Internat. J. Math. & Math. Sci.
Vol. 22, No. 2 (1999) 251–258
S 0161-1712(99)22251-X

TWO COUNTABLE, BICONNECTED, NOT WIDELY CONNECTED HAUSDORFF SPACES

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(Received 5 November 1996 and in revised form 6 July 1998)

ABSTRACT. We construct two countable, biconnected spaces, not widely connected, not having a dispersion point, and not being strongly connected. The first is Hausdorff and the second is Urysohn and almost regular.

Keywords and phrases. Countable connected, biconnected, almost regular.

1991 Mathematics Subject Classification. 54D05, 54D10, 54G15.

1. Introduction. The first example of a biconnected space with a dispersion point was constructed by B. Knaster and K. Kuratowski in [23], and the first example of a biconnected space without a dispersion point by E. W. Miller in [26]. Two stronger examples of biconnected spaces without a dispersion point were constructed by M. E. Rudin in [30, 31]. The example in [30] has the property that the complement of every connected subset containing more than one point is at most countable and the example in [31] has the property of being widely connected. All spaces in [26, 30, 31], are subsets of the plane. The first two are constructed under the Continuum Hypothesis and the third one under Martin's Axiom. In [7], G. Gruenhage constructed a countable connected Hausdorff space under Martin's Axiom and a perfectly normal connected space under the Continuum Hypothesis in which the complement of every connected subspace containing more than one point is finite. In [36], we constructed a countable widely connected Hausdorff space and a countable widely connected and biconnected Hausdorff space.

Now, we construct two countable spaces which are biconnected without being widely connected and without a dispersion point. The first is Hausdorff, and the second is Urysohn almost regular. In addition, as it is the case with widely connected spaces and spaces with a dispersion point, both have the property of not being strongly connected [13]. The construction is based on a modification of [16] or [20]. It can be also based on [37]. From the construction, it follows that there exist 2^c non-homeomorphic such spaces.

A space X is called

(1) *Urysohn* if every pair of distinct points of X have disjoint closed neighborhoods.

(2) *Almost regular* if X contains a dense subset at every point of which the space is regular.

A connected space X is called

(1) *Biconnected* (K. Kuratowski [24]) if it admits no decomposition into two connected disjoint proper subsets containing more than one point.

(2) *Widely connected* (P. M. Swingle [34]) if every connected subset, containing more than one point, is dense.

A point x of a connected space X is called

(1) A *cut* point if $X\setminus\{x\}$ is disconnected.

(2) A *dispersion* point if $X\setminus\{x\}$ is totally disconnected.

A connected space (X,τ) is called

(1) *Maximal connected* if, for every strictly finer topology σ, the space (X,σ) is not connected.

(2) *Strongly connected* if it has a finer maximal connected topology.

Biconnected spaces (countable or not, with or without a dispersion point) are considered in [26, 37, 1, 2, 3, 4, 6, 9, 10, 11, 18, 19, 21, 22, 25, 27, 28, 29, 33, 38, 39] and maximal connected spaces in [13, 1, 5, 8, 12, 14, 15, 32, 35].

2. Results

The space T. For the construction of the countably, biconnected and not widely connected Hausdorff space T, we first construct an appropriate countable Hausdorff totally disconnected space X containing a specific point p and a closed discrete subspace $\mathbb{N}$ which cannot be separated by disjoint open sets. Then keeping fixed the subspace $\mathbb{N}$ and condensing the point p (instead of condensing pairs of points as in [16, 20], or [37]), we construct the space T.

On the set

$$X=\{a_{ki}:k,i=1,2,\dots\}\cup\mathbb{N}\cup\{p\}, \tag{2.1}$$

where $\mathbb{N}$ is the space of natural numbers, we define the following topology: every point a_{ki} is isolated. For the points of $\mathbb{N}$ a basis of open neighborhoods in X is defined as follows: let $\mathcal{P}$ be a free ultrafilter on $\mathbb{N}$ and let $\mathcal{P}_k$ be the copy of $\mathcal{P}$ in $\{a_{ki}:i=1,2,\dots\}$. If $U\in\mathcal{P}$, we denote the copy of U in $\{a_{ki}:i=1,2,\dots\}$ by U_k. Then, for every $k\in\mathbb{N}$, a basis of open neighborhoods is the collection of sets

$$U(k)=\{k\}\cup\{a_{ki}:a_{ki}\in U_k\},\quad U\in\mathcal{P}. \tag{2.2}$$

For the point p, a basis of open neighborhoods is the collection of sets

$$U(p)=\{p\}\cup\{a_{ki}:k\in U\},\quad U\in\mathcal{P}. \tag{2.3}$$

Obviously, the space X is Hausdorff and totally disconnected but not regular since the point p and the closed subset $\mathbb{N}$ cannot be separated by disjoint open sets.

We observe that every basic open neighborhood of p is defined by some $U\in\mathcal{P}$, and every $U\in\mathcal{P}$ defines a basic open neighborhood $U(p)$. Obviously, $\overline{U(p)}\setminus U(p)=U$.

Let $X^1(n)$, $n=1,2,\dots$ be disjoint copies of X and let $\mathbb{N}^1(n)$ and $p^1(n)$ be the copies of $\mathbb{N}$ and p, respectively, in $X^1(n)$. The copies of $U(k)$ and $U(p)$ in $X^1(n)$ are denoted by $U(k^1(n))$ and $U(p^1(n))$, respectively. Since the set $P^1=\{p^1(n):n=1,2,\dots\}$ and the dense subset $D=X\setminus\mathbb{N}\cup\{p\}$ of isolated points of X are countable, there exists one-to-one function f_1 of P^1 onto D. We attach the spaces $X^1(n)$, $n=1,2,\dots$ to the space X identifying simultaneously each point $p^1(n)$ with the point $f_1(p^1(n))$ of D and each set $\mathbb{N}^1(n)$ with $\mathbb{N}$ (by putting $k^1(n)$ on k).

On the set

$$T^1 = X \cup \bigcup_{n=1}^{\infty} \left(X^1(n) \setminus (\mathbb{N}^1(n) \cup \{p^1(n)\}) \right), \tag{2.4}$$

we define the following topology: every point of $T^1 \setminus X$ is isolated. For every $k \in \mathbb{N}$, a basis of open neighborhoods is the collection of sets

$$\begin{aligned} O_U^1(k) = U(k) \cup \bigcup_{n=1}^{\infty} \left(U(k^1(n)) \setminus \{k^1(n)\} \right) \\ \cup \bigcup_{f_1(p^1(j)) \in U(k)} \left(U(p^1(j)) \setminus \{p^1(j)\} \right), \quad U \in \mathcal{P}. \end{aligned} \tag{2.5}$$

For every isolated point x of X, a basis of open neighborhoods is the collection of sets

$$O_U^1(x) = \{x\} \cup \left(U(p^1(j)) \setminus \{p^1(j)\} \right), \quad U \in \mathcal{P}, \tag{2.6}$$

where $f_1(p^1(j)) = x$.

For the point p, a basis of open neighborhoods is the collection of sets

$$O_U^1(p) = U(p) \cup \bigcup_{f_1(p^1(j)) \in U(p)} \left(U(p^1(j)) \setminus \{p^1(j)\} \right), \quad U \in \mathcal{P}. \tag{2.7}$$

It can be easily proved that the space T^1 is Hausdorff, totally disconnected, and contains the space X as a closed nowhere dense subset. We observe that every basic open neighborhood in T^1, of every $x \in X$ is defined by some $U \in \mathcal{P}$, and every $U \in \mathcal{P}$ defines in T^1, a basic open neighborhood $O_U^1(x)$, for every $x \in X$. Obviously, $\overline{O_U^1(x)} \setminus O_U^1(x) = U$. Furthermore, for every pair of points x, y of D and every basic open neighborhoods $O_U^1(x), O_V^1(y), U, V \in \mathcal{P}$, of x, y respectively, in T^1, it holds that $\overline{O_U^1(x)} \cap \overline{O_V^1(y)} \neq \emptyset$, which implies that every continuous real-valued function of T^1 is constant on D and, hence, on X since D is dense in X.

We construct by induction the spaces $T^2, T^3, \dots, T^m$, where

$$T^m = T^{m-1} \cup \bigcup_{n=1}^{\infty} \left(X^{m-1}(n) \setminus (\mathbb{N}^{m-1}(n) \cup \{p^{m-1}(n)\}) \right), \tag{2.8}$$

and where $X^{m-1}(n), n = 1, 2, \dots$ are disjoint copies of the initial space X, and $\mathbb{N}^{m-1}(p)$, $P^{m-1}(n)$ are the copies of $\mathbb{N}, p$ in $X^{m-1}(n)$, respectively. Each point $p^{m-1}(n)$ is identified with the point $f_{m-1}(p^{m-1}(n))$, where f_{m-1} is one-to-one function of the set $P^{m-1} = \{p^{m-1}(n) : n = 1, 2, \dots\}$ onto the dense subset of isolated points of T^{m-1}. Each set $N^{m-1}(n)$ is identified with the set $\mathbb{N}$ (by putting $k^{m-1}(n)$ on k).

It can be easily proved that the space T^m is Hausdorff, totally disconnected, and contains the space T^{m-1} as a closed nowhere dense subset. We observe that every basic open neighborhood in T^m, of every $x \in T^{m-1}$ is defined by some $U \in \mathcal{P}$, and every $U \in \mathcal{P}$, defines in T^m, a basic open neighborhood $O_U^m(x)$, for every $x \in T^{m-1}$. Obviously, $\overline{O_U^m(x)} \setminus O_U^m(x) = U$. Furthermore, for every pair x, y of isolated points of T^{m-1} and every basic open neighborhood $O_U^m(x), O_V^m(y), U, V \in \mathcal{P}$ of x, y respectively, in T^m, it holds that $\overline{O_U^m(x)} \cap \overline{O_V^m(y)} \neq \emptyset$, which implies that every continuous real-valued function of T^m is constant on the set of isolated points of T^{m-1} and, hence, it is constant on T^{m-1} since this set is dense in T^{m-1}.

Finally, we consider the set $T = \bigcup_{m-1}^{\infty} T^m$ on which we define the following topology: If $t \in \mathbb{N}$, we first consider the basic open neighborhood $O_U^1(t)$ of t in T^1 and then its corresponding basic open neighborhood in T^m,

$$O_U^m(t) = O_U^{m-1}(t) \cup \bigcup_{n=1}^{\infty} \big(U(k^m(n)) \setminus \{k^m(n)\}\big) \cup \bigcup_{f_m(p^m(j)) \in O_U^{m-1}(t)} \big(U(p^m(j)) \setminus \{p^m(j)\}\big). \tag{2.9}$$

A basis of open neighborhoods of t in T is the collection of sets

$$O_U(t) = \bigcup_{m=1}^{\infty} O_U^m(t), \quad U \in \mathcal{P}. \tag{2.10}$$

If $t \in T \setminus \mathbb{N}$, then either $t \in X \setminus \mathbb{N}$ or t is an isolated point of T^l, $l = 1, 2, \ldots$, where l is the minimal integer for which $t \in T^l$.

In the first case, we first consider the basic open neighborhood $O_U^1(t)$ of t in T^1 and then its corresponding basic open neighborhood in T^m,

$$O_U^m(t) = O_U^{m-1}(t) \cup \bigcup_{f_m(p^m(j)) \in O_U^{m-1}(t)} \big(U(p^m(j)) \setminus \{p^m(j)\}\big). \tag{2.11}$$

A basis of open neighborhoods of t in T is the collection of sets

$$O_U(t) = \bigcup_{m=1}^{\infty} O_U^m(t), \quad U \in \mathcal{P}. \tag{2.12}$$

In the second case, we first consider the basic open neighborhood $O_U^1(t)$ of t in T^l and then its corresponding basic open neighborhood in T^{l+m},

$$O_U^{l+m}(t) = O_U^{l+m-1}(t) \cup \bigcup_{f_{l+m}(p^{l+m}(j)) \in O_U^{l+m-1}(t)} \big(U(p^{l+m}(j)) \setminus \{p^{l+m}(j)\}\big). \tag{2.13}$$

A basis of open neighborhoods of t in T is the collection of sets

$$O_U(t) = \bigcup_{m=1}^{\infty} O_U^m(t), \quad U \in \mathcal{P}. \tag{2.14}$$

From the definition of topology on T, it follows that, for every $t \in T$ and for every $U \in \mathcal{P}$, the set $O_U(t)$ is open-and-closed in $T \setminus \mathbb{N}$ and that $\overline{O_U(t)} \setminus O_U(t) = U$.

Proposition 1. *The space T is countable biconnected Hausdorff not widely connected and without a dispersion point.*

Proof. That T is countable Hausdorff is obvious. To prove that T is connected, we consider two arbitrary points x, y of T and let m be the minimal integer for which both $x, y \in T^m$. But then every continuous real-valued function of T^{m+1} is constant on T^m and, hence, for every continuous real-valued function g of T, $g(x) = g(y)$, which implies that T is connected.

Suppose now that T is not biconnected and let A, B be two connected, proper disjoint subsets containing more than one point and $A \cup B = T$. By the construction of the space T, it follows that $T \backslash \mathbb{N}$ is totally disconnected. Hence, there exists $b \in B \backslash \mathbb{N}$. Let $O_U(b)$ be the basic open neighborhood of b defined by some $U \in \mathcal{P}$. Suppose that $\overline{O_U(b)} \cap B \cap \mathbb{N} = W \neq \emptyset$. If $W \notin \mathcal{P}$, then $\mathbb{N} \backslash W \in \mathcal{P}$ and, hence, for the set $O_{\mathbb{N} \backslash W}(b)$, it holds that $\overline{O_{\mathbb{N} \backslash W}(b)} \cap \mathbb{N} = \mathbb{N} \backslash W$. Therefore, $\overline{O_U(b) \cap O_{\mathbb{N} \backslash W}(b)} \cap B \cap \mathbb{N} = \emptyset$, which implies that the set $O_U(b) \cap O_{\mathbb{N} \backslash W}(b) \cap B$ is open-and-closed in B. Consequently, $B \subseteq O_U(b)$ for every $U \in \mathcal{P}$ and, hence, B is a singleton, which is a contradiction. Hence, $W \in \mathcal{P}$. But then if we consider a point $a \in A \backslash \mathbb{N}$ and the basic open neighborhood $O_U(a)$ of a, it follows, in a similar manner, that the relation $\overline{O_U(a)} \cap A \cap \mathbb{N} = V \neq \emptyset$ implies that $V \in \mathcal{P}$, which is impossible because $B \cap A = \emptyset$. Therefore, either $\overline{O_U(a)} \cap A \cap \mathbb{N} = \emptyset$ or $\overline{O_U(b)} \cap B \cap \mathbb{N} = \emptyset$. Since $\overline{O_U(a)} \backslash O_U(a) \subseteq \mathbb{N}$ and $\overline{O_U(b)} \backslash O_U(b) \subseteq \mathbb{N}$, it follows that either $\overline{O_U(a)} \cap \mathbb{N}$ is open-and-closed in A or $\overline{O_U(b)} \cap \mathbb{N}$ is open-and-closed in B. Hence, either the subset A is a singleton or not connected, or the subset B is a singleton or not connected.

That T is not widely connected is obvious observing that, for every $U \in \mathcal{P}$ and every $t \in T$, the subset $\overline{O_U(t)}$ is connected. That T has no dispersion point is obvious by its construction. □

Corollary 1. *The space T is not strongly connected.*

Proof. let τ denote the topology on T and let $\tau_{\max}$ denote a maximal connected topology finer that τ. By [13, Cor. 14A], it follows that the space $(T, \tau_{\max})$ has infinitely many cut points. Hence, if t is such a point, then there exist two disjoint subsets K and L such that K and L are open-and-closed in $T \backslash \{t\}$, contain more than one point, and $K \cup L = T \backslash \{t\}$. Since the sets $K \cup \{t\}, L \cup \{t\}$, are connected in $(T, \tau_{\max})$, they are also connected in (T, τ). But by the proof of Proposition 1, it follows that, for every pair of connected subsets of (T, τ), which contain more than one point, their intersections include a member of $\mathcal{P}$. Therefore, the set $(K \cup \{t\}) \cap (L \cup \{t\}) = \{t\}$ must be a member of $\mathcal{P}$, which is impossible. □

Corollary 2. *There exists 2^c mutually non-homeomorphic countable biconnected Hausdorff spaces not widely connected and without a dispersion point.*

Proof. Because [19, Thm. 10], there exists 2^c different types of free ultrafilters on the discrete subspace $\mathbb{N}$ of the initial space X. □

The space S. For the construction of the countable biconnected Urysohn almost regular space S, we first construct an appropriate countable Urysohn almost regular non-regular space and then, using the method of F. B. Jones [17], we construct a space Y having the additional property of containing a point ∞ at which the space Y is regular. The condensation process of this regular point is the same as in the construction of the space T.

We consider the initial space X and, for every $n \in \mathbb{N}$, we consider a sequence $\langle b_{ni} \rangle_{i \in \mathbb{N}}$ converging to n and consisting of isolated points not belonging to X. We set $B = \{b_{ni} : n, i = 1, 2, \dots\}$ and we consider the space $C = X \cup B$. Let C_1, C_2 be disjoint copies of C and let p_1, p_2 and $\mathbb{N}_1$, $\mathbb{N}_2$ be the copies of p and $\mathbb{N}$ in C_1, C_2, respectively. We attach the space C_1 to C_2 identifying the point p_1 with p_2. We set $q = p_1 = p_2$ and we consider the space $Z = (C_1 \backslash \{p_1\}) \cup \{q\} \cup (C_2 \backslash \{p_2\})$ which is obviously Hausdorff but not regular since the point q and the closed subset $\mathbb{N}_1 \cup \mathbb{N}_2 = K$ cannot be separated by disjoint open sets.

Let Z_n, $n = 1,2,\dots$ be disjoint copies of Z and let $\bigcup_{n=1}^{\infty} Z_n$ be their disjoint union (topological sum). We add one more point r and, on the set $L = \{r\} \cup \bigcup_{n=1}^{\infty} Z_n$, we define a basis of open neighborhoods of r as follows: we consider the copies B_1, B_2 of B in C_1, C_2, respectively. We set $B_1 \cup B_2 = R$ and let R_n, $n = 1,2,\dots$ be the copy of R in Z_n. Let $\mathcal{R}$ be a free ultrafilter on the closed discrete subspace $Q = \{q_n : n = 1,2,\dots\}$, where q_n is the copy of q in Z_n. Then, for every $U \in \mathcal{R}$, a basis of open neighborhoods of r is the collection of sets $U(r) = \{r\} \cup \{\cup R_i : q_i \in U\}$.

It can be easily verified that the space L is Urysohn but not normal since the closed subsets Q and $\bigcup_{n=1}^{\infty} K_n$, ($K_n$ is the copy of K in Z_n) cannot be separated by disjoint open sets. Also, the subsets $\bigcup_{n=1}^{\infty} K_n$, and the point r cannot be separated by disjoint open sets, while Q and r can be separated by disjoint open sets but not by disjoint open-and-closed sets. However, L is not regular at r. Since the closed subsets Q and $\{r\}$ of L cannot be separated by a continuous real-valued function, it follows that if we consider L_n, $n = 1,2,\dots$ disjoint copies of L, we can apply the construction in [17] and obtain a space Y with the following properties

(1) It is countable Urysohn containing a dense subset of isolated points.

(2) It contains a point ∞ at which Y is regular.

(3) The point ∞ and each copy Q_n, $n = 1,2,\dots$ of the subset Q, in L_n cannot be separated by disjoint open-and-closed subsets, that is they cannot be separated by a continuous real-valued function of Y.

Proposition 2. *There exists 2^c mutually non-homeomorphic countable biconnected Urysohn almost regular spaces, not widely connected, not having a dispersion point, and not being strongly connected.*

Proof. We imitate the condensation process that we used in the construction of the space T using the space Y in place of the space X and the point ∞ and the set Q_1 in place of p and $\mathbb{N}$, respectively. Let S^m, $m = 1,2,\dots$ and $S = \bigcup_{m=1}^{\infty} S^m$ be the corresponding spaces to T^m and T, respectively. It can be easily proved that S is Urysohn. Since the different copies of the regular point ∞ are attached in each step to the isolated points of S^m, $m = 1,2,\dots$, it follows that these points remain regular in the final space S. Obviously, the set of all these points is dense in S and, hence, S is almost regular. □

All the other properties of S are proved as in Proposition 1 and Corollaries 1 and 2.

Remark. In [37], E. K. van Dowen constructed a regular space with a dispersion point on which every continuous real-valued function is constant. We can modify his method and construct a countable biconnected Hausdorff space not widely connected, not having a dispersion point, and not being strongly connected. For this, we consider again the initial space X and let X_i, $i = 1,2,\dots$ be disjoint copies of X. We denote by x_i the copy of $x \in X$ in X_i, and by $\mathbb{N}_i$ the copy of $\mathbb{N}$. We attach the spaces X_i, $i = 2,3,\dots$ to X_1 identifying each copy $\mathbb{N}_i$ with $\mathbb{N}_1$, that is by putting each n_i to n_1. We denote this point by n. In the space $Z = \mathbb{N} \cup \bigcup_{i=1}^{\infty} (X_i \backslash N_i)$, the subset $P = \{p_i : i = 1,2,\dots\}$ and the subset D consisting of all isolated points of the copies X_i are countable and, therefore, there exists a one-to-one function g of P onto D. On the quotient space $T_X = \mathbb{N} \cup \{(p_i, g(p_i)) : i = 1, 2, \ldots\}$, we define a second topology τ in a similar manner as in the construction of the space T. Obviously, the topology τ is weaker than the quotient topology of T_X. It can be proved, as in Proposition 1 and Corollaries 1 and 2, that (T_X, τ) is the required space.

In a similar manner, we can construct a Urysohn almost regular space having all the above properties. For this, it suffices to consider space Y as the initial space.

Acknowledgement. The author is grateful to the referee for his suggestions and comments.

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