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CORRIGENDUM

A NOTE ON SELF-CONJUGATE n-COLOR PARTITIONS

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A theorem involving self-conjugate n-color partitions was proved by A. K. Agarwal and R. Balasubrananian in their paper entitled n-Color Partitions with Weighted Differences Equal to Minus Two, *Internat. J. Math. Math. Sci.*, Vol. 20, No. 4 (1997), 759-768. There are some errors in this theorem which are corrected as follows:

THEOREM 1. Let $A(\nu)$ denote the number of n-color self-conjugate partitions of ν such that each part is self-conjugate. Let $B(\nu)$ denote the number of ordinary partitions of ν into odd parts. Then $A(\nu) = B(\nu)$, for all $\nu \geq 0$. Hence

$$1+\sum_{\nu=1}^{\infty} A(\nu)q^{\nu} = \prod_{n=1}^{\infty} \frac{1}{1-q^{2n-1}}. \tag{1}$$

EXAMPLE. $A(5) = 3$, since the relevant partitions are 5_3, $3_1 1_1 1_1$ and $1_1 1_1 1_1 1_1 1_1$. Also, $B(5) = 3$, since in this case the relevant partitions are 5, 311, 11111.

THEOREM 2. Let $C(\nu)$ denote the number of all n-color self-conjugate partitions of ν. Let $D(\nu)$ denote the number of partitions of ν such that each even part $2n$ can come in $[n/2]$, where $[\ ]$ is the greatest integer function, different colors denoted by

$$(2n)_1, (2n)_2, \cdots, (2n)_{[n/2]}.$$

Then $C(\nu) = D(\nu)$, for all $\nu \geq 0$. Hence,

$$1+\sum_{\nu=1}^{\infty} C(\nu)q^{\nu} = \prod_{n=1}^{\infty} (1-q^{2n-1})^{-1}(1-q^{2n})^{-[n/2]}. \tag{2}$$

EXAMPLE. $C(5) = 4$, since the relevant partitions are 5_3, $3_2 1_1 1_1$, $2_1 2_2 1_1$, $1_1 1_1 1_1 1_1 1_1$. Also, $D(5) = 4$. In this case the relevant partitions are $5 4_1 1$, 3 1 1, 1 1 1 1 1.

PROOF OF THEOREM 1. Let Π be an n-color partition enumerated by $A(\nu)$. Then in each part m_i of it, m must be odd. Because $m_i = m_{m-i+1} \Rightarrow m = 2i - 1$. Thus if we ignore the subscripts of all parts in Π, we get a unique ordinary partition of ν into odd parts. Conversely, if we consider an ordinary partition of ν into odd parts and replace each part $2a-1$ by $(2a-1)_a$ we get a unique partition enumerated by $A(\nu)$. This bijection proves Theorem 1.

PROOF OF THEOREM 2. Let σ be an n-color partition enumerated by $C(\nu)$. This implies that the parts of σ are either self-conjugate or they appear in pairs of mutually conjugate parts. It was observed in the proof of Theorem 1 that a part can be self-conjugate if it is odd. Also, the number of pairs of mutually conjugate parts corresponding to any even integer $2n$ is $[n/2]$. These arguments together prove Theorem 2

REMARK. It was shown in [1] that $C(\nu)$ of Theorem 2 also equals the number of symmetric plane partition of ν.

REFERENCES

1. A.K. Agarwal, Proceedings of NERCOM (1997), 35-46, Assam Academy of Mathematics.