xi, \eta, g)$ .

If on  $(M, \phi, \xi, \eta, g)$  the exterior derivative of 1-form  $\eta$  satisfies

$$d\eta(X, Y) = g(X, \phi Y), \quad (2.3)$$

then  $(\phi, \xi, \eta, g)$  is said to be a contact metric structure, and  $M$  equipped with a contact metric structure is called a contact metric manifold.

If moreover  $\xi$  is killing vector field on  $M$ , then,  $M$  is called a  $K$ -contact Riemannian manifold [19, 20]. A  $K$ -contact Riemannian manifold is called Sasakian [19], if the relation

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X \quad (2.4)$$

holds, where  $\nabla$  denotes the operator of covariant differentiation with respect to  $g$ .

In a  $K$ -contact manifold  $M$ , the following relations holds:

$$\nabla_X \xi = -\phi X, \quad (2.5)$$

$$g(R(X, Y)Z, \xi) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \quad (2.6)$$

$$S(X, \xi) = (n - 1)\eta(X), \quad (2.7)$$

for all vector fields  $X, Y$ , and  $Z$ . Here  $R$  and  $S$  are the Riemannian curvature tensor and the Ricci tensor of  $M$ , respectively.

*Definition 2.1.* A  $K$ -contact manifold  $M$  is said to be  $\phi$ -recurrent if there exists a nonzero 1-form  $A$  such that,

$$\phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z, \quad (2.8)$$

where  $A$  is defined by  $A(W) = g(W, \rho)$ , and  $\rho$  is a vector field associated with the 1-form  $A$ .

*Definition 2.2.* A  $K$ -contact manifold  $M$  is said to be concircular  $\phi$ -recurrent [12] if there exists a non-zero 1-form  $A$  such that,

$$\phi^2\left(\left(\nabla_W \bar{C}\right)(X, Y)Z\right) = A(W)\bar{C}(X, Y)Z, \quad (2.9)$$

where  $\bar{C}$  is a concircular curvature tensor given by [21] as follows:

$$\bar{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y], \quad (2.10)$$

where  $R$  is the Riemannian curvature tensor and  $r$  is the scalar curvature.

A linear connection  $\tilde{\nabla}$  in an  $n$ -dimensional differentiable manifold  $M$  is said to be a semisymmetric connection if its torsion tensor  $T$  is of the form

$$T(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y] = \eta(Y)X - \eta(X)Y, \quad (2.11)$$

for all  $X, Y$  on  $TM$ . A semisymmetric connection  $\tilde{\nabla}$  is called semisymmetric metric connection, if it further satisfies  $\tilde{\nabla}g = 0$ .

### 3. Semisymmetric Metric Connection in a $K$ -Contact Manifold

A semisymmetric metric connection  $\tilde{\nabla}$  in a  $K$ -contact manifold can be defined by

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi, \quad (3.1)$$

where  $\nabla$  is the Levi-Civita connection on  $M$  [3].

A relation between the curvature tensor of  $M$ , with respect to the semisymmetric metric connection  $\tilde{\nabla}$  and the Levi-Civita connection,  $\nabla$  is given by

$$\begin{aligned} \tilde{R}(X, Y)Z &= R(X, Y)Z + [g(\phi Y, Z)X - g(\phi X, Z)Y] + [g(Y, Z)\phi X - g(X, Z)\phi Y] \\ &\quad + [g(\phi X, \phi Z)Y - g(\phi Y, \phi Z)X] + [\eta(X)g(Y, Z) - \eta(Y)g(X, Z)]\xi, \end{aligned} \quad (3.2)$$

where  $\tilde{R}$  and  $R$  are the Riemannian curvatures of the connections  $\tilde{\nabla}$  and  $\nabla$ , respectively.

From (3.2), it follows that

$$\tilde{S}(Y, Z) = S(Y, Z) - (n-2)g(Y, Z) + (n-2)g(\phi Y, Z) + (n-2)\eta(Y)\eta(Z), \quad (3.3)$$

where  $\tilde{S}$  and  $S$  are the Ricci tensors of the connections  $\tilde{\nabla}$  and  $\nabla$ , respectively.

Contracting (3.3), we get

$$\tilde{r} = r - (n-1)(n-2), \quad (3.4)$$

where  $\tilde{r}$  and  $r$  are the scalar curvatures of the connections  $\tilde{\nabla}$  and  $\nabla$ , respectively.

#### 4. $\phi$ -Recurrent $K$ -Contact Manifold with respect to Semisymmetric Metric Connection

A  $K$ -contact manifold is called  $\phi$ -recurrent with respect to the semisymmetric metric connection if its curvature tensor  $\tilde{R}$  satisfies the following condition:

$$\phi^2\left(\left(\tilde{\nabla}_W \tilde{R}\right)(X, Y)Z\right) = A(W)\tilde{R}(X, Y)Z. \quad (4.1)$$

By virtue of (2.1) and (4.1), we have

$$-\left(\tilde{\nabla}_W \tilde{R}\right)(X, Y)Z + \eta\left(\left(\tilde{\nabla}_W \tilde{R}\right)(X, Y)Z\right)\xi = A(W)\tilde{R}(X, Y)Z, \quad (4.2)$$

from which, it follows that

$$-g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(X, Y)Z, U\right) + \eta\left(\left(\tilde{\nabla}_W \tilde{R}\right)(X, Y)Z\right)g(\xi, U) = A(W)g\left(\tilde{R}(X, Y)Z, U\right). \quad (4.3)$$

Let  $\{e_i\}$ ,  $i = 1, 2, \dots, n$  be an orthonormal basis of the tangent space at any point of the manifold. Then putting  $X = U = e_i$  in (4.3) and taking summation over  $i$ ,  $1 \leq i \leq n$ , we get

$$-\left(\tilde{\nabla}_W \tilde{S}\right)(Y, Z) + \sum_{i=1}^n \eta\left(\left(\tilde{\nabla}_W \tilde{R}\right)(e_i, Y)Z\right)\eta(e_i) = A(W)\tilde{S}(Y, Z). \quad (4.4)$$

Put  $Z = \xi$ , then the second term of (4.4) takes the following form:

$$\begin{aligned} g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(e_i, Y)\xi, \xi\right) &= g\left(\tilde{\nabla}_W \tilde{R}(e_i, Y)\xi, \xi\right) - g\left(\tilde{R}\left(\tilde{\nabla}_W e_i, Y\right)\xi, \xi\right) \\ &\quad - g\left(\tilde{R}\left(e_i, \tilde{\nabla}_W Y\right)\xi, \xi\right) - g\left(\tilde{R}(e_i, Y)\tilde{\nabla}_W \xi, \xi\right). \end{aligned} \quad (4.5)$$

On simplification, we obtain  $g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(e_i, Y)\xi, \xi\right) = 0$ .

Now (4.4) implies that

$$\left(\tilde{\nabla}_W \tilde{S}\right)(Y, \xi) = -A(W)\tilde{S}(Y, \xi). \quad (4.6)$$

We know that

$$\left(\tilde{\nabla}_W \tilde{S}\right)(Y, \xi) = \tilde{\nabla}_W \tilde{S}(Y, \xi) - \tilde{S}\left(\tilde{\nabla}_W Y, \xi\right) - \tilde{S}\left(Y, \tilde{\nabla}_W \xi\right). \quad (4.7)$$

Using (3.3), (2.5), and (2.7) in the above relation, we get

$$\begin{aligned} \left(\tilde{\nabla}_W \tilde{S}\right)(Y, \xi) &= S(Y, \phi W) - S(Y, W) - (n-1)g(Y, \phi W) \\ &+ (n-1)g(Y, W) + 2(n-2)g(\phi Y, \phi W). \end{aligned} \quad (4.8)$$

In view of (4.6) and (4.8), we have

$$S(Y, W) - S(Y, \phi W) + (n-1)g(Y, \phi W) - (n-1)g(Y, W) - 2(n-2)g(\phi Y, \phi W) = (n-1)A(W)\eta(Y). \quad (4.9)$$

Again putting  $Y = \phi Y$  in (4.9), we get

$$S(\phi Y, W) - S(\phi Y, \phi W) + (n-1)g(\phi Y, \phi W) - (n-1)g(\phi Y, W) + 2(n-2)g(Y, \phi W) = 0. \quad (4.10)$$

Interchanging  $Y$  and  $W$  in (4.10), we obtain

$$S(\phi W, Y) - S(\phi W, \phi Y) + (n-1)g(\phi W, \phi Y) - (n-1)g(\phi W, Y) + 2(n-2)g(W, \phi Y) = 0. \quad (4.11)$$

Adding (4.10) and (4.11) which on simplification, we have

$$S(Y, W) = (n-1)g(Y, W). \quad (4.12)$$

Therefore, we can state the following.

**Theorem 4.1.** *A  $\phi$ -recurrent  $K$ -contact manifold with respect to semisymmetric metric connection is an Einstein manifold.*

## 5. Conircular $\phi$ -Recurrent $K$ -Contact Manifold with respect to Semisymmetric Metric Connection

Let us consider a conircular  $\phi$ -recurrent  $K$ -contact manifold with respect to the semisymmetric metric connection defined by

$$\phi^2 \left( \left( \tilde{\nabla}_W \tilde{C} \right) (X, Y) Z \right) = A(W) \tilde{C}(X, Y) Z, \quad (5.1)$$

where  $\tilde{C}$  is a conircular curvature tensor with respect to the semisymmetric metric connection given by

$$\tilde{C}(X, Y) Z = \tilde{R}(X, Y) Z - \frac{\tilde{r}}{n(n-1)} [g(Y, Z) X - g(X, Z) Y]. \quad (5.2)$$

By virtue of (2.1) and (5.1), we have

$$-\left(\tilde{\nabla}_W \tilde{\tilde{C}}\right)(X, Y)Z + \eta\left(\left(\tilde{\nabla}_W \tilde{\tilde{C}}\right)(X, Y)Z\right)\xi = A(W)\tilde{\tilde{C}}(X, Y)Z, \quad (5.3)$$

from which, it follows that

$$-g\left(\left(\tilde{\nabla}_W \tilde{\tilde{C}}\right)(X, Y)Z, U\right) + \eta\left(\left(\tilde{\nabla}_W \tilde{\tilde{C}}\right)(X, Y)Z\right)g(\xi, U) = A(W)g\left(\tilde{\tilde{C}}(X, Y)Z, U\right), \quad (5.4)$$

where

$$\begin{aligned} \left(\tilde{\nabla}_W \tilde{\tilde{C}}\right)(X, Y)Z = & ((\nabla_W R)(X, Y)Z) + 3[g(Y, W)\eta(Z)X - g(X, W)\eta(Z)Y] \\ & + 3[g(Y, Z)g(W, X) - g(X, Z)g(W, Y)]\xi \\ & + 2[\eta(X)g(\phi W, Z)Y - \eta(Y)g(\phi W, Z)X] \\ & + 2[\eta(Y)g(X, Z) - \eta(X)g(Y, Z)]\phi W + [g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]W \\ & + 2\eta(W)[\eta(Y)g(X, Z) - \eta(X)g(Y, Z)]\xi \\ & + 2\eta(Z)\eta(W)[\eta(X)Y - \eta(Y)X] + g(Z, W)[\eta(Y)X - \eta(X)Y] \\ & - g(W, R(X, Y)Z)\xi - \eta(X)R(W, Y)Z \\ & - \eta(Y)R(X, W)Z - \eta(Z)R(X, Y)W \\ & - \frac{\nabla_W r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \quad (5.5)$$

Let  $\{e_i\}$ ,  $i = 1, 2, \dots, n$  be an orthonormal basis of the tangent space at any point of the manifold. Then putting  $X = U = e_i$  in (5.4) and taking summation over  $i$ ,  $1 \leq i \leq n$ , we get

$$\left(\tilde{\nabla}_W \tilde{\tilde{S}}\right)(Y, Z) - \frac{\tilde{\nabla}_W \tilde{r}}{n}g(Y, Z) = -\frac{\tilde{\nabla}_W \tilde{r}}{n(n-1)}[g(Y, Z) - \eta(Y)\eta(Z)] - A(W)\left[\tilde{\tilde{S}}(Y, Z) - \frac{\tilde{r}}{n}g(Y, Z)\right]. \quad (5.6)$$

Replacing  $Z$  by  $\xi$  in (5.6), we obtain

$$\left(\tilde{\nabla}_W \tilde{\tilde{S}}\right)(Y, \xi) = \frac{\tilde{\nabla}_W \tilde{r}}{n}\eta(Y) - A(W)\left[\tilde{\tilde{S}}(Y, \xi) - \frac{\tilde{r}}{n}\eta(Y)\right]. \quad (5.7)$$

We know that

$$\left(\tilde{\nabla}_W \tilde{\tilde{S}}\right)(Y, \xi) = \tilde{\nabla}_W \tilde{\tilde{S}}(Y, \xi) - \tilde{S}(\tilde{\nabla}_W Y, \xi) - \tilde{S}(Y, \tilde{\nabla}_W \xi). \quad (5.8)$$

Using (3.3), (2.5) and (2.7), the above relation becomes

$$\left(\tilde{\nabla}_W \tilde{S}\right)(Y, \xi) = S(Y, \phi W) - S(Y, W) - (n-1)g(Y, \phi W) + (n-1)g(Y, W) + 2(n-2)g(\phi Y, \phi W). \quad (5.9)$$

In view of (5.7) and (5.9), we obtain

$$\begin{aligned} S(Y, \phi W) - S(Y, W) - (n-1)g(Y, \phi W) + (n-1)g(Y, W) + 2(n-2)g(Y, W) - 2(n-2)\eta(Y)\eta(W) \\ = \frac{\nabla_W r}{n}\eta(Y) - A(W)\left[\frac{2(n-1)^2 - r}{n}\eta(Y)\right]. \end{aligned} \quad (5.10)$$

Replacing  $Y$  by  $\phi Y$  in (5.10), we have

$$S(\phi Y, \phi W) - S(\phi Y, W) - (n-1)g(\phi Y, \phi W) + (n-1)g(\phi Y, W) + 2(n-2)g(\phi Y, W) = 0. \quad (5.11)$$

Interchanging  $Y$  and  $W$  in (5.11), we get

$$S(\phi W, \phi Y) - S(\phi W, Y) - (n-1)g(\phi W, \phi Y) + (n-1)g(\phi W, Y) + 2(n-2)g(\phi W, Y) = 0. \quad (5.12)$$

Adding (5.11) and (5.12), which on simplification, we have

$$S(Y, W) = (n-1)g(Y, W). \quad (5.13)$$

Thus, we obtain the following theorem.

**Theorem 5.1.** *A Concircular  $\phi$ -recurrent  $K$ -contact manifold with respect to semisymmetric metric connection is an Einstein manifold.*

Next, from (5.3), one has

$$\left(\tilde{\nabla}_W \tilde{C}\right)(X, Y)Z = \eta\left(\left(\tilde{\nabla}_W \tilde{C}\right)(X, Y)Z\right)\xi - A(W)\tilde{C}(X, Y)Z. \quad (5.14)$$

Now, using (3.2), (3.4), (5.5), and Bianchi's identity in (5.14), one obtains

$$\begin{aligned} A(W)\eta(R(X, Y)Z) + A(X)\eta(R(Y, W)Z) + A(Y)\eta(R(W, X)Z) \\ = -A(W)[g(\phi Y, Z)\eta(X) - g(\phi X, Z)\eta(Y)] \\ - A(X)[g(\phi W, Z)\eta(Y) - g(\phi Y, Z)\eta(W)] \\ - A(Y)[g(\phi X, Z)\eta(W) - g(\phi W, Z)\eta(X)] \end{aligned}$$

$$\begin{aligned}
& + \frac{r - (n-1)(n-2)}{n(n-1)} A(W) [g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\
& + \frac{r - (n-1)(n-2)}{n(n-1)} A(X) [g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\
& + \frac{r - (n-1)(n-2)}{n(n-1)} A(Y) [g(X, Z)\eta(W) - g(W, Z)\eta(X)].
\end{aligned} \tag{5.15}$$

Putting  $Y = Z = e_i$  in (5.15) and taking summation over  $i$ ,  $1 \leq i \leq n$ , one gets

$$\begin{aligned}
& \left[ \frac{-n(n-1)(n-2) + r(n-2) - (n-1)(n-2)^2}{n(n-1)} \right] A(X)\eta(W) \\
& + \left[ \frac{n(n-1)(n-2) - r(n-2) + (n-1)(n-2)^2}{n(n-1)} \right] A(W)\eta(X) \\
& = A(\phi W)\eta(X) - A(\phi X)\eta(W).
\end{aligned} \tag{5.16}$$

Replacing  $X$  by  $\xi$  in (5.16), one gets

$$\left[ \frac{[r(n-2) + 2(n-1)(n-2)]^2 + n^2(n-1)^2}{n(n-1)[r(n-2) + 2(n-1)(n-2)]} \right] [A(W) - A(\xi)\eta(W)] = 0, \tag{5.17}$$

therefore

$$A(W) = \eta(W)\eta(\rho), \tag{5.18}$$

for any vector field  $W$ .

Hence, one states the following.

**Theorem 5.2.** *In a concircular  $\phi$ -recurrent  $K$ -contact manifold admitting semisymmetric metric connection the characteristic vector field  $\xi$  and the vector field  $\rho$  associated to the 1-form  $A$  are co-directional and the 1-form  $A$  is given by (5.18).*

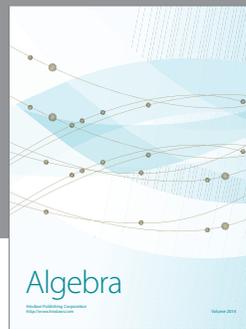
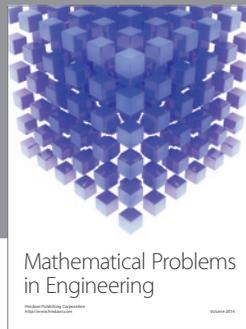
## Acknowledgment

The authors express their thanks to DST (Department of Science and Technology), Government of India for providing financial assistance under major research project (no. SR/S4/MS:482/07).

## References

- [1] A. Friedmann and J. A. Schouten, "Über die Geometrie der halbsymmetrischen Übertragungen," *Mathematische Zeitschrift*, vol. 21, no. 1, pp. 211–223, 1924.

- [2] H. A. Hayden, "Subspaces of a space with torsion," *Proceedings London Mathematical Society*, vol. 34, pp. 27–50, 1932.
- [3] K. Yano, "On semi-symmetric metric connection," *Revue Roumaine de Mathématiques Pures et Appliquées*, vol. 15, pp. 1579–1586, 1970.
- [4] S. Golab, "On semi-symmetric and quarter-symmetric linear connections," *Tensor*, vol. 29, no. 3, pp. 249–254, 1975.
- [5] A. Sharfuddin and S. I. Husain, "Semi-symmetric metric connexions in almost contact manifolds," *Tensor*, vol. 30, no. 2, pp. 133–139, 1976.
- [6] K. Amur and S. S. Pujar, "On submanifolds of a Riemannian manifold admitting a metric semisymmetric connection," *Tensor*, vol. 32, no. 1, pp. 35–38, 1978.
- [7] S. C. Rastogi, "On quarter-symmetric metric connection," *Comptes Rendus de l'Académie Bulgare des Sciences*, vol. 31, no. 7, pp. 811–814, 1978.
- [8] S. C. Rastogi, "On quarter-symmetric metric connections," *Tensor*, vol. 44, no. 2, pp. 133–141, 1987.
- [9] R. S. Mishra and S. N. Pandey, "On quarter symmetric metric  $F$ -connections," *Tensor*, vol. 34, no. 1, pp. 1–7, 1980.
- [10] C. S. Bagewadi, "On totally real submanifolds of a Kählerian manifold admitting semisymmetric metric  $F$ -connection," *Indian Journal of Pure and Applied Mathematics*, vol. 13, no. 5, pp. 528–536, 1982.
- [11] C. S. Bagewadi, N. S. Basavarajappa, D. G. Prakasha, and Venkatesha, "Some results on  $K$ -contact and trans-Sasakian manifolds," *European Journal of Pure and Applied Mathematics*, vol. 1, no. 2, pp. 21–31, 2008.
- [12] Venkatesha and C. S. Bagewadi, "On concircular  $\phi$ -recurrent LP-Sasakian manifolds," *Differential Geometry—Dynamical Systems*, vol. 10, pp. 312–319, 2008.
- [13] K. T. Pradeep Kumar, C. S. Bagewadi, and Venkatesha, "Projective  $\phi$ -symmetric  $K$ -contact manifold admitting quarter-symmetric metric connection," *Differential Geometry—Dynamical Systems*, vol. 13, pp. 128–137, 2011.
- [14] K. T. Pradeep Kumar, Venkatesha, and C. S. Bagewadi, "On  $\phi$ -recurrent para-Sasakian manifold admitting quarter-symmetric metric connection," *ISRN Geometry*, vol. 2012, Article ID 317253, 10 pages, 2012.
- [15] G. Pathak and U. C. De, "On a semi-symmetric metric connection in a Kenmotsu manifold," *Bulletin of the Calcutta Mathematical Society*, vol. 94, no. 4, pp. 319–324, 2002.
- [16] A. K. Mondal and U. C. De, "Some properties of a quarter-symmetric metric connection on a Sasakian manifold," *Bulletin of Mathematical Analysis and Applications*, vol. 1, no. 3, pp. 99–108, 2009.
- [17] T. Takahashi, "Sasakian  $\phi$ -symmetric spaces," *The Tohoku Mathematical Journal*, vol. 29, no. 1, pp. 91–113, 1977.
- [18] U. C. De, A. A. Shaikh, and S. Biswas, "On  $\phi$ -recurrent Sasakian manifolds," *Novi Sad Journal of Mathematics*, vol. 33, no. 2, pp. 43–48, 2003.
- [19] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Vol. 509, Springer-Verlag, Berlin, Germany, 1976.
- [20] S. Sasaki, *Lecture note on almost contact manifolds*, Part I, Tohoku University, 1965.
- [21] K. Yano, "Concircular geometry—I. Concircular transformations," *Proceedings of the Japan Academy*, vol. 16, pp. 195–200, 1940.



# Hindawi

Submit your manuscripts at  
<http://www.hindawi.com>

