SOME OPERATIONS ON LATTICE IMPLICATION ALGEBRAS

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ABSTRACT. We introduce the concept of a ⊗-closed set and a ⊗-homomorphism in lattice implication algebras, and we discuss some of their properties. Next, we introduce the fuzzy implicative filter and obtain equivalent conditions. Finally, we discuss the operation ⊗, fuzzy filters, and fuzzy implicative filters.

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1. Introduction. In order to research the logical system whose propositional value is given in a lattice from the semantic viewpoint, Xu [3] proposed the concept of lattice implication algebras, and discussed some of their properties in [3, 4]. Xu and Qin [5] introduced the notion of filters in a lattice implication algebra, and investigated their properties. In [8], Xu and Qin defined the fuzzy filter in a lattice implication algebra, and they discussed some of their properties. Recently, Xu et al. [9] defined a congruence relation on lattice implication algebras induced by fuzzy filters and they proved the fuzzy homomorphism fundamental theorem, and Liu and Xu [1] introduced the notion of the new binary operation ⊗ on lattice implication algebras and they investigated their properties.

2. Preliminaries. We recall a few definitions and properties.

DEFINITION 2.1 (see [4]). By a *lattice implication algebra* we mean a bounded lattice $(L, \vee, \wedge, 0, 1)$ with order-reversing involution "$'$" and a binary operation "$\rightarrow$" satisfying the following axioms:

(I1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,

(I2) $x \rightarrow x = 1$,

(I3) $x \rightarrow y = y' \rightarrow x'$,

(I4) $x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y$,

(I5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,

(L1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,

(L2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$,

for all $x, y, z \in L$. If $(L, \vee, \wedge, 0, 1)$ satisfies the conditions (I1), (I2), (I3), (I4), and (I5), is called a *quasi lattice implication algebra*.

We can define a partial ordering $\leq$ on a lattice implication algebra L by $x \leq y$ if and only if $x \rightarrow y = 1$.

In a lattice implication algebra L, the following hold: for all $x, y, z \in L$,

(1) $0 \rightarrow x = 1$, $1 \rightarrow x = x$, and $x \rightarrow 1 = 1$,

(2) $x \leq y$ implies $z \rightarrow x \leq z \rightarrow y$ and $x \rightarrow z \geq y \rightarrow z$,

(3) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$,

(4) $x \rightarrow ((x \rightarrow y) \rightarrow y) = 1$,

(5) $((x \rightarrow y) \rightarrow y) \rightarrow y = x \rightarrow y$.

DEFINITION 2.2 (see [7])**.** Let $(L, \vee, \wedge, \prime, \rightarrow)$ be a lattice implication algebra. A subset F of L is called a *filter* of L if it satisfies for all $x, y \in L$

(F1) $1 \in F$,

(F2) $x \in F$ and $x \rightarrow y \in F$ imply $y \in F$.

A subset F of L is called an *implicative filter* of L, if it satisfies (F1) and

(F3) $x \rightarrow (y \rightarrow z) \in F$ and $x \rightarrow y \in F$ imply $x \rightarrow z \in F$

for all $x, y, z \in L$.

The following propositions are clear.

PROPOSITION 2.3. *In a lattice implication algebra, every implicative filter is a filter.*

PROPOSITION 2.4. *Every filter F of a lattice implication algebra L has the following property:*

$$x \leq y, \quad x \in F \text{ imply } y \in F. \tag{2.1}$$

DEFINITION 2.5 (see [6])**.** Let $(L, \vee, \wedge, \prime, \rightarrow)$ be a lattice implication algebra and $S \subseteq L$. If S satisfies the following condition: S is called a *subalgebra* of L if

(i) $0 \in S$,

(ii) for all $x, y \in S, x \rightarrow y \in S$.

DEFINITION 2.6 (see [3])**.** Let $(L, \vee, \wedge, \prime, \rightarrow, \otimes, 0, 1)$, and $(L^*, \vee, \wedge, \prime, \rightarrow, \otimes, 0, 1)$ be lattice implication algebras and $f : L \rightarrow L^*$ be a mapping. Then f is called the *implication homomorphism* if for all $x, y \in L$,

$$f(x \longrightarrow y) = f(x) \longrightarrow f(y), \tag{2.2}$$

and f is called the *lattice implication homomorphism* if for all $x, y \in L$,

$$f(x \vee y) = f(x) \vee f(y), \qquad f(x \wedge y) = f(x) \wedge f(y), \qquad f(x') = f(x)'. \tag{2.3}$$

LEMMA 2.7 (see [3])**.** *Let $(L, \vee, \wedge, \prime, \rightarrow, 0, 1)$ and $(L^*, \vee, \wedge, \prime, \rightarrow, 0, 1)$ be lattice implication algebras. If $f : L \rightarrow L^*$ is a function. Then f is a lattice implication homomorphism if and only if f is an implication homomorphism and $f(a') = f(a)'$ for all $a \in L$.*

DEFINITION 2.8 (see [1])**.** Let $(L, \vee, \wedge, \prime, \rightarrow, 0, 1)$ be a quasi-lattice implication algebra and given elements a, b of L, we define

$$A(a, b) := \{x \in L \mid a \leq b \longrightarrow x\}. \tag{2.4}$$

If for all $x, y \in L$, $A(x, y)$ has a least element, written $x \otimes y$, then the quasi-lattice implication algebra is said to be *with property (P)*.

LEMMA 2.9 (see [1])**.** *Any lattice implication algebra is with property (P), in fact $a \otimes b = (a \rightarrow b')'$.*

LEMMA 2.10 (see [1])**.** *Let* $(L, \vee, \wedge, ', \to, 0, 1)$ *be a lattice implication algebra. Then the following hold: for all* $a, b, c \in L$,

(6) $a \otimes b \leq a \wedge b \leq a$,

(7) $a \leq b$ *if and only if* $a \otimes b' = 0$,

(8) $a \otimes b = b \otimes a$,

(9) $(a \to b) \otimes a \leq b$,

(10) $(a \otimes b) \to c = b \to (a \to c)$.

We now review some fuzzy logic concepts. Let X be a set. A function $\mu : X \to [0,1]$ is called a *fuzzy subset* on X.

In the following, it will be convenient to write simply L for $(L, \vee, \wedge, ', \to, \otimes, 0, 1)$, and L^* for $(L^*, \vee, \wedge, ', \to, \otimes, 0, 1)$.

3. $\otimes$-closed sets and $\otimes$-homomorphisms. In this section, we introduce the concept of a $\otimes$-closed set and a $\otimes$-homomorphism in lattice implication algebras, and we discuss their properties. If L is a lattice implication algebra, then by Lemma 2.9, we know that $a \otimes b \in L$ for all $a, b \in L$. Now we define the $\otimes$-closed sets as follows.

DEFINITION 3.1. Let L be a lattice implication algebra and A a nonempty subset of L. Then A is said to be $\otimes$-*closed* of L if $a \otimes b \in A$ whenever $a, b \in A$.

THEOREM 3.2. *Let F be a filter of a lattice implication algebra L. Then F is a $\otimes$-closed set in L.*

PROOF. Let F be a filter of L and $a, b \in F$. Then by Proposition 2.4, we have $b \to (a \otimes b) \in F$, and so we get $a \otimes b \in F$. Therefore F is $\otimes$-closed in L. □

THEOREM 3.3. *In a lattice implication algebra, every subalgebra is $\otimes$-closed.*

PROOF. Let S be a subalgebra of a lattice implication algebra L and $a, b \in S$. Then, since $x' = x \to 0 \in S$ for all $x \in S$, we have

$$a \otimes b = (a \longrightarrow b')' \in S, \tag{3.1}$$

and so S is $\otimes$-closed in L. □

REMARK 3.4. The converse of Theorems 3.2 and 3.3 are not true, and so we know that $\otimes$-closed sets are the generalization of subalgebras and filters in a lattice implication algebra.

EXAMPLE 3.5. Let $L := \{0, a, b, c, 1\}$. Define the partially ordered relation on L as $0 < a < b < c < 1$, and define $x \wedge y := \min\{x, y\}$, $x \vee y := \max\{x, y\}$ for all $x, y \in L$, and "$'$" and "$\to$" as follows:

x	x'
0	1
a	c
b	b
c	a
1	0

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then $(L, \vee, \wedge, ', \rightarrow)$ is a lattice implication algebra and we can find the following $\otimes$-table:

$\otimes$	0	a	b	c	1
0	0	0	0	0	0
a	0	0	0	0	a
b	0	0	0	a	b
c	0	0	a	b	c
1	0	a	b	c	1

Now we consider the $\otimes$-closed set $A := \{0, a, b\}$, which is neither a filter nor a sub-algebra of L since $0 \in A$ and $b \rightarrow a = c \notin A$.

DEFINITION 3.6. Let L and L^* be lattice implication algebras and let $f : L \rightarrow L^*$ be a function. Then F is called a $\otimes$-*homomorphism* if for all $a, b \in L$,

$$f(a \otimes b) = f(a) \otimes f(b). \tag{3.2}$$

THEOREM 3.7. *Let $f : L \rightarrow L^*$ be a $\otimes$-homomorphism of lattice implication algebras. If $f^{-1}(0) := \{a \in L \mid f(a) = 0\} \neq \emptyset$, then $f^{-1}(0)$ is $\otimes$-closed in L.*

PROOF. Let $a, b \in f^{-1}(0)$. Then we have $f(a \otimes b) = f(a) \otimes f(b) = 0 \otimes 0 = 0$, and so $a \otimes b \in f^{-1}$, that is, $f^{-1}(0)$ is a $\otimes$-closed set in L. $\qquad \square$

We discuss the connection between lattice implication homomorphisms and $\otimes$-homomorphisms.

THEOREM 3.8. *Let $f : L \rightarrow L^*$ be a function of lattice implication algebras. If f is a lattice implication homomorphism, then f is a $\otimes$-homomorphism.*

PROOF. For any $a, b \in L$, we have

$$f(a \otimes b) = f((a \rightarrow b')') = f(a \rightarrow b')' = (f(a) \rightarrow f(b)')' = f(a) \otimes f(b), \tag{3.3}$$

and hence f is a $\otimes$-homomorphism. $\qquad \square$

We consider the converse of Theorem 3.8, so we have the following theorem.

THEOREM 3.9. *Let $f : L \rightarrow L^*$ be a function of lattice implication algebras. If f is a $\otimes$-homomorphism and $f(a') = f(a)'$ for all $a \in L$, then f is a lattice implication homomorphism.*

PROOF. For any $a, b \in L$, we have

$$f(a \rightarrow b) = f((a \otimes b')') = f(a) \rightarrow f(b). \tag{3.4}$$

Thus by Lemma 2.7, we obtain that f is a lattice implication homomorphism. $\qquad \square$

By Theorems 3.8 and 3.9, we have the following corollary.

COROLLARY 3.10. *Let $f : L \to L^*$ be a function of lattice implication algebras. Then f is a lattice implication homomorphism if and only if f is a $\otimes$-homomorphism and $f(a') = f(a)'$ for all $a \in L$.*

THEOREM 3.11. *Let L and L^* be lattice implication algebras, and f and g be functions from L into L^*. If $f(a \otimes b) = g(a \otimes b)$ for all $a, b \in L$, then $f = g$.*

PROOF. For any $x \in L$, we have $f(x) = f(x \otimes 1) = g(x \otimes 1) = g(x)$, and so $f = g$. $\square$

THEOREM 3.12. *Let $f : L \to L^*$ be a surjective $\otimes$-homomorphism of lattice implication algebras. If A is $\otimes$-closed in L, then $f(A)$ is $\otimes$-closed in L^*.*

PROOF. For any $a, b \in f(A)$, there exist x, y in A such that $f(x) = a$ and $f(y) = b$. Thus we have $a \otimes b = f(x) \otimes f(y) = f(x \otimes y)$. Since A is $\otimes$-closed in L, we get $a \otimes b \in f(A)$, and hence $f(A)$ is $\otimes$-closed in L^*. $\square$

THEOREM 3.13. *Let $f : L \to L^*$ be a $\otimes$-homomorphism of lattice implication algebras. If M^* is $\otimes$-closed in L^*, then $f^{-1}(M^*)$ is $\otimes$-closed in L.*

PROOF. Suppose that M^* is $\otimes$-closed in L^* and let $a, b \in f^{-1}(M^*)$. Then we have $f(a) \in M^*$ and $f(b) \in M^*$. Since M^* is $\otimes$-closed in L^*, we get $f(a \otimes b) = f(a) \otimes f(b) \in M^*$, and so $a \otimes b \in f^{-1}(M^*)$, that is, $f^{-1}(M^*)$ is $\otimes$-closed in L. $\square$

4. The operation $\otimes$ and the fuzzy (implicative) filters. In this section, we introduce the fuzzy implicative filter and obtain equivalent conditions of implicative filters. Moreover, we discuss the operation $\otimes$, fuzzy filters and fuzzy implicative filters.

DEFINITION 4.1 (see [8])**.** A fuzzy subset μ of a lattice implication algebra L is called a *fuzzy filter* if it satisfies for any $x, y \in L$,
(FF1) $\mu(1) \geq \mu(x)$,
(FF2) $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$.

PROPOSITION 4.2 (see [8])**.** *Let μ be a fuzzy filter of a lattice implication algebra L and $x, y \in L$. Then $x \leq y$ implies $\mu(x) \leq \mu(y)$.*

THEOREM 4.3. *If μ is a fuzzy filter of a lattice implication algebra L, then for any $x, y \in L$,*
(FF3) $\mu(x \otimes y) \geq \min\{\mu(x), \mu(y)\}$.

PROOF. Let μ be a fuzzy filter of L. Then for all $x, y \in L$, we have

$$\mu(x \otimes y) \geq \min\{\mu(y \longrightarrow (x \otimes y)), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}. \qquad (4.1)$$
$$\square$$

Let μ be a fuzzy filter of a lattice implication algebra L and $x, y \in L$. Then, using Proposition 4.2 and (FF3), we have $\mu(1) \geq \mu(x)$ and

$$\mu(y) \geq \mu(x \wedge y) = \mu(x \otimes (x \longrightarrow y)) \geq \min\{\mu(x), \mu(x \longrightarrow y)\}. \qquad (4.2)$$

Thus by Theorem 4.3, we have the following theorem.

THEOREM 4.4. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy filter if and only if μ is an order preserving set and (FF3) holds.*

We provide an equivalent condition for a fuzzy set to be a fuzzy filter.

THEOREM 4.5. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy filter if and only if it satisfies for all $x, y, z \in L$,*
(FF4) $x \geq y \otimes z$ *implies* $\mu(x) \geq \min\{\mu(y), \mu(z)\}$.

PROOF. Suppose that μ is a fuzzy filter of L and let $x, y, z \in L$ be such that $x \geq y \otimes z$. Then by Theorem 4.3, we have

$$
\begin{aligned}
\mu(x) &\geq \min\{\mu((y \otimes z) \longrightarrow x), \mu(y \otimes z)\} \\
&= \min\{\mu(1), \mu(y \otimes z)\} \\
&\geq \min\{\mu(y), \mu(z)\},
\end{aligned}
\tag{4.3}
$$

and so (FF4) holds.

Conversely, suppose that μ satisfies (FF4). Since $1 \geq x \otimes x$ for all $x \in L$, it follows from (FF4) that

$$
\mu(1) \geq \min\{\mu(x), \mu(x)\} = \mu(x) \quad \forall x \in L.
\tag{4.4}
$$

Note that $(y \rightarrow x) \otimes y \leq x$ for all $x, y \in L$. Hence, by (FF4), we have $\mu(x) \geq \min\{\mu(y \rightarrow x), \mu(y)\}$, which proves (FF2). Therefore μ is a fuzzy filter of L. $\square$

Now, we give an equivalent condition of implicative filters as follows.

LEMMA 4.6 (see [2]). *Let F be an implicative filter of L. Then for any $x, y \in L$,*

$$
(x \longrightarrow y) \longrightarrow x \in F \text{ implies } x \in F.
\tag{4.5}
$$

THEOREM 4.7. *A nonempty subset F of L is an implicative filter if and only if it satisfies (F1) and*
(F4) $z \rightarrow ((x \rightarrow y) \rightarrow x) \in F$ *and* $z \in F$ *imply* $x \in F$ *for all* $x, y, z \in L$.

PROOF. Suppose that F satisfies (F1) and (F4). Let $x, z \in L$ be such that $z \rightarrow x \in F$ and $z \in F$. In (F4), we take $y = x$, then

$$
z \longrightarrow ((x \longrightarrow x) \longrightarrow x) = z \longrightarrow (1 \longrightarrow x) = z \longrightarrow x \in F.
\tag{4.6}
$$

Thus by (F4), we have $x \in F$. This says that F is a filter of L. Let $x, y, z \in L$ be such that $z \rightarrow (y \rightarrow x) \in F$ and $z \rightarrow y \in F$. Since $z \rightarrow (y \rightarrow x) = y \rightarrow (z \rightarrow x) \leq (z \rightarrow y) \rightarrow (z \rightarrow (z \rightarrow x))$, we have $z \rightarrow (z \rightarrow x) \in F$. As $((z \rightarrow x) \rightarrow x) \rightarrow (z \rightarrow x) = z \rightarrow (((z \rightarrow x) \rightarrow x) \rightarrow x) = z \rightarrow (z \rightarrow x) \in F$, it follows that $1 \rightarrow (((z \rightarrow x) \rightarrow x) \rightarrow (z \rightarrow x)) \in F$. Combining (F1) and (F4) we obtain $z \rightarrow x \in F$. This means that F is an implicative filter of L.

Conversely, suppose that F is an implicative filter of L. Let $x, y, z \in L$ be such that $z \to ((x \to y) \to x) \in F$ and $z \in F$. Then by [Proposition 2.4](#), we have $(x \to y) \to x \in F$. By [Lemma 4.6](#), we get $x \in F$. $\qquad\square$

We state the fuzzification of implicative filters.

DEFINITION 4.8. A fuzzy subset μ of a lattice implication algebra L is called a *fuzzy implicative filter* if it satisfies (FF1) and

(FF5) $\mu(x) \geq \min\{\mu(z \to ((x \to y) \to x)), \mu(z)\}$ for all $x, y, z \in L$.

THEOREM 4.9. *In a lattice implication algebra, every fuzzy implicative filter is a fuzzy filter.*

PROOF. In (FF5), we take $y = x$, then we have

$$
\begin{aligned}
\mu(x) &\geq \min\{\mu(z \longrightarrow ((x \longrightarrow x) \longrightarrow x)), \mu(z)\} \\
&= \min\{\mu(z \longrightarrow (1 \longrightarrow x)), \mu(z)\} \\
&= \min\{\mu(z \longrightarrow x), \mu(z)\}.
\end{aligned}
\tag{4.7}
$$

This means that F is a filter of L. $\qquad\square$

We state an equivalent condition for a fuzzy set to be a fuzzy implicative filter.

THEOREM 4.10. *Let μ be a fuzzy filter of a lattice implication algebra L. Then the following are equivalent:*

(i) *μ is a fuzzy implicative filter.*

(ii) *$\mu(x) \geq \mu((x \to y) \to x)$ for all $x, y \in L$.*

(iii) *$\mu(x) = \mu((x \to y) \to x)$ for all $x, y \in L$.*

PROOF. (i)$\Rightarrow$(ii). Let μ be a fuzzy implicative filter of L. Then by (FF5), we have

$$
\mu(x) \geq \min\{\mu(1 \longrightarrow ((x \longrightarrow y) \longrightarrow x)), \mu(1)\} = \mu((x \longrightarrow y) \longrightarrow x)
\tag{4.8}
$$

for all $x, y \in L$. Hence the condition (ii) holds.

(ii)$\Rightarrow$(iii). Observe that $x \leq (x \to y) \to x$ for all $x, y \in L$. Then, by [Proposition 2.4](#), we have $\mu(x) \leq \mu((x \to y) \to x)$. It follows from (ii) that $\mu(x) = \mu((x \to y) \to x)$. Hence the condition (ii) holds.

(iii)$\Rightarrow$(i). Suppose that the condition (iii) holds. Since μ is a fuzzy filter, by (FF2) we have

$$
\mu((x \longrightarrow y) \longrightarrow x) \geq \min\{\mu(z \longrightarrow ((x \longrightarrow y) \longrightarrow x)), \mu(z)\}.
\tag{4.9}
$$

Combining (iii) we obtain

$$
\mu(x) \geq \min\{\mu(z \longrightarrow ((x \longrightarrow y) \longrightarrow x)), \mu(z)\},
\tag{4.10}
$$

thus μ satisfies (FF5). Therefore μ is a fuzzy implicative filter of L. $\qquad\square$

THEOREM 4.11. *Let L be a lattice implication algebra. A fuzzy set μ in L is a fuzzy implicative filter if and only if it satisfies*

(FF6) *$z \otimes u \leq (x \to y) \to x$ in L implies $\mu(x) \geq \min\{\mu(z), \mu(u)\}$ for all $x, y, z, u \in L$,*

PROOF. Suppose that μ is a fuzzy implicative filter of L and let $x,y,z,u \in L$ be such that $z \otimes u \leq (x \to y) \to x$. Since μ is a fuzzy filter of L by Theorem 4.9, it follows from Theorems 4.5 and 4.10 that

$$\mu(x) = \mu\big((x \longrightarrow y) \longrightarrow x\big) \geq \min\{\mu(z),\mu(u)\}. \tag{4.11}$$

Conversely, suppose that μ satisfies (FF6). Obviously, μ satisfies (FF1). Since $(z \to ((x \to y) \to x)) \otimes z \leq (x \to y) \to x$, (FF6) implies that

$$\mu(x) \geq \min\{\mu(z \longrightarrow ((x \longrightarrow y) \longrightarrow x)),\mu(z)\}, \tag{4.12}$$

which proves (FF5). Therefore μ is a fuzzy implicative filter of L. $\qquad\square$

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